

# Anexas Consultancy Services

ANEXAS EUROPE ©  
CERTIFICATION TRAINING

**Lean Six Sigma Black Belt**

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# Anexas Consultancy Services

## Course Overview

# Introductions and Expectations

Introduce yourself to the group:

- Name
- Job
- Any one thing not many people know about you!
- Expectations for the session



# Ground Rules

- Be on time
- Start and end on time
- Take a spelling holiday
- Participate, don't spectate
- Track issues and questions for later on the parking lot
- ON-LINE TRAINING
  - Keep cameras OFF during the training
  - Please interact using the chat box or comment box
  - If you wish to speak, please unmute yourself and speak and afterwards mute yourself once again
- CLASSROOM TRAINING
  - Don't conduct any side-conversations
  - It's OK to stand up and stretch
  - Turn off your cell phones or set them on vibrate
  - Conduct business at breaks and lunch
- Have fun!



# Anexas Consultancy Services

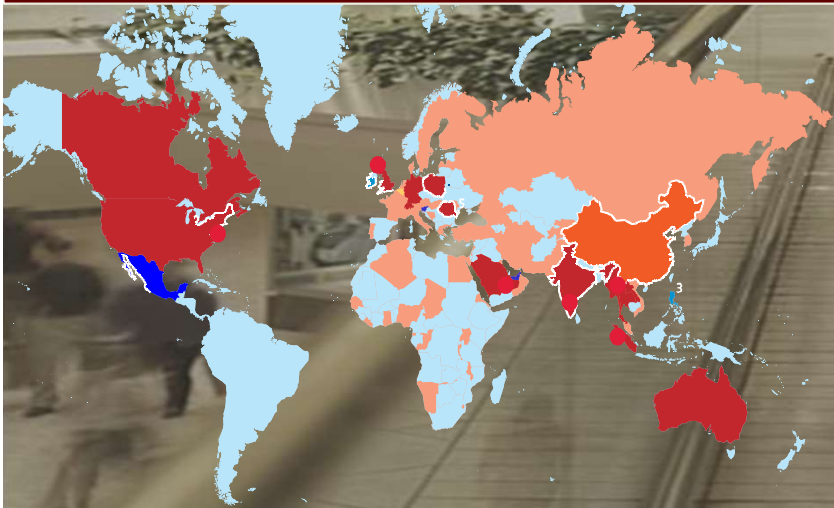
## Introduction to Anexas

# Objectives

- To introduce you to Anexas Consultancy Services
- Introduce the benefits available to Anexas alumni
- Project support details
- Certification process

## About Anexas

*Enabling individuals and organizations  
achieve excellence  
since 2006*



Anexas Consulting

- Anexas is a global network of lean six sigma and project professionals serving the wide spectrum of industries. We operate in 10 countries and have 25 professionals in the team.
- Our mission is to help organizations and individuals achieve excellence.
- Trained more than 150,000 professionals in Lean and Six Sigma, Project Management and quality related trainings across the world from various industries.
- Lean Green Belts and Black Belts certified by Anexas have completed more than 5000 successful projects under our guidance.

# Completed Projects for Worldwide Organizations by Anexas

## Technip



BHARAT PETROLEUM CORP. LTD.

**UAE XCHANGE**  
Service is our Currency



### Some of Our Clients

- Al Futtaim
- DP World
- ADNOC
- Concentriix
- UAE Exchange
- Novartis
- Colgate Palmolive
- International Medical Centre
- And many more...



सेल SAIL

स्टील अथॉरिटी ऑफ इण्डिया लिमिटेड  
STEEL AUTHORITY OF INDIA LIMITED

- Reduction of cycle times by more than 50%, at times to more than 90%
- Error reduction ranging from 50% to 100%
- Total project savings worth more than 50 Million Dollars
- Certified more than 10,000 Black Belts, 20,000 Green Belts and trained more than 50,000 professionals in Lean and Six Sigma, project management and strategic management

## Al Futtaim

Anexas has won many Project and Quality Management Awards  
Anexas projects presented in World Conferences in ASQ, IQPC, Dubai  
Quality Awards, ADIPEC, etc.



# Satisfying Customers — The Problem

- Best Flour Milling produces and distributes white corn flour to a wide range of customers within the tortilla industry
- The product is shipped in 50-pound multiwall paper bags that are loaded and weighted at the end of the production line
- **Defining the problem:** Approximately 4% of the bags are found to be either overweight or underweight, an issue that the organization spends an average of approximately US\$5 million per year to address

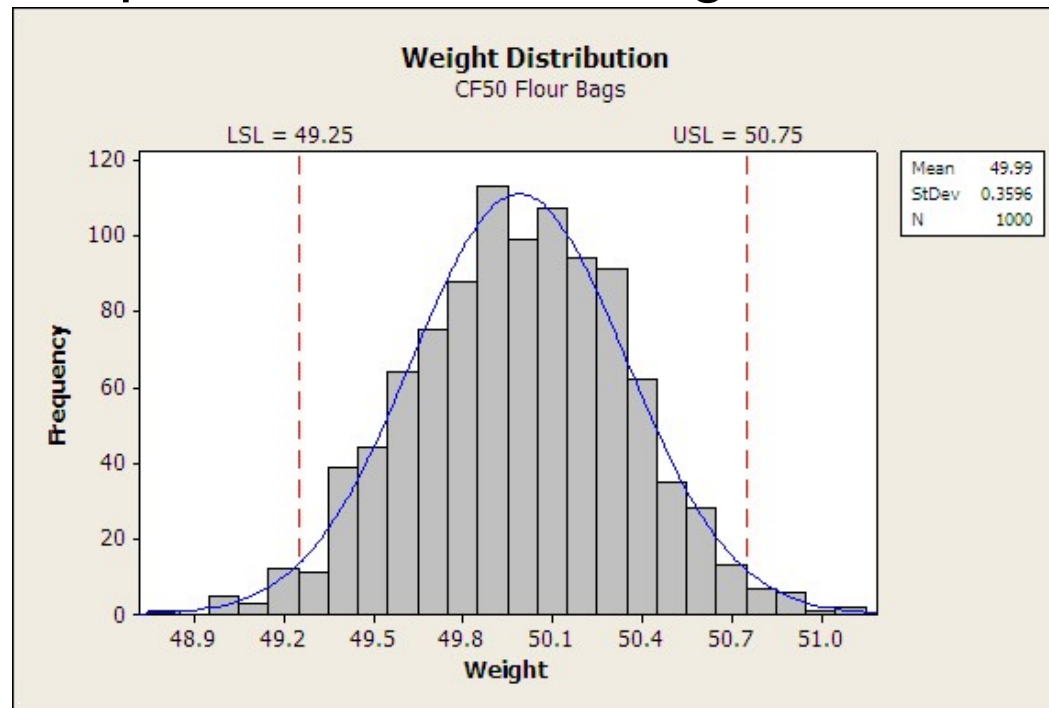




# Satisfying Customers — The Measure

- Best Flour knows that customers will be satisfied if the bag weight remains within agreed-upon limits
- The chart below shows the weight distribution of 1,000 bags taken from the production line, along with the spec limits

4% fall beyond the spec limits



# Measuring Process Capability

We will use the Best Flour example to illustrate how we can use Six Sigma to measure process capability

Processes have

→ A level (e.g., average or mean)

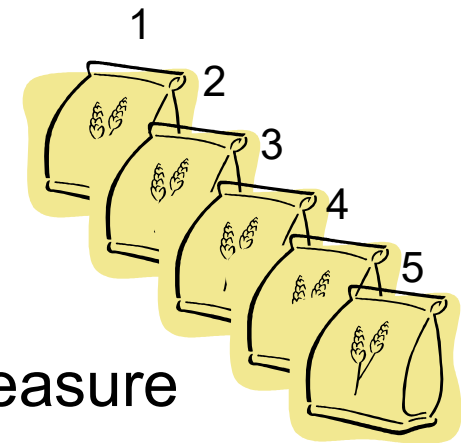
→ Some measure of variation

How do we measure level and variation?



# Measuring the Average

Consider the production of the 50-pound flour bags



Let's take five bags from production and measure their weight

Bag No.	1	2	3	4	5
Weight (pounds)	48	49	50	51	52

# Measuring the Average, cont.

We can then calculate the average as follows:

$$\text{Average} = \frac{48 + 49 + 50 + 51 + 52}{5} = 50 \text{ lb}$$

Or, using the formula...

$$\text{Average} = \frac{\sum X_i}{n} \quad \rightarrow \quad (i = 1, 2, 3, \dots n)$$

Also known as **mean**

Number of points in the data set

# Measuring the Variation Around the Average

Let “S” be the variation around the average

$$S = \frac{\overbrace{(48 - 50)}^{-2} + \overbrace{(49 - 50)}^{-1} + \overbrace{(50 - 50)}^0 + \overbrace{(51 - 50)}^1 + \overbrace{(52 - 50)}^2}{5} = \text{ZERO}$$

But, if we square all quantities in the **numerator**...

$$S = \frac{(48 - 50)^2 + (49 - 50)^2 + (50 - 50)^2 + (51 - 50)^2 + (52 - 50)^2}{5} = 2 \text{ sq-lb}$$

# Measuring the Variation Around the Average, cont.

Any issues with this?

This quantity does not add or “subtract any value. There are only 4 (i.e., 5 – 1) “independent” values in the data set

Variation is measured in squared units

$$S = \frac{(48 - 50)^2 + (49 - 50)^2 + (50 - 50)^2 + (51 - 50)^2 + (52 - 50)^2}{5} = 2 \text{ sq-lb}$$

# Measuring the Variation Around the Average, cont.

So...

the square root brings it back to the original unit

$$S = \sqrt{\frac{(48 - 50)^2 + (49 - 50)^2 + (50 - 50)^2 + (51 - 50)^2 + (52 - 50)^2}{(5 - 1)}} = 1.58 \text{ lb}$$

The variation calculated this way is called **standard deviation (S)**

Subtracting 1 from “n” eliminates the “non-value-adding” quantity

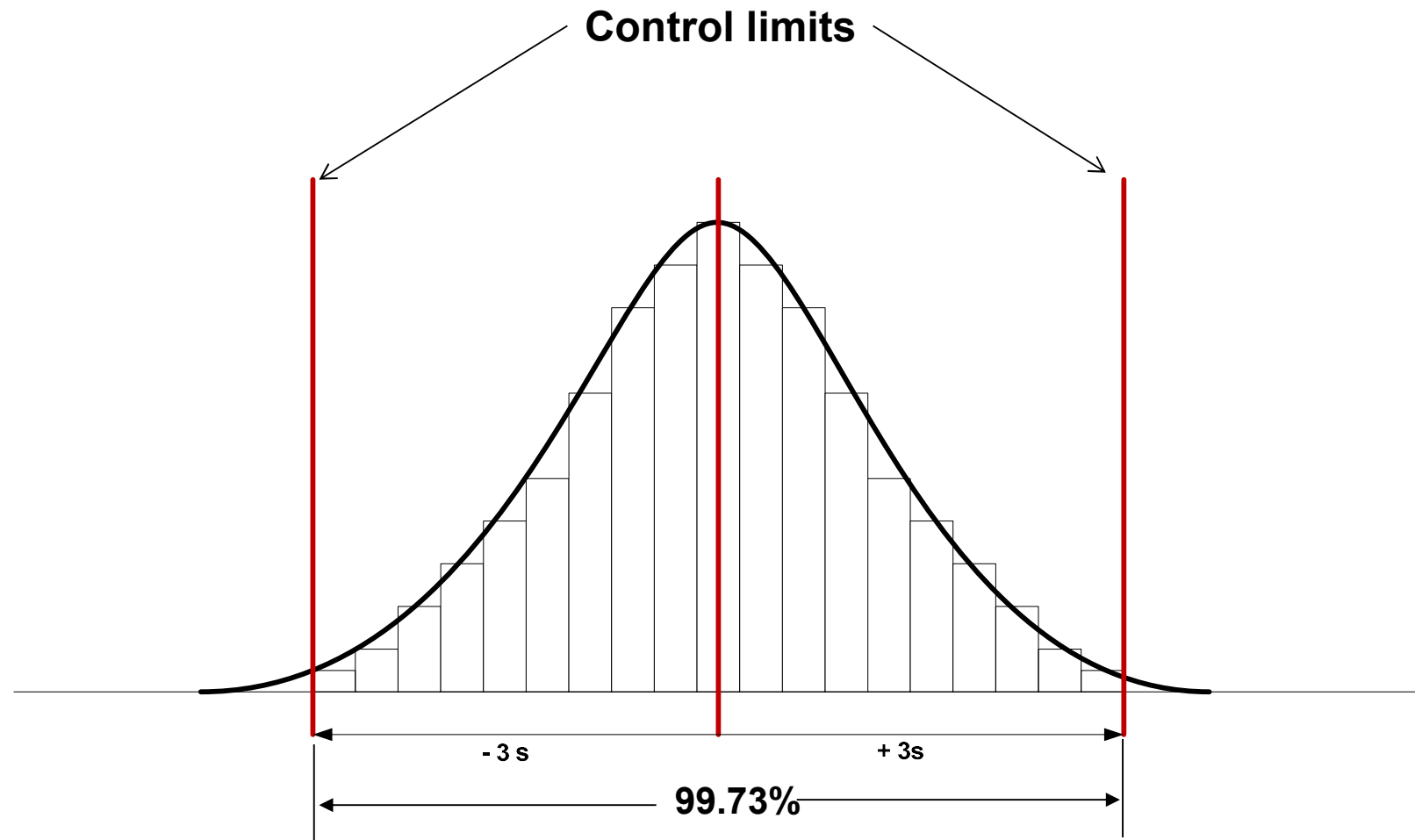
# Measuring the Variation Around the Average, cont.

The standard deviation formula...

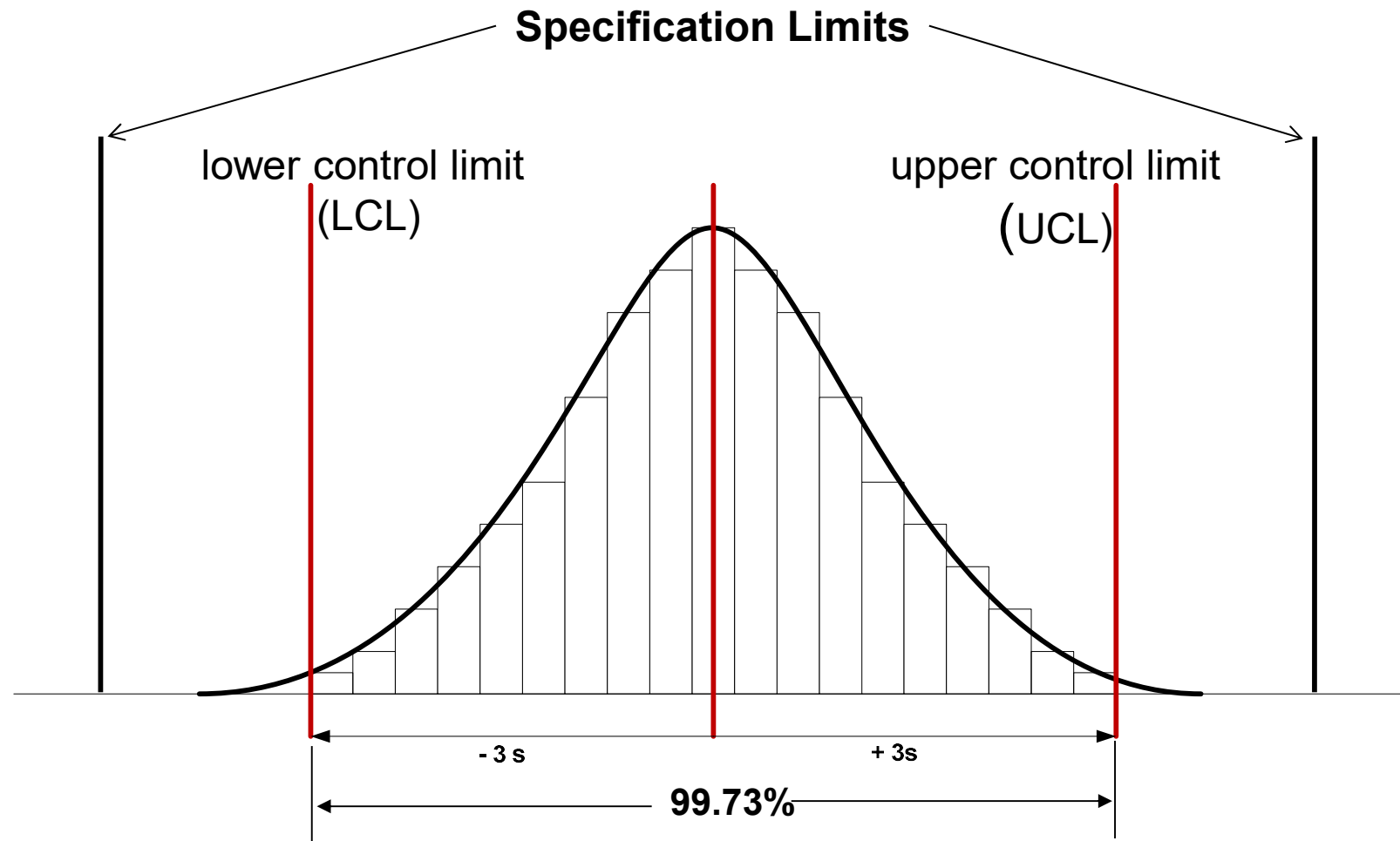
$$S = \sqrt{\frac{\sum (X_i - X_{\text{bar}})^2}{n - 1}} \quad (i = 1, 2, \dots, n)$$

Notation		
	Sample	Population
Mean	Xbar	$\mu$
Std. Dev.	S	$\sigma$

# Understanding “6 Sigma”



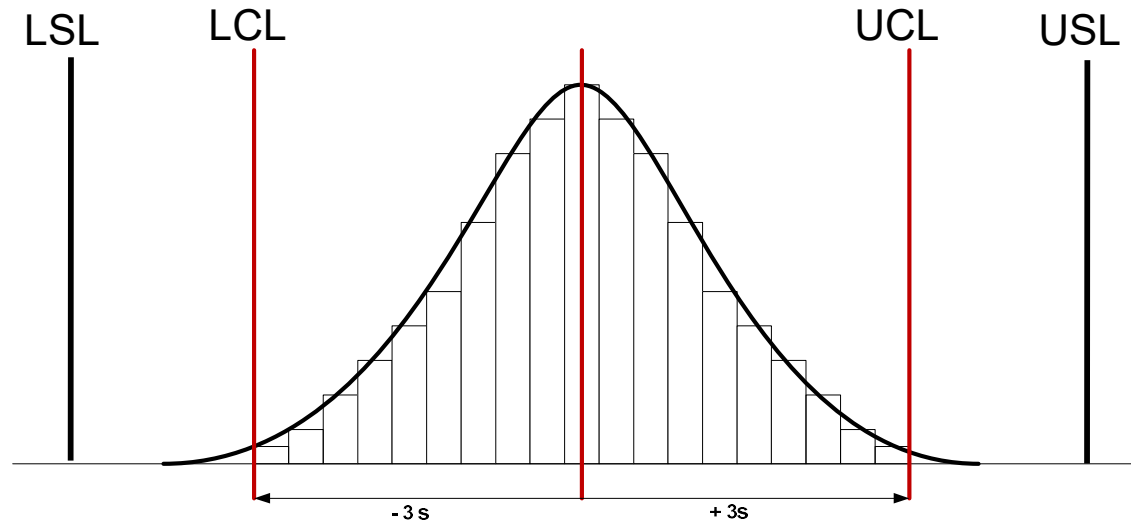
# Understanding “6 Sigma,” cont.





# Understanding “6 Sigma”—Conclusion

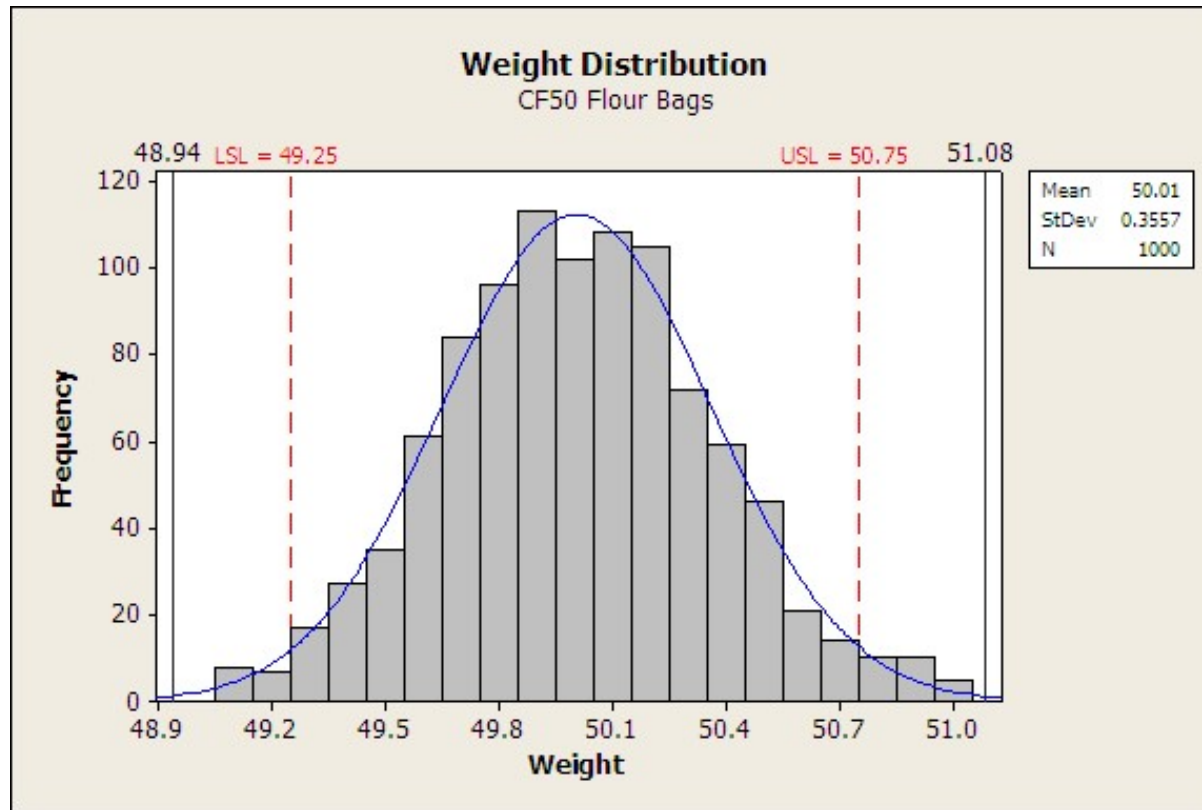
- Process sigma (a.k.a. sigma quality level):
  - An expression of process “yield” in relation to specification limits
  - It measures in  $\sigma$  units the distance between the mean and the spec limits
  - $6\sigma$  away from the mean  $\rightarrow$  3.4 defects per million opportunities
- Key concepts:
  - Control limits = Voice of the Process
  - Specification limits = Voice of the Customer



# The Best Flour Problem

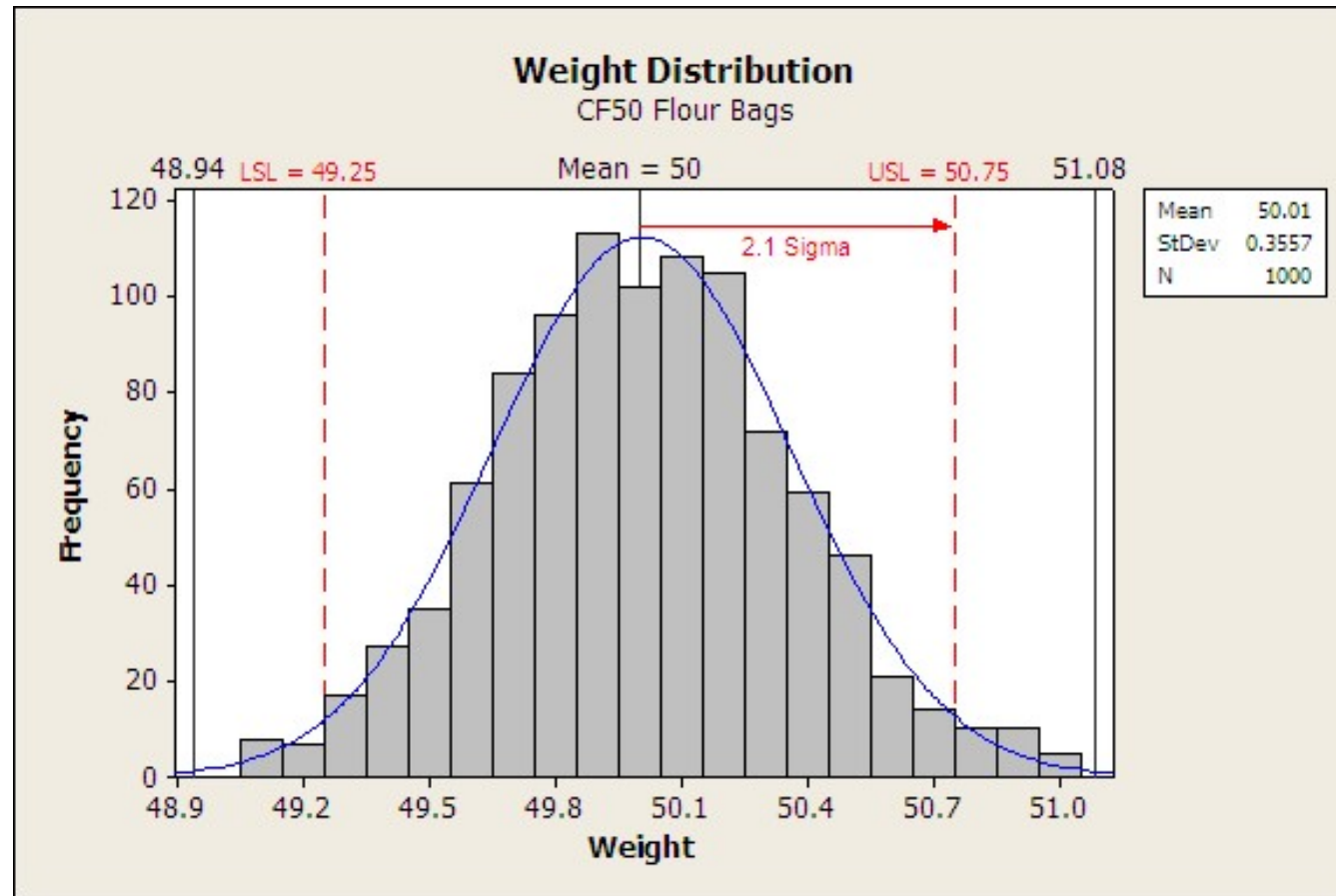
Here are the control limits and the specification limits

- What is happening with the process?
- What should Best Flour do?



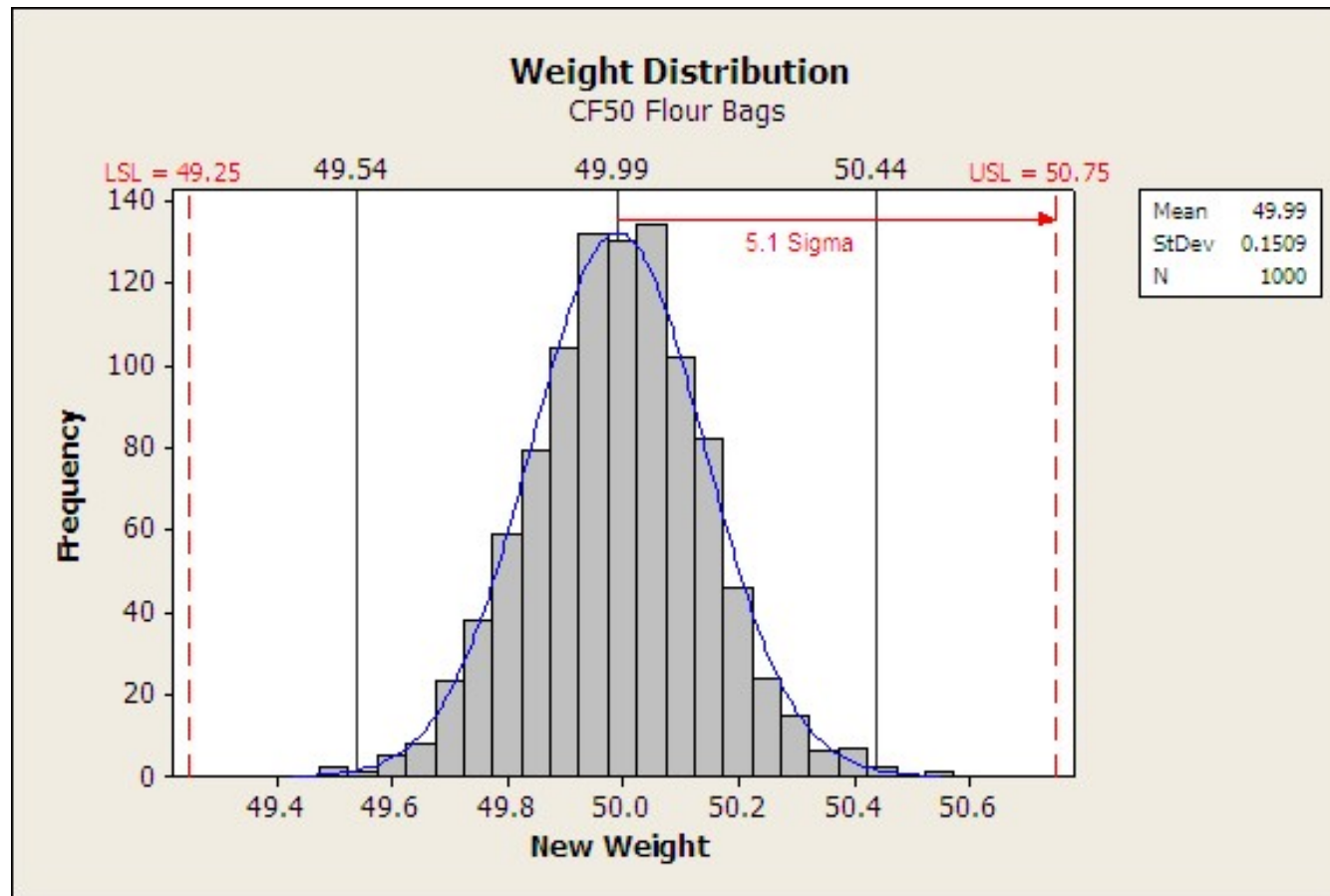
# The Best Flour Problem, cont.

Before Improvement: sigma quality level at 2.1



# The Best Flour Problem, cont.

After Improvement: sigma quality level at 5.1



# Anexas Consultancy Services

## The Six Sigma Metric

Discrete Variables

# Variables: Continuous vs. Discrete

## ■ Continuous Variables

- Can be measured on a continuous scale:
  - Length (inches)
  - Temperature (F)
  - Pressure (PSI)
  - Revenues (US\$)
  - Time (hours)
  - Cholesterol (mg/dL)

## ■ Discrete Variables

- Cannot be measured, only observed, counted, or verified
  - Percent of nonconforming product or service
  - Number of customer complaints
  - Customer satisfaction on a scale from 1 to 10

In this module, we will focus on discrete variables only  
Later modules address process sigma for continuous variables

# Process Sigma — Discrete Variables

- First we calculate the process yield by obtaining the percent of items conforming to specifications over some given period

$$\text{Yield} = \frac{\# \text{ in spec}}{\# \text{ in spec} + \# \text{ out of spec}} = \% \text{ in spec}$$

- Then, we obtain process sigma from the Six Sigma table (below)
  - Example: over a 7-day period, we produced 200 batches of product and 20 were out of spec

$$\text{Yield} = \frac{180}{200} = 90\% \quad \longrightarrow \quad \text{Sigma} \approx 2.8$$

# Different Levels of Process Sigma

- From a broad perspective, consider the following when reviewing different process sigma values:



Too much variation	Hard to produce output within specs	Low sigma values (0 – 2)
Moderate variation	Most output within specs	Middle sigma values (2 – 4.5)
Very little variation	Virtually all output within specs	Hi sigma values (4.5 – 6)



# Process Sigma and DPMO

Increase in sigma requires exponential nonconformity reduction

DPMO	$\sigma$
308,537	2
66,807	3
6210	4
233	5
3.4	6

# Why Use Six Sigma as a Metric?

Process sigma is a more sensitive indicator than percentage

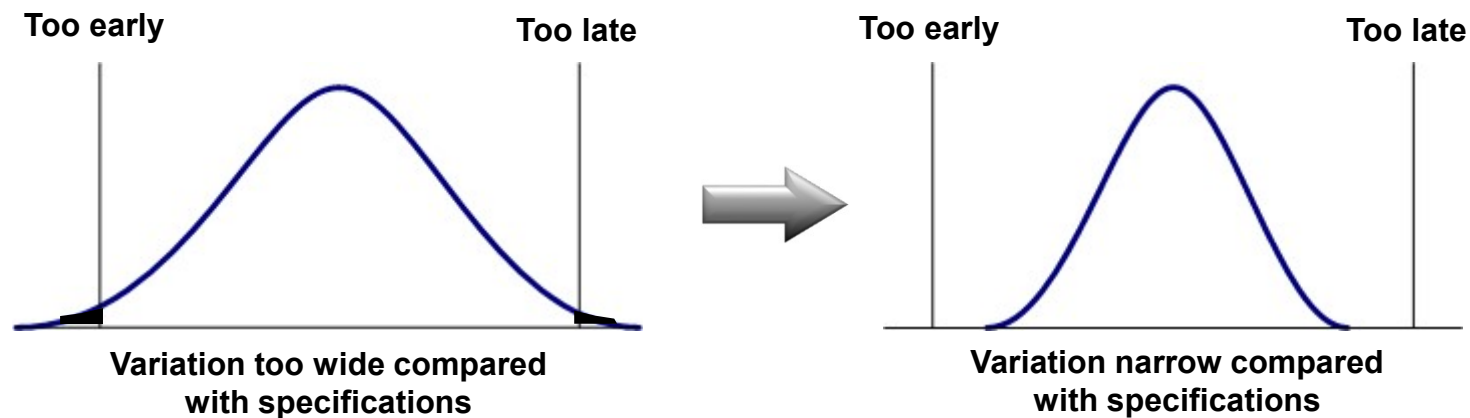
Percent	DPMO	$\sigma$
93	66,807	3.0
98	22,750	3.5
99	6,210	4.0
99.87	1,350	4.5
99.977	233	5.0
99.9997	3.4	6.0

# Actual Sigma or Rate of Improvement?

- What is most important, the actual sigma level or the rate of improvement in sigma?
  - The calculated process sigma level is influenced by what you define as an “opportunity”—which can be subject to much debate
  - Often, a more enlightening measure of good management is to see how process sigma is changing
    - Increased customer focus and process improvement will be reflected with an increase in process sigma
    - A process sigma that goes up by one unit every year is much more indicative of a customer focus than a process sigma that is constant at 4

# Importance of Reducing Variation

- To increase a process sigma level, you have to decrease variation



- Less variation means:
  - Greater predictability in the process
  - Less waste and rework, which lowers costs
  - Products and services that perform better and last longer
  - Happier customers

# Anexas Consultancy Services

## Overview of DMAIC

The Methodology for Process Improvement

# The DMAIC Process

## Monitor/Measure:

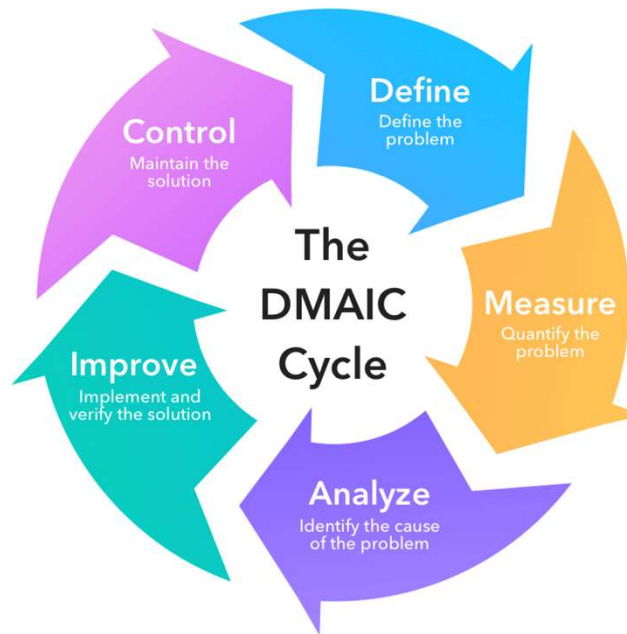
- Standardize process
- Train personnel
- Monitor performance

## Generate solutions:

- Set criteria
- Develop/select solutions
- Conduct risk analysis

## Implement solutions:

- Plan and implement
- Consider the human side



## Define:

- Problem
- Team
- Customer

## Data:

- Collect
- Plot and analyze

## Process:

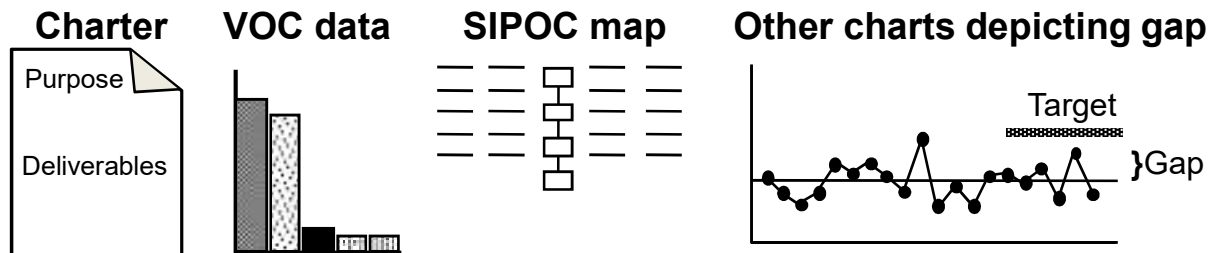
- Map process
- Narrow the focus

## Root-cause analysis:

- Investigate root cause
- Verify potential causes

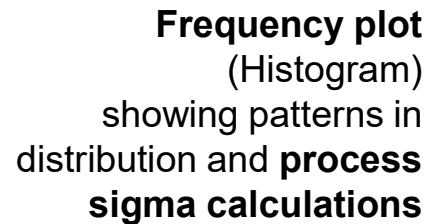
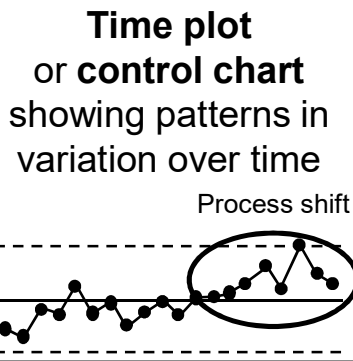
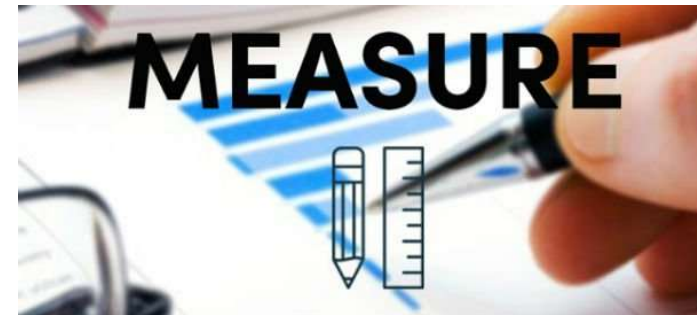
# How Six Sigma Works — DEFINE

- **Goal:** Define the project's purpose and scope
- **Output:** A clear statement of the intended improvement and how it is to be measured
- **Sample tools:**

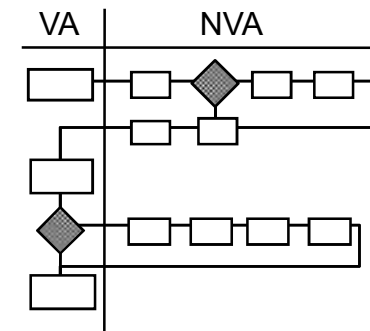


# How Six Sigma Works — MEASURE

- **Goal:** Gather information on the current situation
- **Output:** A focused problem statement and baseline process sigma
- **Sample tools:**



**Process map**  
depicting  
current  
process

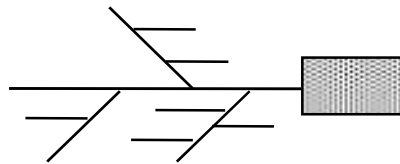




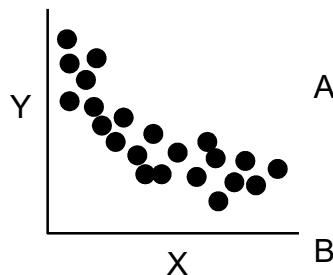
# How Six Sigma Works — ANALYZE

- **Goal:** Identify deep causes and confirm them with data
- **Output:** A theory that has been tested and confirmed
- **Sample tools:**

**Cause-and-effect diagram**  
(or other tool showing potential causes identified)



**Scatter plot, stratified frequency plots, or tables**  
showing verification data



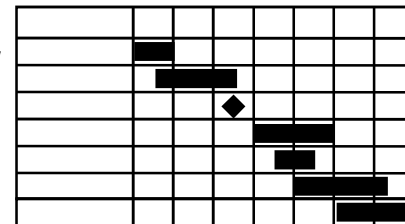
# How Six Sigma Works — IMPROVE

- **Goal:** Develop, test, and implement solutions that address deep causes
- **Output:** Planned, tested actions that should eliminate identified root causes or reduce their impact
- **Sample tools:**

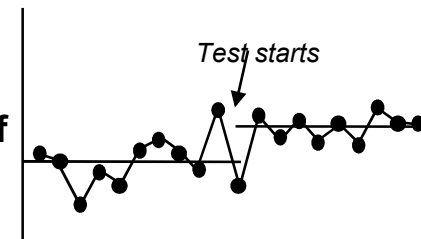
**Solution matrix**  
showing ranked criteria and scoring for each solution option

A	●		○	4
B	●	●	●	1
C	△	○	●	3
D	●	△	●	2

**Plans for implementation**



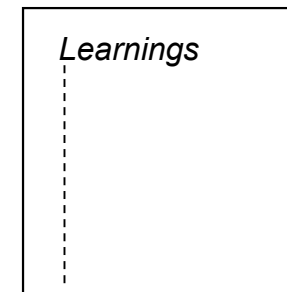
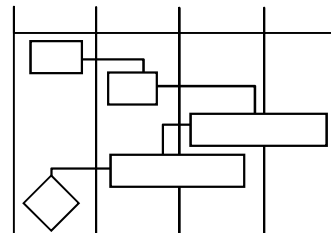
**Results of pilot tests**



# How Six Sigma Works — CONTROL

- **Goal:** Maintain the gains by consistently implementing the new work methods or processes
- **Output:**
  - Documentation of the new standardized method
  - Training in the new method
  - A system for monitoring consistent use of the new method and checking the results
- **Sample Tools:**

**Samples of standardization documentation**  
including revised maps, process management chart, etc.



# Anexas Consultancy Services

## Overview of DMADV

The Methodology for Process Design

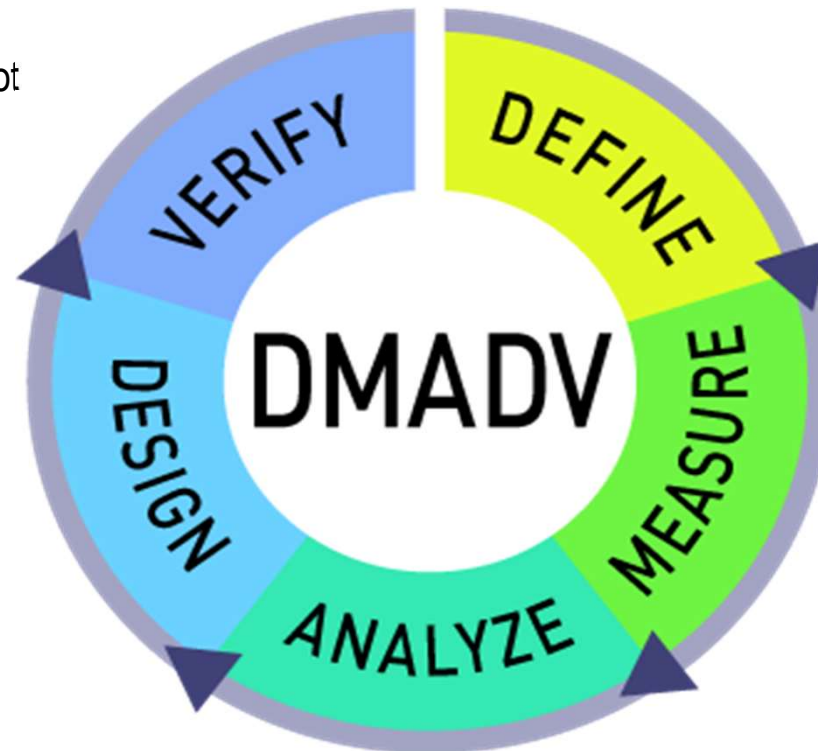
# The DMADV Process

## Verification/validation:

- Conduct and evaluate pilot
- Implement design
- Close project

## Product/service design:

- Identify and prioritize design elements
- Develop design requirements
- Design product and process
- Predict design performance
- Optimize design
- Plan pilot



## Define project:

- Develop charter
- Develop project plans
- Develop change plans
- Assess risks
- Review tollgate requirements

## Get data:

- Collect VOC data
- Translate VOC into CTQs
- Prioritize CTQs
- Reassess risk

## Conceptual design:

- Identify and prioritize key functions
- Generate, evaluate, select, and review key concepts

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Overview of DMAIC—The Process  
Improvement Methodology

# Goals

- Identify the steps and their sequence in DMAIC
- Match each of the steps with its outputs
- Be aware of tools and concepts useful in each step

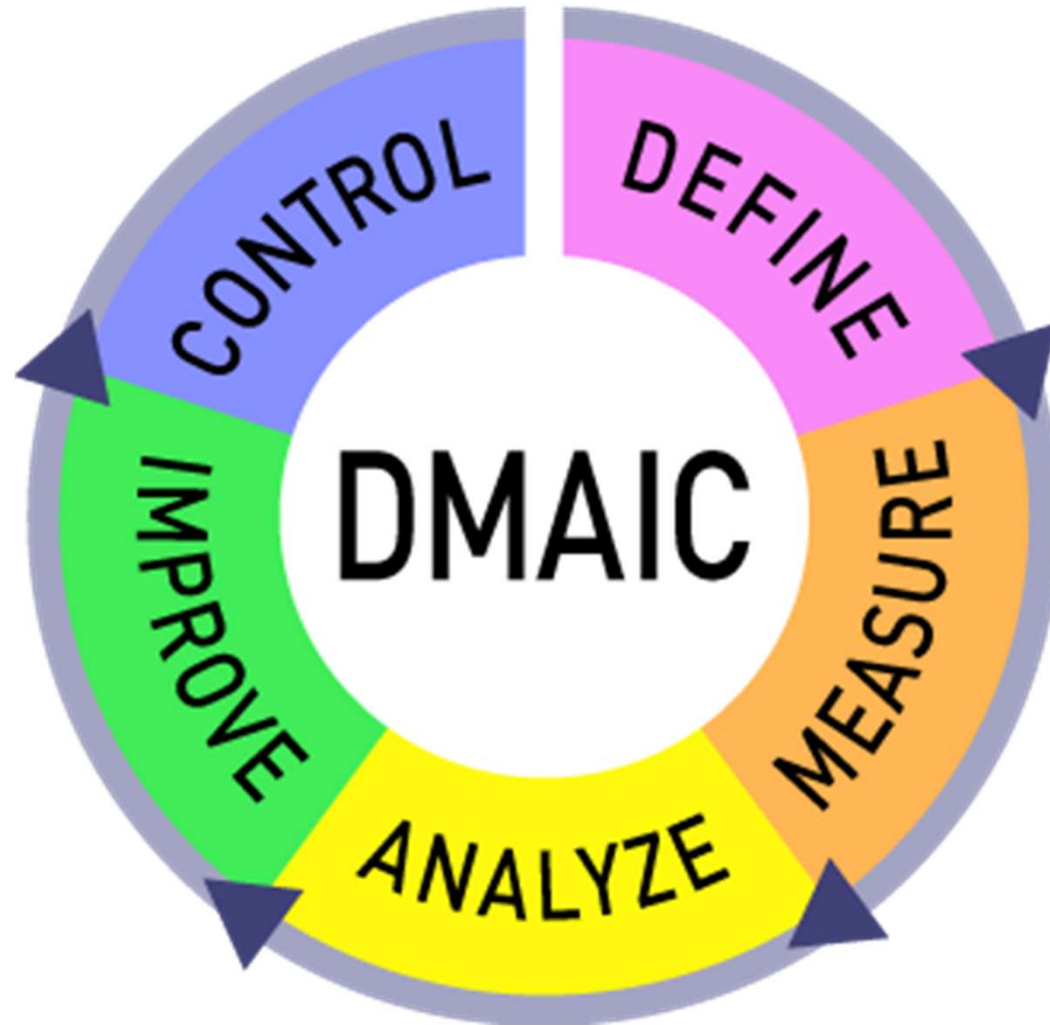
# Why Use an Improvement Methodology?

- Provides a framework
- Provides a common language
- Provides a checklist to prevent skipping critical steps
- Allows you to improve how you improve





# Overview of DMAIC



# Anexas Consultancy Services

## DEFINE: Define the Project

### Overview

# You Are Here — DEFINE

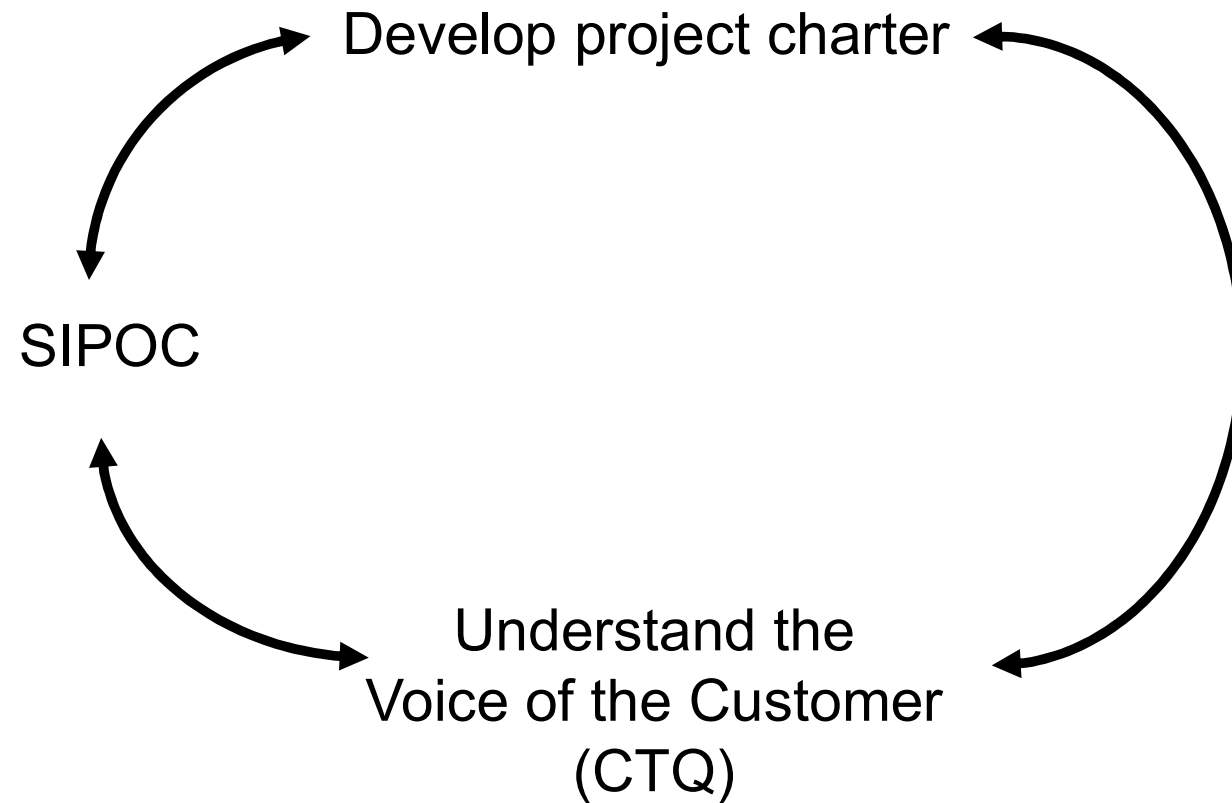


# DEFINE: Define the Project

- **Goal**
  - Define the purpose and scope of the project and get background on the process and customer
- **Outputs**
  - A clear statement of the intended improvement and how it is to be measured
  - A high-level map of the process
  - A translation of the VOC into CTQ
  - An initial project plan



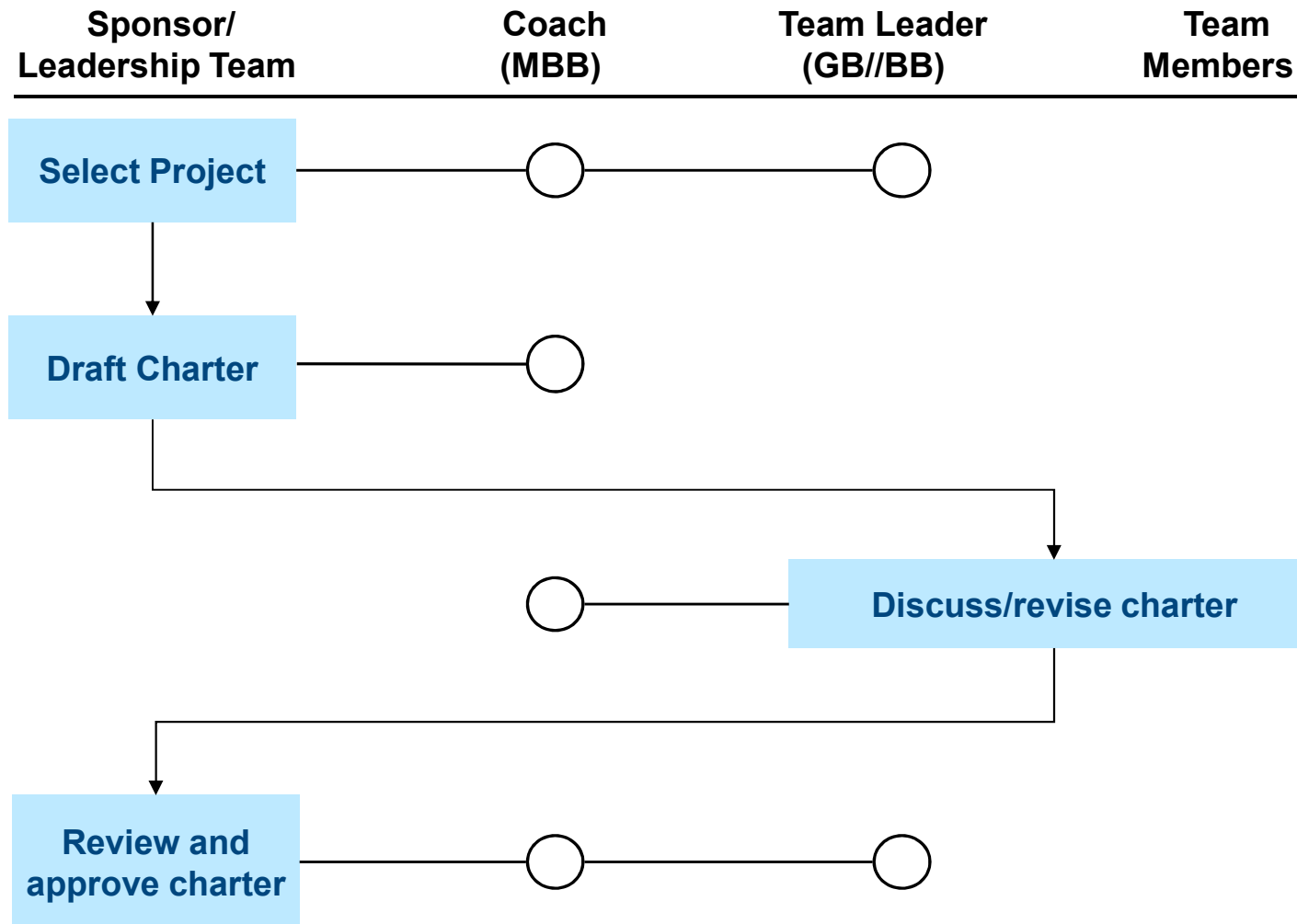
# Approach to DEFINE



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## Starting Your Project

# How This Work Gets Done



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## Project Criteria



# DMAIC Project Selection Criteria

DMAIC projects will likely have greater success if:

- The problem is related to a key business issue
- The problem is linked to a clearly defined process (you can identify the starting and ending points)
- You can identify the customers who use or receive the output from this process

## DMAIC Project Selection Criteria, cont.

DMAIC projects will likely have greater success if (cont.):

- You can clearly identify what a defect is and count its occurrences
- You can demonstrate how improvements could enhance financial performance
- There is appropriate organizational support

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## Drafting Your Charter

# DMAIC Project Charter

Project No.: \_\_\_\_\_

<b>Project Name:</b>	<b>Core Process:</b>
<b>Resource Plan:</b>	<b>Team Members/Support Resources:</b>
<b>Black Belt :</b> <b>Sponsor:</b> <b>Process Owner:</b> <b>Six Sigma Leader:</b> <b>Black Belt and Master Black Belt:</b>	Text
<b>Problem Statement</b>	<b>Scope</b>
For the last 2 years, procurement time in our organization is 120 days for XYZ type of items/services.	<i>Corporate services included</i> <i>Projects not included</i>
<b>Goal Statement</b>	<b>Customer CTQ's</b>
Reduce procurement from 120 days to 90 days by Feb 2021.	Text
<b>Estimate Financial Opportunities / Intangible Benefits</b>	<b>High Level Project Milestone</b>
<i>Reduce the procurement time.</i> <i>Ensures that we comply with the plan</i> <i>Ensures timely revenue</i>	<i>Define: 15<sup>th</sup> Nov</i> <i>Measure: 15<sup>th</sup> Dec</i> <i>Analyze: 5<sup>th</sup> Jan</i> <i>Improve: 31<sup>st</sup> Jan</i> <i>Control: 15<sup>th</sup> Feb</i>
Functional Manager/Process Owner: _____ Date: _____	Sponsor: _____ Date: _____
Black/Green Belt: _____ Date: _____	Six Sigma Leader: _____ Date: _____
	Financial Analyst: _____ Date: _____
	Other: _____ Date: _____

# DMAIC Project Charter

Project No.: \_\_\_\_\_

<b>Project Name:</b> Reduce Change-over Time For Oil Types	<b>Core Process:</b> Oil Refinery Process
<b>Resource Plan:</b>	<b>Team Members/Support Resources:</b>
<b>Green Belt :</b> <b>Sponsor:</b> <b>Process Owner:</b> <b>Six Sigma Leader:</b> <b>Black Belt and Master Black Belt:</b>	Text
<b>Problem Statement</b>	<b>Scope</b>
For the last one year, The Refinery Change-over Time is 6 hours, which reduces the productivity.	Oil refinery change over processes
<b>Goal Statement</b>	<b>Customer CTQ's</b>
Reduce change-over time from 6 hours to 2 hours by 30 <sup>th</sup> April, 2021.	Change-over Time
<b>Estimate Financial Opportunities / Intangible Benefits</b>	<b>High Level Project Milestone</b>
Productivity improvement (( XXXX SAR of additional revenue) Plan Adherence Enhanced OEE, Power saving, Utilities saving.	Define: 15 <sup>th</sup> Nov Measure: 15 <sup>th</sup> Dec Analyze: 5 <sup>th</sup> Jan Improve: 31 <sup>st</sup> Jan Control: 15 <sup>th</sup> Feb
Functional Manager/Process Owner: _____ Date: _____	Sponsor: _____ Date: _____
Black/Green Belt: _____ Date: _____	Six Sigma Leader _____ Date: _____
_____	Financial Analyst: _____ Date: _____
_____	Other: _____ Date: _____

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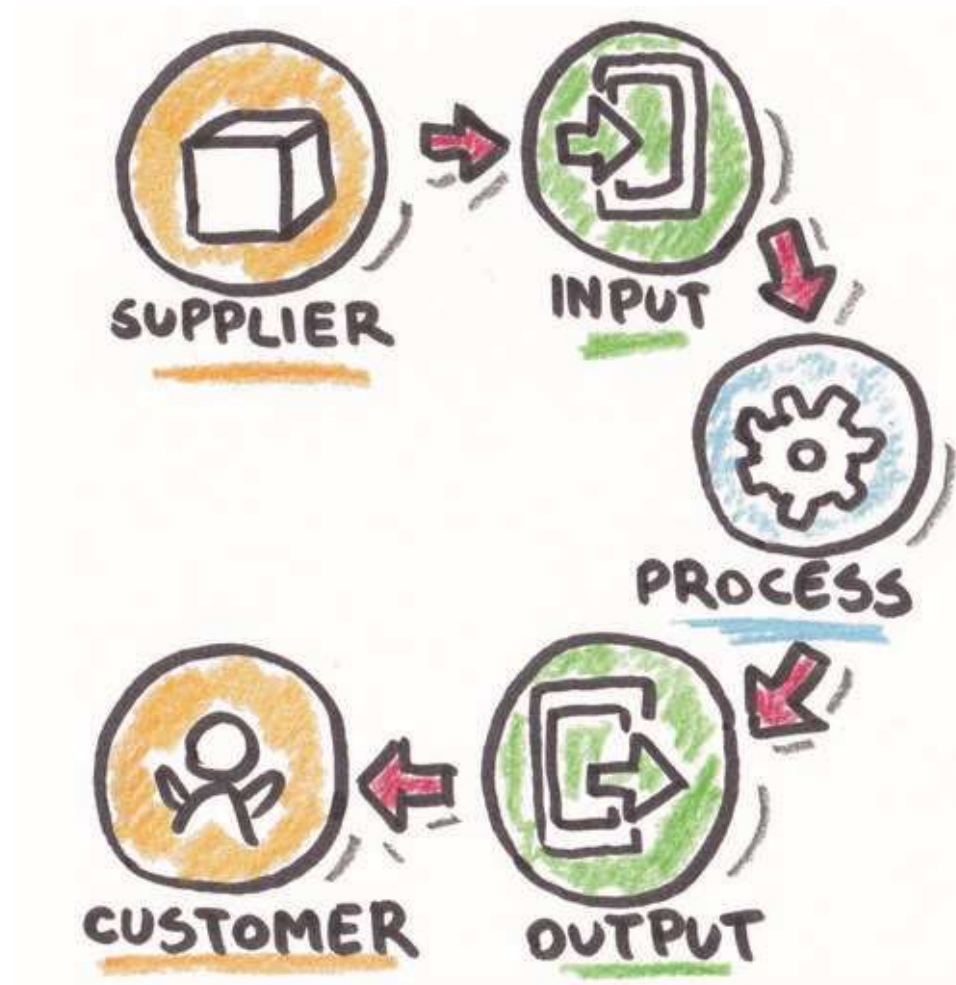
## Understanding and Mapping Processes

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## High-Level Process Mapping

SIPOC

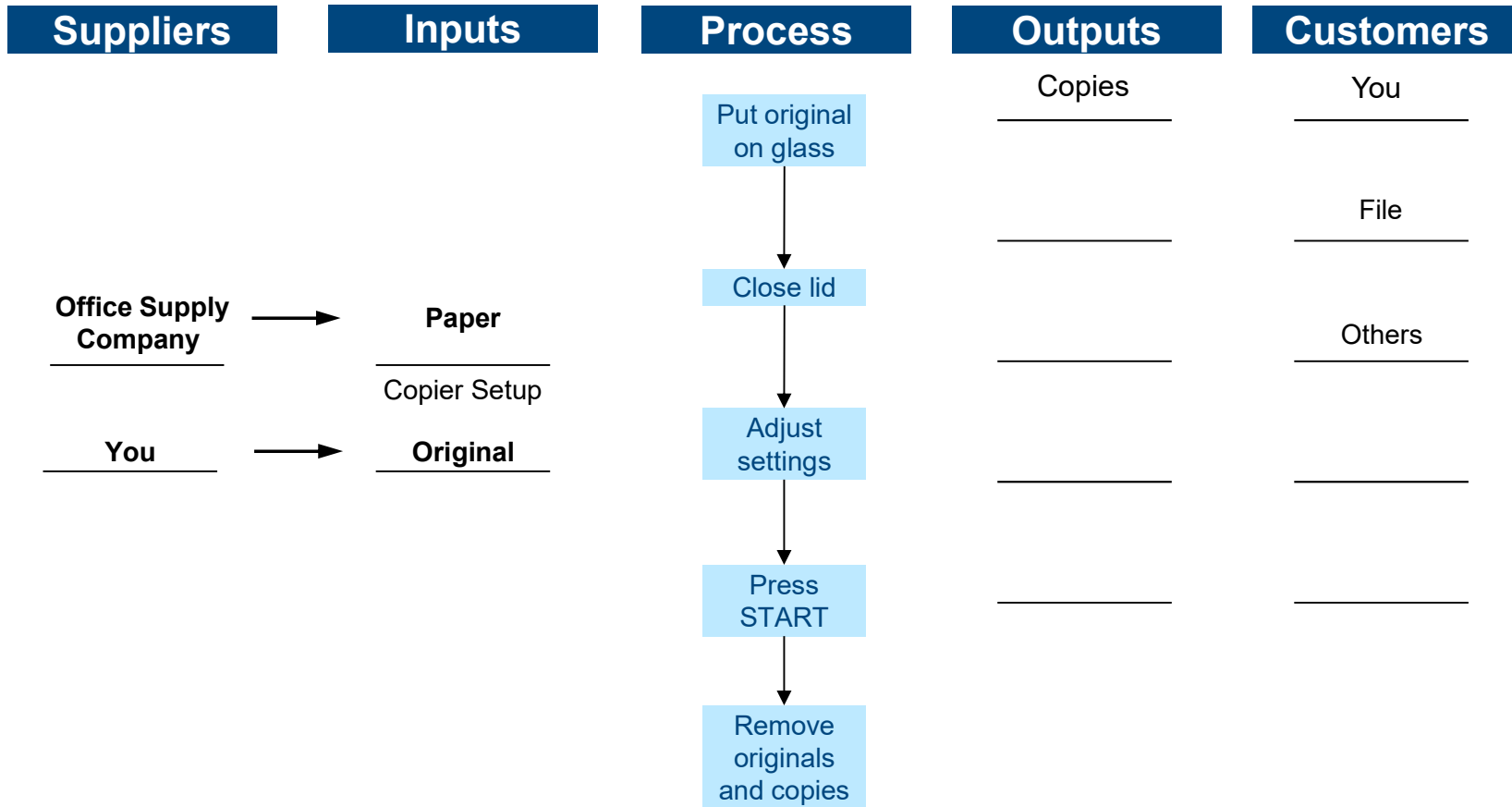
# SIPOC



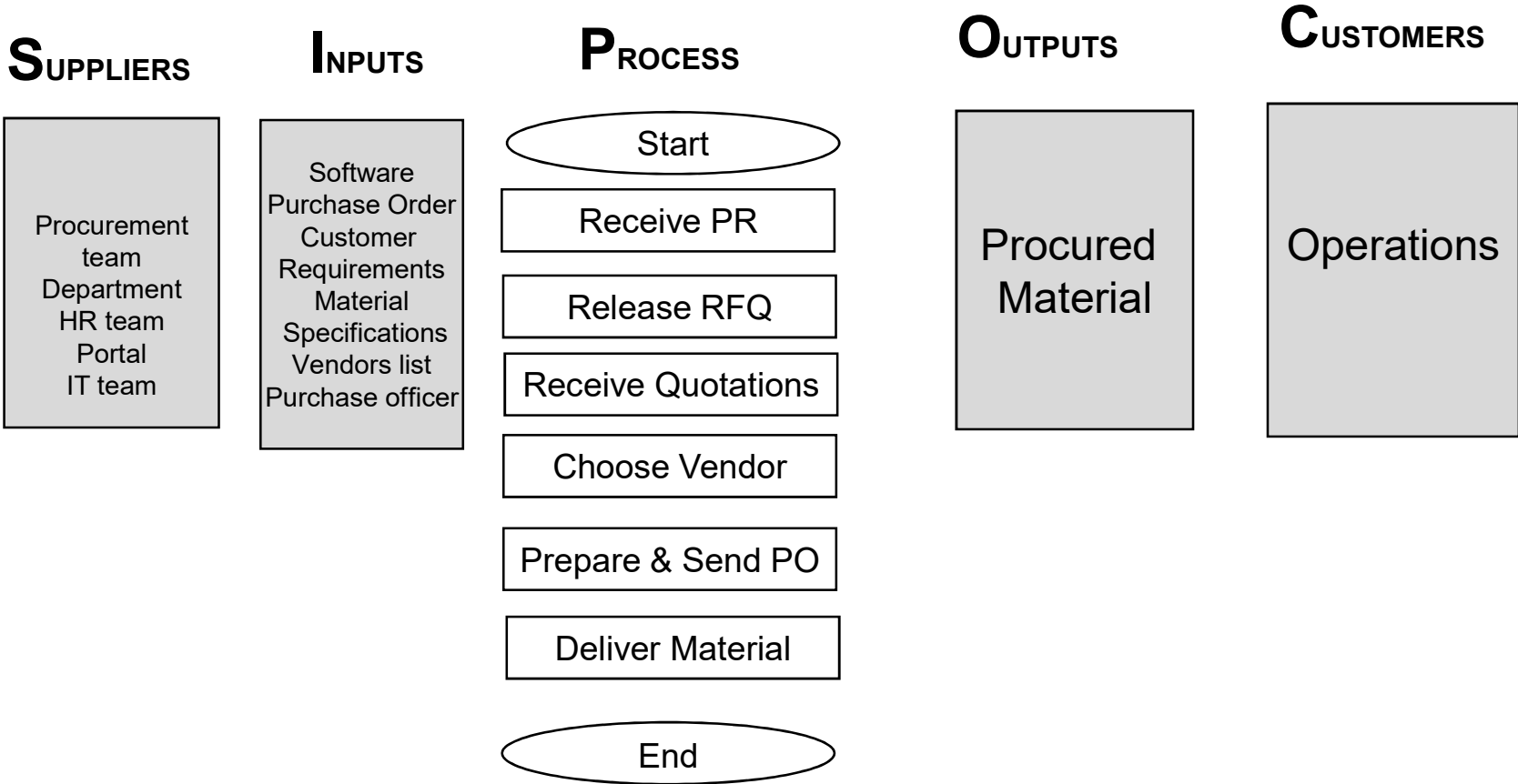


# SIPOC Example

## Making a Photocopy



# SIPOC



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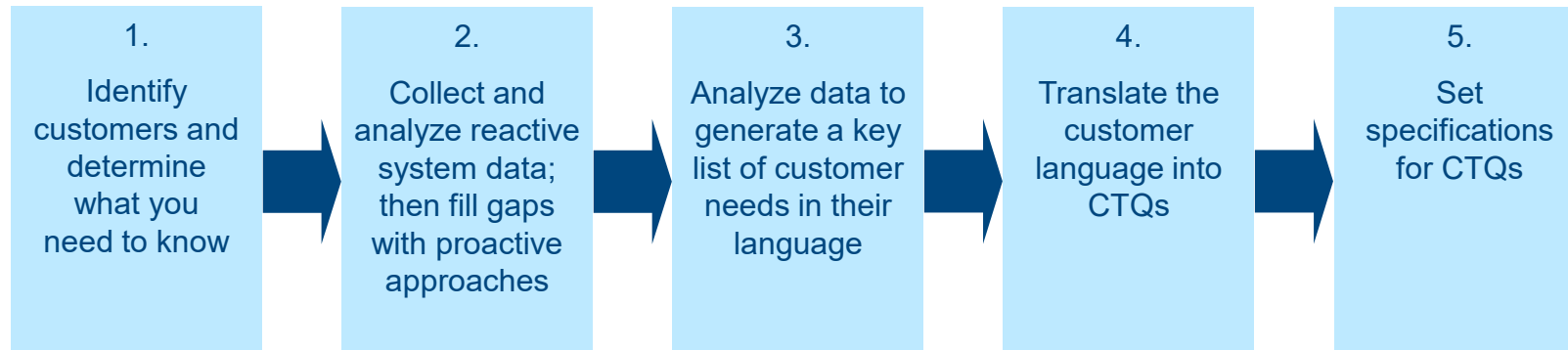
Understanding the  
Voice of the Customer

# What Is the Voice of the Customer?

- The term Voice of the Customer (VOC) is used to describe customers' needs and their perceptions of your product or service



# VOC Process



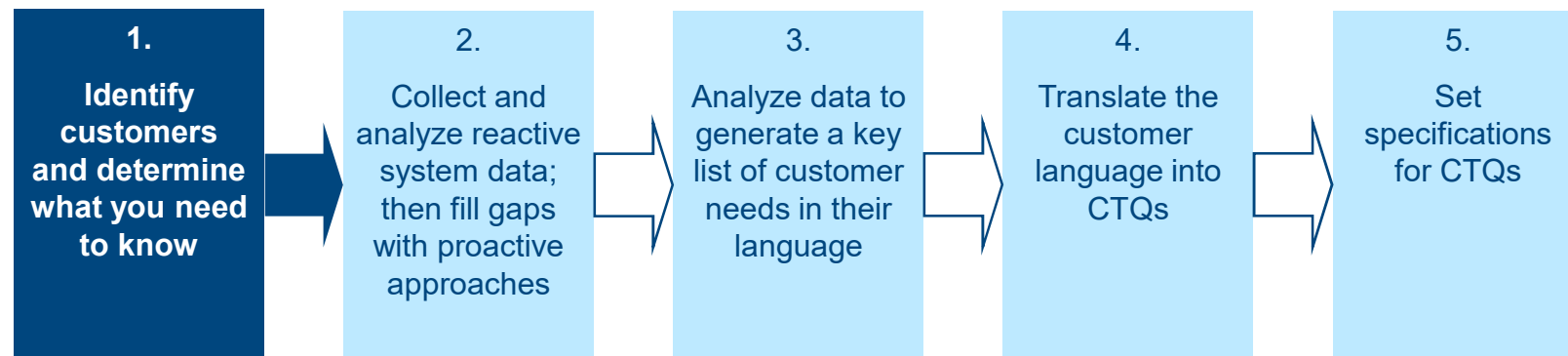
## Outcomes

- A list of customers and customer segments
- Identification of relevant reactive and proactive sources of data
- Verbal (or numerical) data that identify customer needs
- Defined Critical to Quality characteristics
- Specifications for each CTQ

# Anexas Consultancy Services

## VOC Data Collection, Step 1

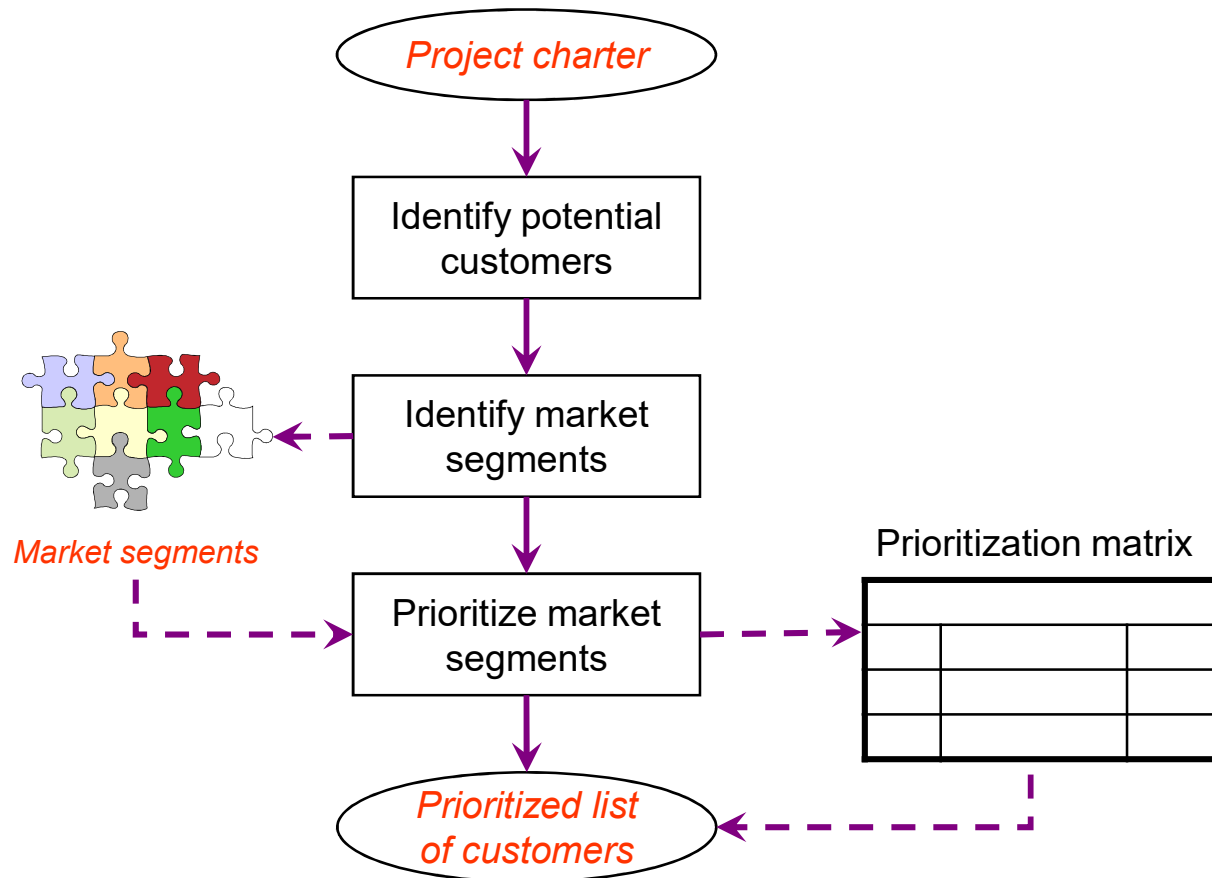
# Step 1: Identify Customers and Determine What You Need to Know



## Goal

- Identify your customers
- Decide what you need to know about their needs
- Decide when and how you will get this information

# Identify Customers: Process





# Project Worksheet: VOC Data Collection Plan

**PROJECT:**

Who	What and Why
Customers and Segments	<p>Indicate specifically what you want to know about your customers. Develop customized versions of the following questions, which you can ask during face-to-face interviews.</p> <ul style="list-style-type: none"> <li>• What's important to you?</li> <li>• What's a defect?</li> <li>• How are we doing? How do we compare with our competitors?</li> <li>• What do you like? What don't you like?</li> </ul>

**Sources**

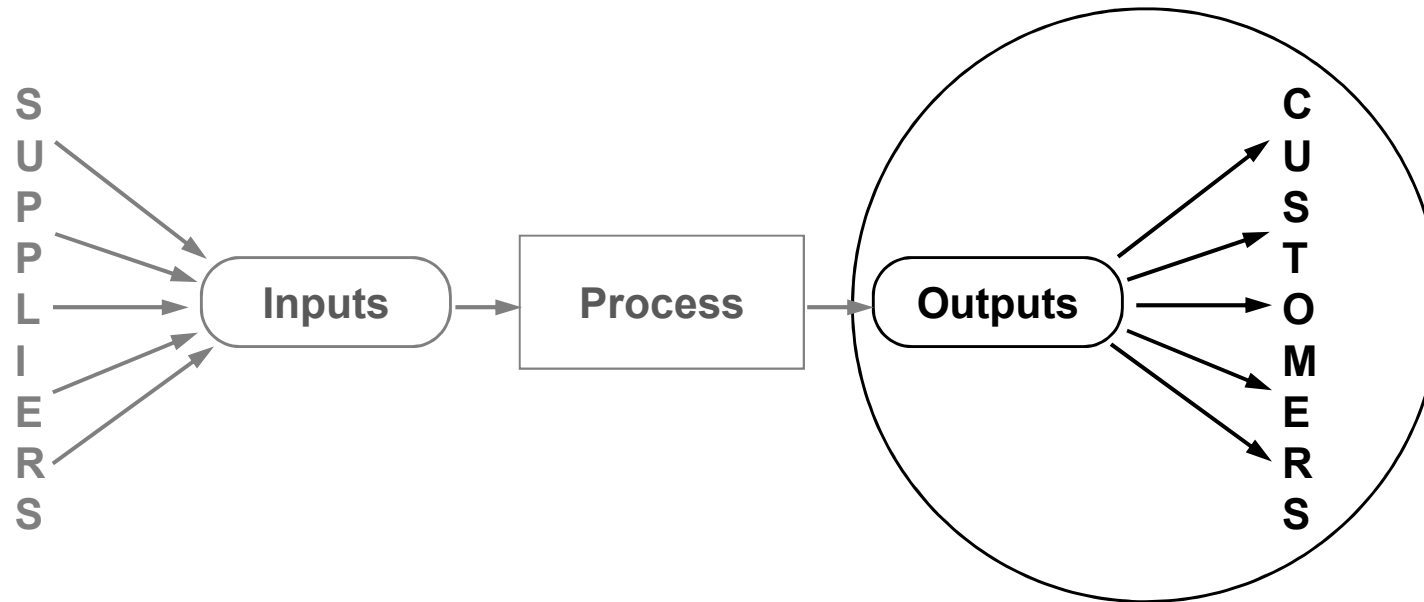
*Type an X next to the data sources you think will be useful for this project.*

REACTIVE SOURCES	PROACTIVE SOURCES
<input type="checkbox"/> Complaints	<input type="checkbox"/> Interviews
<input type="checkbox"/> Problem or service hotlines	<input type="checkbox"/> Focus groups
<input type="checkbox"/> Technical support calls	<input type="checkbox"/> Surveys
<input type="checkbox"/> Customer service calls	<input type="checkbox"/> Comment cards
<input type="checkbox"/> Claims, credits	<input type="checkbox"/> Sales visits/calls
<input type="checkbox"/> Sales reporting	<input type="checkbox"/> Direct observation
<input type="checkbox"/> Product return information	<input type="checkbox"/> Market research/monitoring
<input type="checkbox"/> Warranty claims	<input type="checkbox"/> Benchmarking
<input type="checkbox"/> Web page activity	<input type="checkbox"/> Quality scorecards
<input type="checkbox"/> Other:	<input type="checkbox"/> Other:

**Summary: Which, How Many, How, and When**

On a separate sheet, summarize your plans to gather and use both reactive and proactive sources. Indicate how much data you will get, how you will get it, and when. Include, for instance, the number of interviews or surveys you plan to use, which customers you will contact, when you will start and end the data collection, and so on.

# Who Are Your Customers?



- What are the outputs of your process? Who are the customers of that output?
- Are there particular groups of customers whose needs are especially important to your organization?

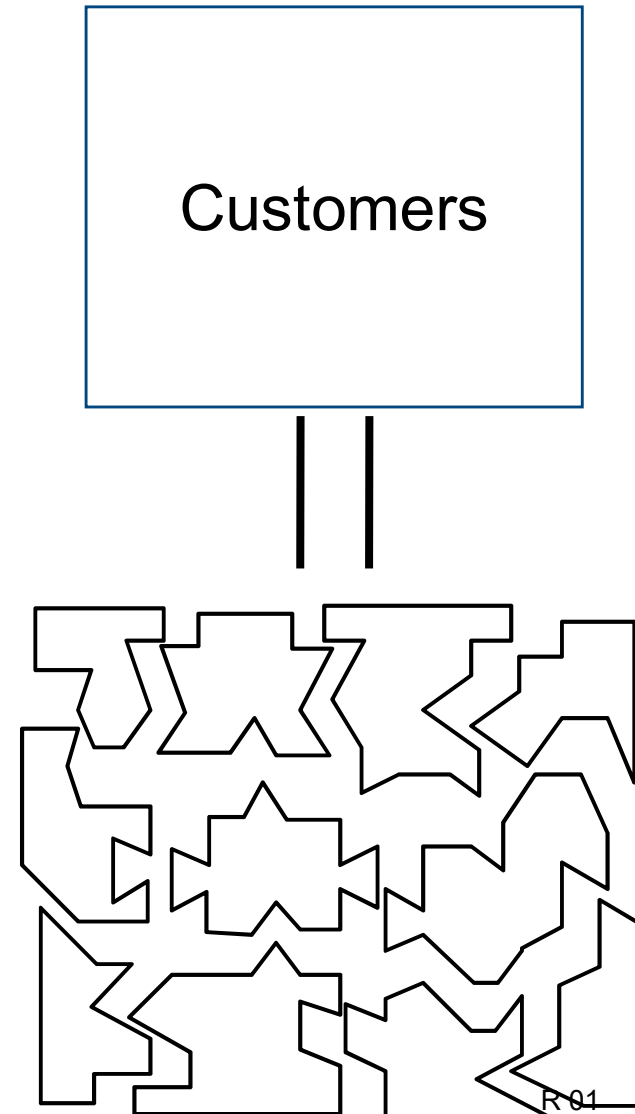
# Identify Potential Customers

Identify whose interests are key and whose perspectives can add value:

- **Current customers** (buy **your** existing products)
- **Former customers** (**stopped** buying your products)
- **Competitors' customers** (buy **alternative** products)
- **Lead thinkers** and other experts
- **Technology** leaders in the industry
- **Strategic partners**
- **Internal customers**
- **Stakeholders** (internal and external)

# Customer and Market Segmentation

- Often there is no single Voice of the Customer; different customers or types of customers usually have different needs and priorities
- You should include a wide variety of customers in your initial customer research efforts
- Different types of customers are often referred to as **market segments**



# Common Market Segments

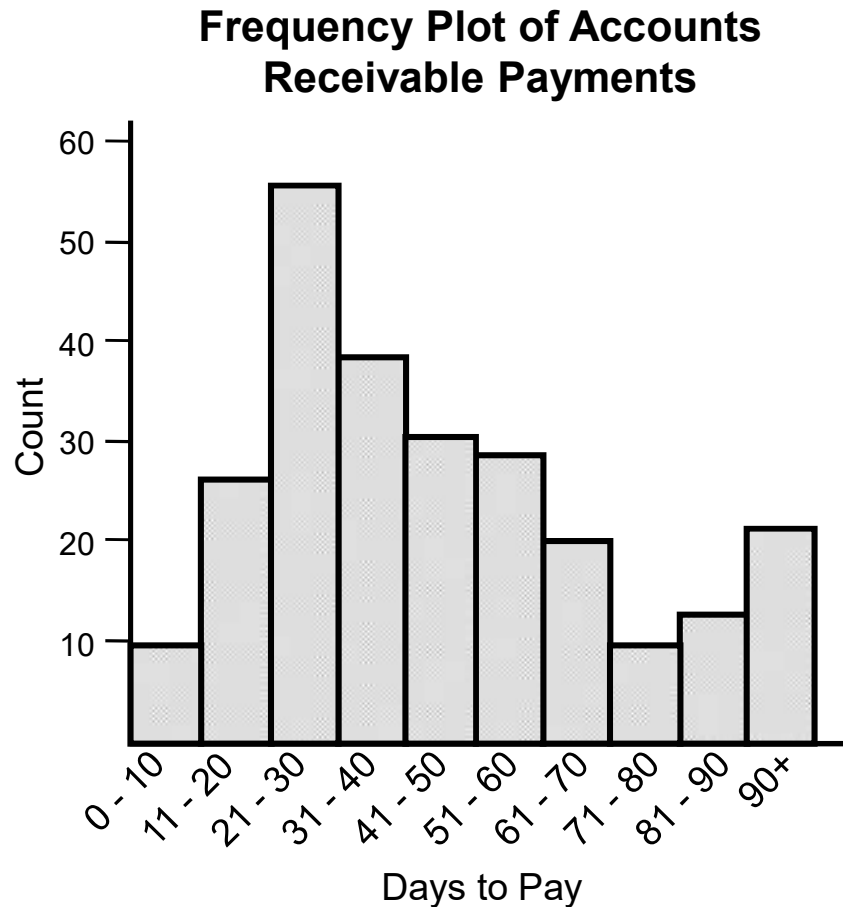
- Customer status: former customers, current customers, customers of competitors, substitute customers
- Where customers are in the “customer chain”
  - Internal user → distributor → end user
- Type of product or service customers buy from you
- Volume of purchases: high, medium, low
- Geography
- Reason for buying
- Industry, division, or department
- Demographics, such as gender or age

# Do You Have Market Segments?

- If your customers seem to have similar needs across the board, you don't necessarily need to divide them into segments
- If you suspect that different groups will have significantly different needs and that these differences will influence how you structure your process, product, or service, then it will be worthwhile to think in terms of segments

# Case Study: Receivables

- A company that sells both products and associated services is trying to improve its billing process and payment of accounts receivables
- A study of how long it takes customers to pay their bills shows the average at about 48 days, but many extend to 90 days or more



# Receivables VOC Plan: Who

## Who

### Customers and Segments

Position in organization

Accounts payable vs. purchasing agents vs. end users

Type of business

Government and people who work with government vs. non-government vs. foreign vs. distributors

Type of payment

Credit card vs. purchase order vs. invoice vs. prepay by check

Payment history

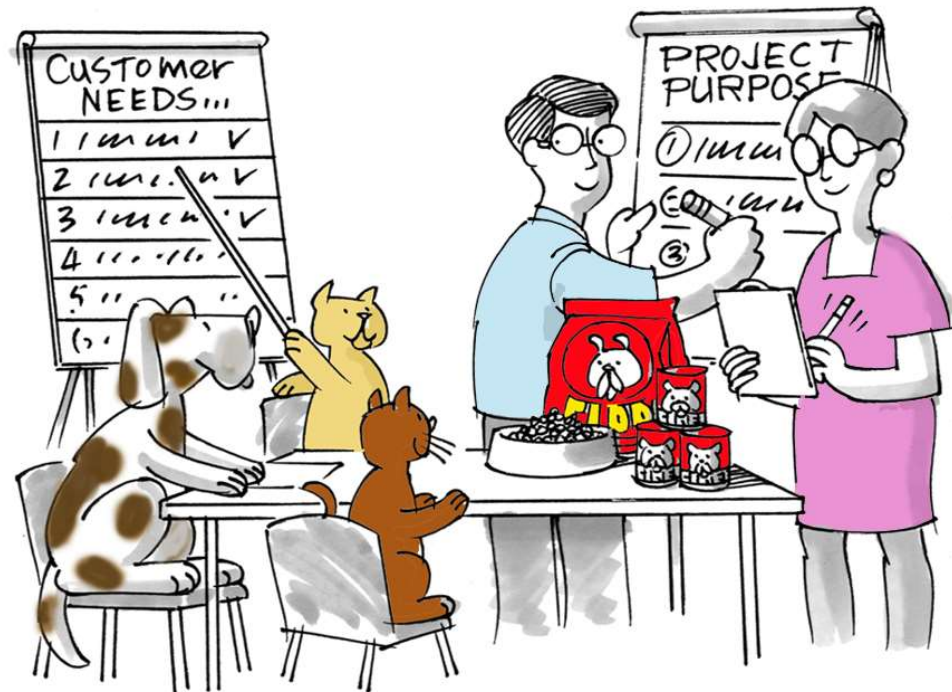
On-time vs. late

Organizations vs. individuals



# Deciding the What and Why

- Revisit your charter: what is the purpose of your project?
- How does your purpose relate to customer needs?
- What do you need to know about the needs of the customers you've identified to make sure your project purpose is on track?



# Receivables VOC Plan: What and Why

## What and Why

Indicate specifically what you want to know about your customers. Develop customized versions of the following questions, which you can ask during interviews.

- What's important to you?
- What problems do you have with \_\_\_\_? (What's a defect?)
- How are we doing? How do we compare with our competitors?
- What do you like? What don't you like?

What are your invoice requirements?

For each requirement, follow up with: If we can't meet that requirement, what does that mean to you? (Try to determine which requirements are minor, which cause delayed payments, etc.)

How well do our processes and policies currently meet your requirements?

How do our processes and policies compare with other vendors?

What do you like about our invoicing processes and policies?

What don't you like?

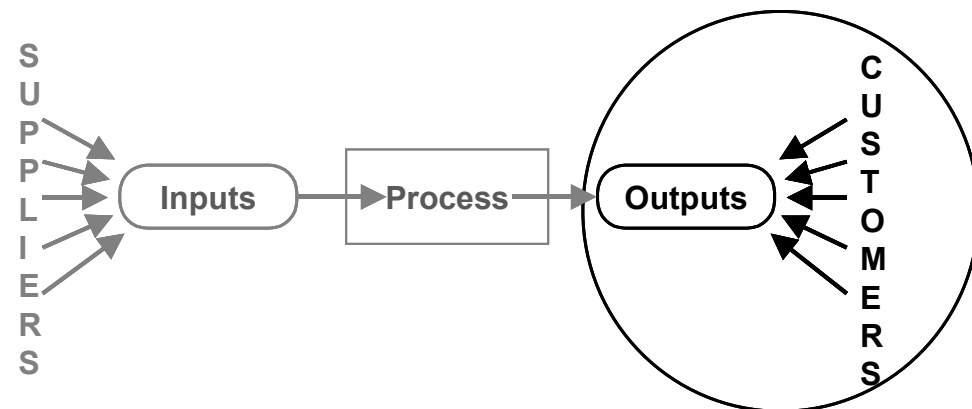
What specific changes would you like to recommend?

# Your Own VOC Plan: Who, What, and Why

## Instructions:

- Use the SIPOC diagram you created to begin developing your own VOC plan
- Identify the customers of the output of your process
  - Think about what customer characteristics might help you separate them into relevant segments
- Identify key questions you have about each of the customer groups
- You may want to take notes on the portion of the worksheet reprinted below and then transfer your final decisions to Project Worksheet: VOC Data Collection Plan

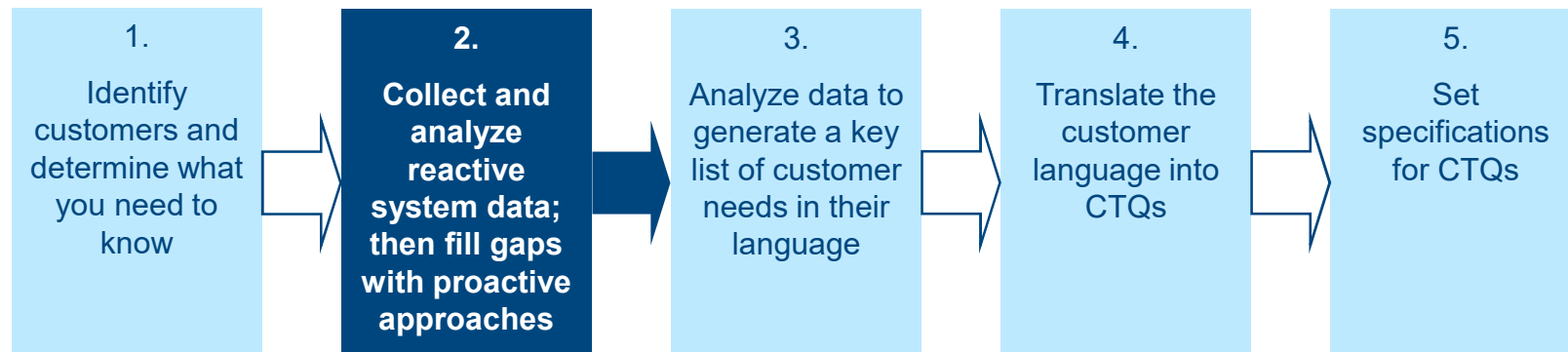
**Time:** 15 minutes



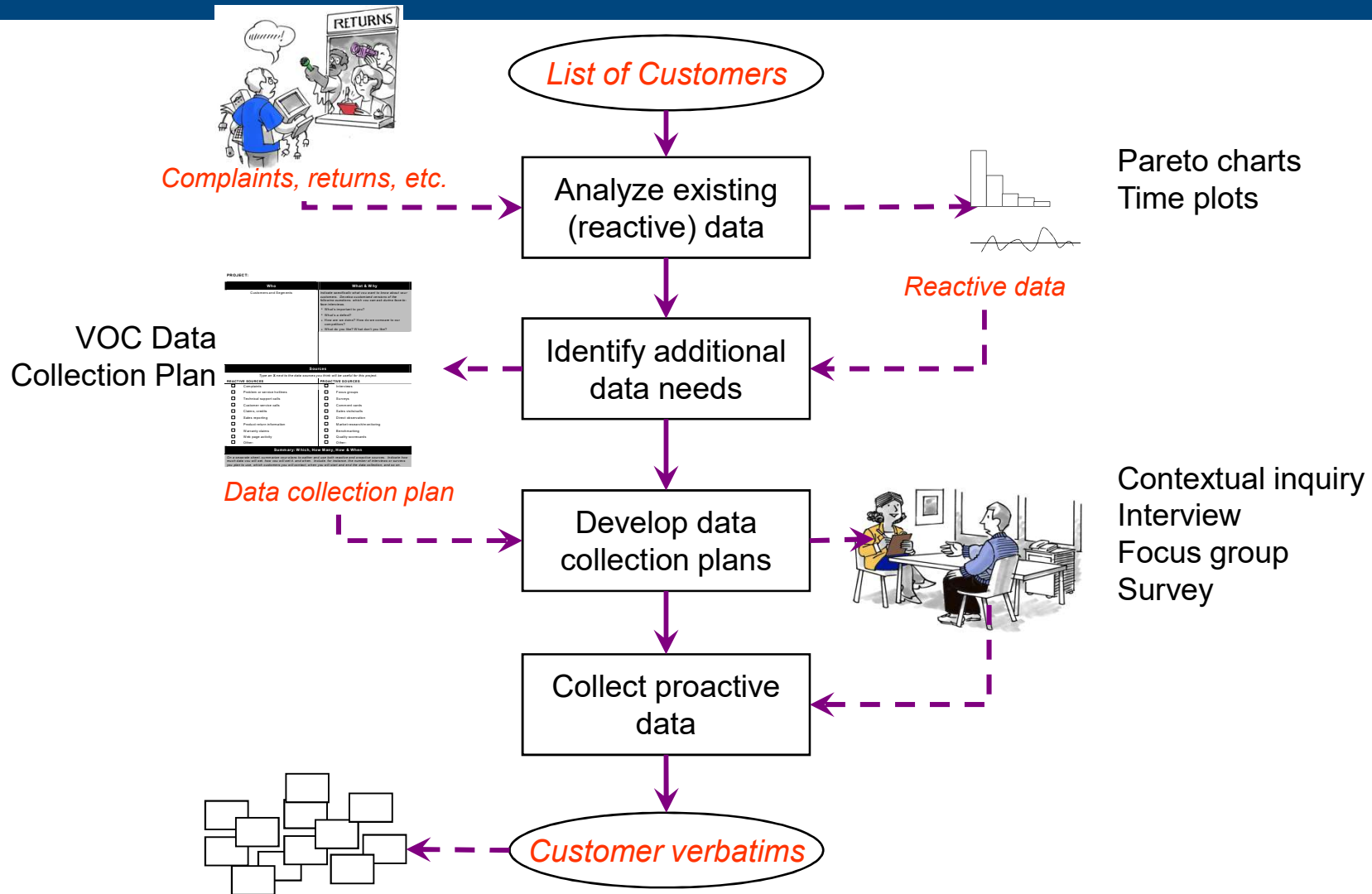
# Anexas Consultancy Services

## VOC Data Collection, Step 2

# Step 2: Collect and Analyze Reactive and Proactive Data



# Collect Needs Data: Process



# Basic VOC Systems

## 1. Reactive systems

- Information comes to you whether or not you take action



## 2. Proactive systems

- You need to put effort into gathering the information



# Typical Reactive VOC Systems

- Customer complaints (phone or written)
- Technical support calls
- Claims, credits, contested payments
- Product return information
- Problem or service hotlines
- Customer service calls
- Sales reporting
- Warranty claims
- Webpage activity



## Typical Reactive VOC Systems, cont.

Reactive systems generally gather data on:

- Current and former customers' issues or problems
- Current and former customers' unmet needs
- Current and former customers' interest in particular products or services

# Proactive VOC Systems

- Direct customer observation
- Interviews
- Focus groups
- Surveys
- Comment cards
- Data gathering during sales visits or calls
- Market research, market monitoring
- Benchmarking

# Analyze Existing Data

## **1. Collect and summarize existing data**

- “Reactive” data will already be on hand, but will most likely need to be organized and summarized

## **2. Analyze these data using appropriate tools, e.g., Pareto charts and control charts**

- Draw preliminary conclusions from relevant existing data

## **3. Determine which preliminary conclusions need to be validated with new customer research**

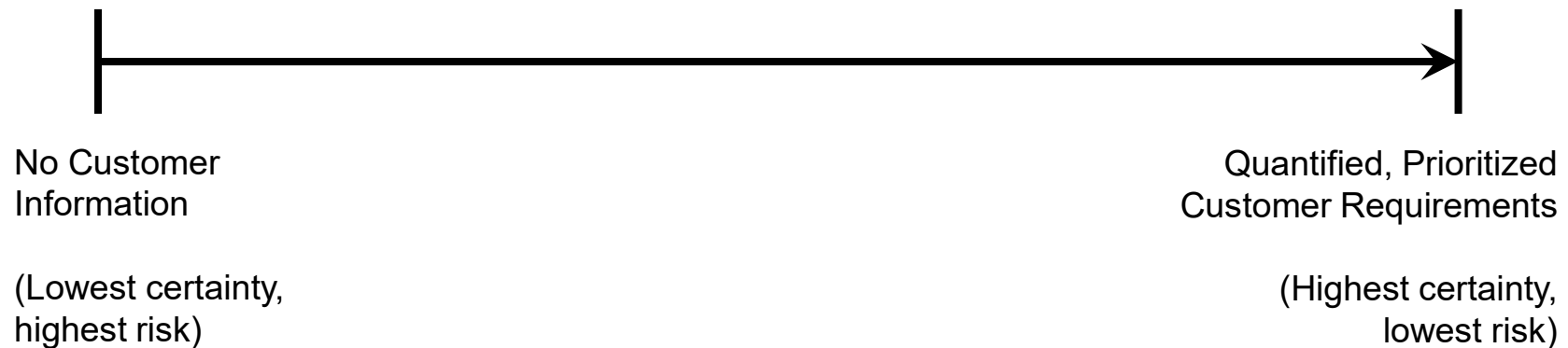
- Did we draw the right conclusions from our existing data?

## **4. Consider additional broad questions to understand customers’ needs and extend thinking beyond existing products and services**

- What else do we need to learn?

# Identify Proactive Data Needed

- After analyzing existing data, assess how much new information is needed
- Consider how to progress from where you are to where you need to be, based on
  - Level of **certainty** you can **afford**
  - Level of **uncertainty** you can **risk**



# Face-to-Face Interaction

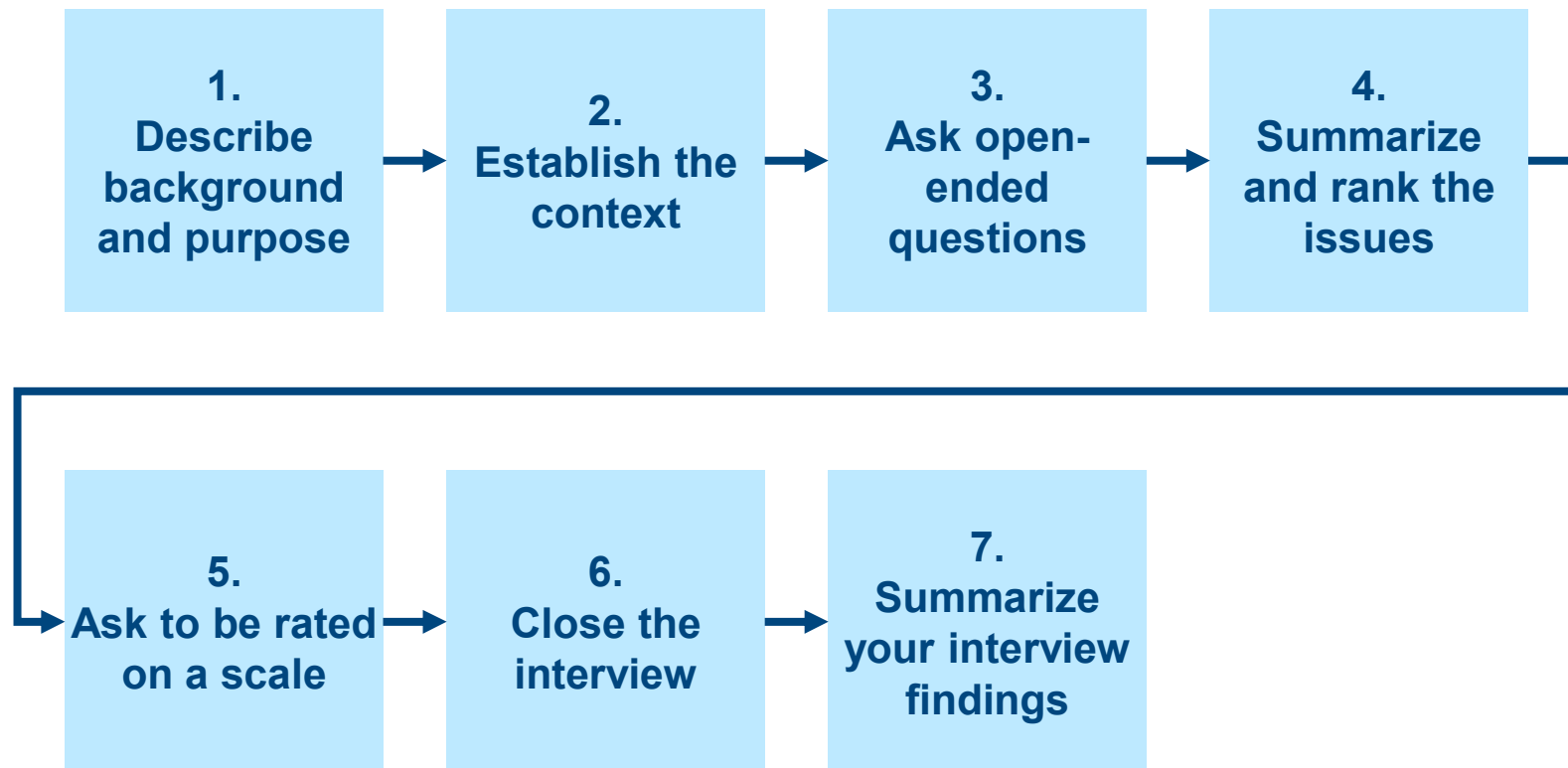
- Face-to-face interaction with customers can provide a wealth of data and knowledge that is unobtainable by other means
- A good first step in proactive data collection is **customer observation**
  1. Get them to show you, in detail, how they use your product or service (talk with them peer to peer, on-site)
  2. Ask “What are three things we could do to improve our product or service to you?” or, if time permits, conduct a lengthier interview
- Ideally, you should have face-to-face interviews prior to any other data collection or extensive phone interviews

# Developing an Interview Guide

- If you are going to conduct interviews, compile questions into an interview guide
- Organize your questions
- Write a standard introduction for all interviewers to use
- Test the guide before using it with actual customers



# Interview Flow



# Guidelines for Conducting Interviews

- Be clear about the purpose of the interview
- Use open-ended, unbiased questions
- Ask simply worded questions
- Let the interviewee do most of the talking
- Listen actively



# Sample Interview Questions

1. What is important to you about \_\_\_\_\_ ?(Name the output of your process, product, or service)
  - If the answer reveals more than one need, ask if some needs are more important than others
2. What problems do you have with our product/service? What impact does it have?
3. How are we performing in the areas you consider important?
4. What do you like about \_\_\_\_\_ ?
5. What can we improve about \_\_\_\_\_ ? What can we do to make your job easier?
6. What specific recommendations would you make to us?

In addition, if your customers are external to your organization, ask questions such as:

7. How do we compare relative to our competitors?
8. How easy is it to do business with us?

## Open-Ended Questions

- The receivables team originally wrote, “Do our invoices meet your requirements?”—a question that asks for a yes or no answer
- The team reworded it as, “How well do our processes and policies meet your requirements? How do they not meet ...?”
- If the customer cannot articulate what is important to him or her, you may want to ask about:
  - Specific products/services by name
  - Sales support—knowledge and helpfulness of sales staff
  - Responsiveness
  - Ease of doing business
  - How use of your product/service has supported the customer’s business goals

# Receivables VOC Plan: Sources

## Sources

Check the data sources you think will be useful for this project.

### REACTIVE SOURCES

- Complaints
- Problem or service hot lines
- Technical support calls
- Customer service calls
- Claims, credits
- Sales reporting
- Product return information
- Warranty claims
- Web page activity
- Other \_\_\_\_\_

### PROACTIVE SOURCES

- Interviews
- Focus groups
- Surveys
- Comment cards
- Sales visits/calls
- Direct observation
- Market research/monitoring
- Benchmarking
- Other Customer contact calls

# VOC Plan: Final Touches

- The last step in developing your data collection plan is to decide specifically which customers you want to interview, how many you want to interview, how you will interview them, and within what time frame the interviews should take place
- If you have a large customer base, these decisions may not be easy; ask for help from your manager, sponsor, advisor, or marketing specialist

## Summary: Which, How Many, How, and When

On the back of this form or a separate sheet, summarize your plans to gather and use both reactive and proactive sources. Indicate how much data you will get, how you will get it, and when. Include, for instance, the number of interviews or surveys you plan to use, which customers you will contact, when you will start and end the data collection, and so on.

# Receivables VOC Plan: Summary

## Summary: Which, How Many, How, and When

On the back of this form or a separate sheet, summarize your plans to gather and use both reactive and proactive sources. Indicate how much data you will get, how you will get it, and when. Include, for instance, the number of interviews or surveys you plan to use, which customers you will contact, when you will start and end the data collection, and so on.

- Will look at five segmentation characteristics. Try to do at least 20 face-to-face interviews first, then follow up phone interviews and/or surveys for additional data. Work with statistician to identify appropriate sample size needed from base of 5,000 customer organizations.
  - Carlos and LaShawn will pull together current reactive data we already have.
  - Tina will work on the bookstore angle.
  - Maury will work with customer service on adding info to customer contact calls.
- Work to begin this Friday and extend for three weeks. Deadline is July 26.

# Your Own VOC Plan: Sources and Summary

## Instructions:

- Check off each of the VOC data sources you think could provide you with information relevant to your project
  - Indicate which already exist and which you may need to create
  - You may not want or need to use all the sources listed, so it's OK to leave some items unchecked
- Complete your VOC plan by jotting down your preliminary thoughts about exactly how you will collect the VOC data
- You may want to take notes on the portion of the worksheet reprinted below and then transfer your final decisions to Project Worksheet: VOC Data Collection Plan

**Time:** 15 minutes

# Gathering the VOC Data

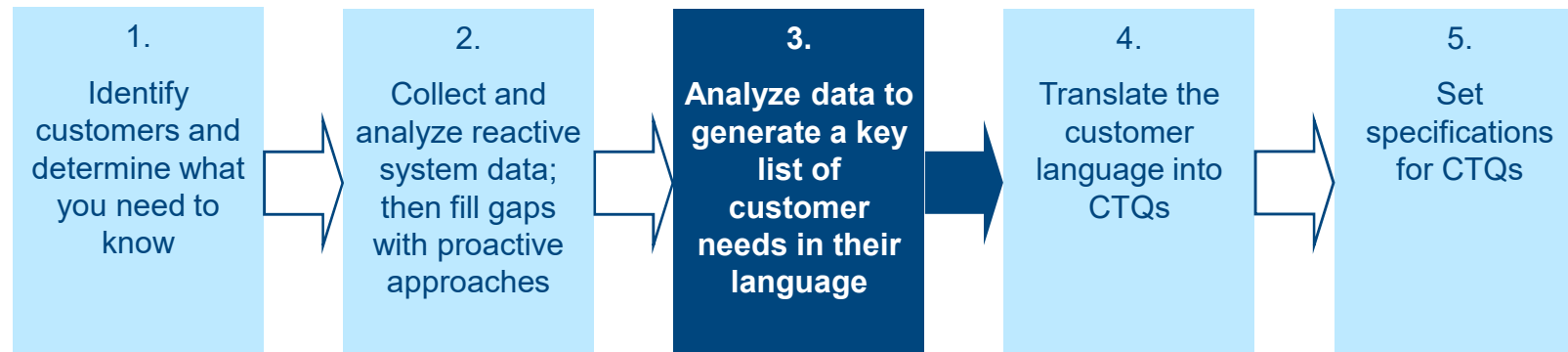
- Train everyone who will be gathering the VOC data
- Practice the interviews before talking with external customers
- Periodically monitor the data collection; make adjustments as necessary

# Anexas Consultancy Services

## VOC Data Collection, Step 3

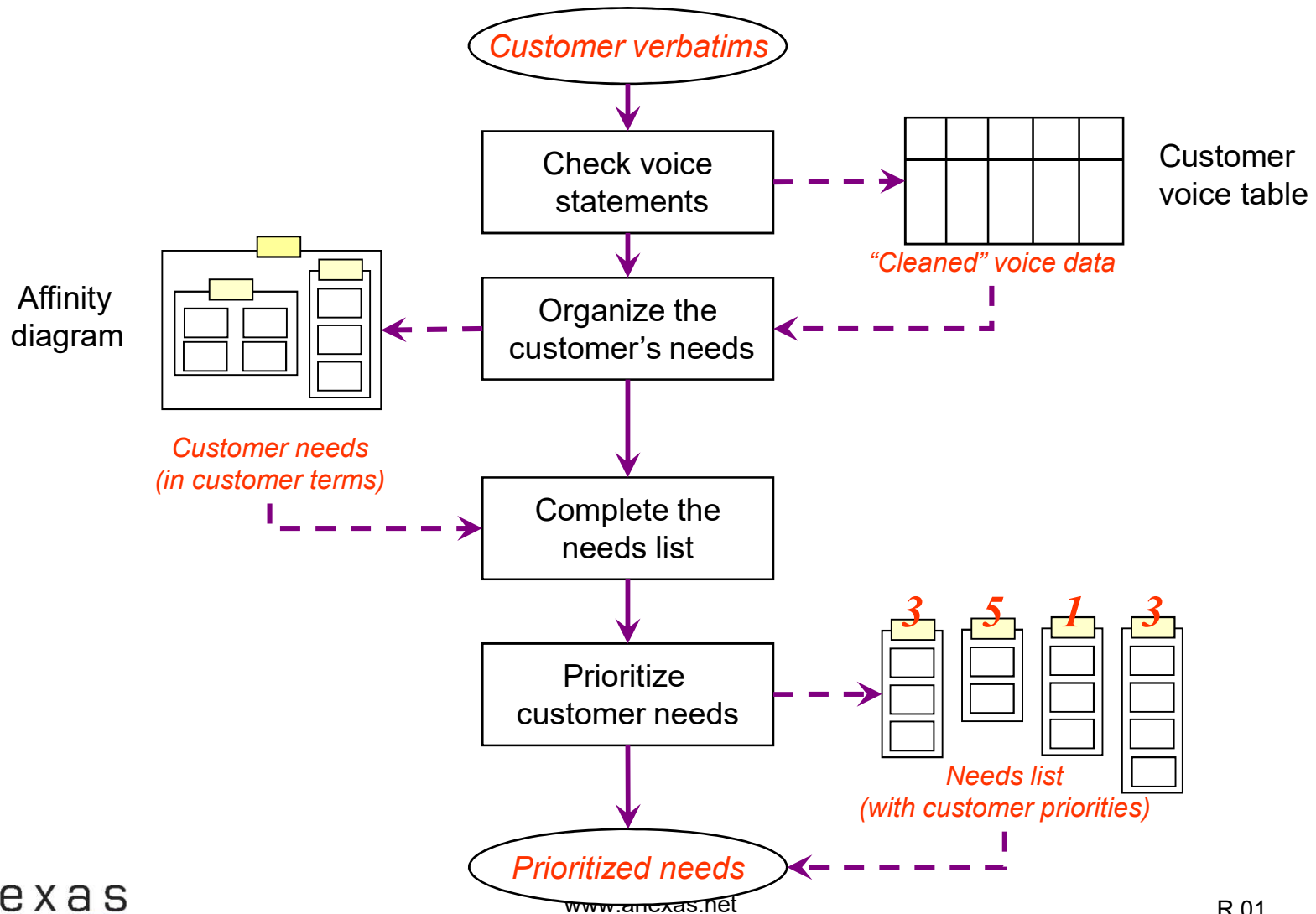


## Step 3: Analyzing Customer Data



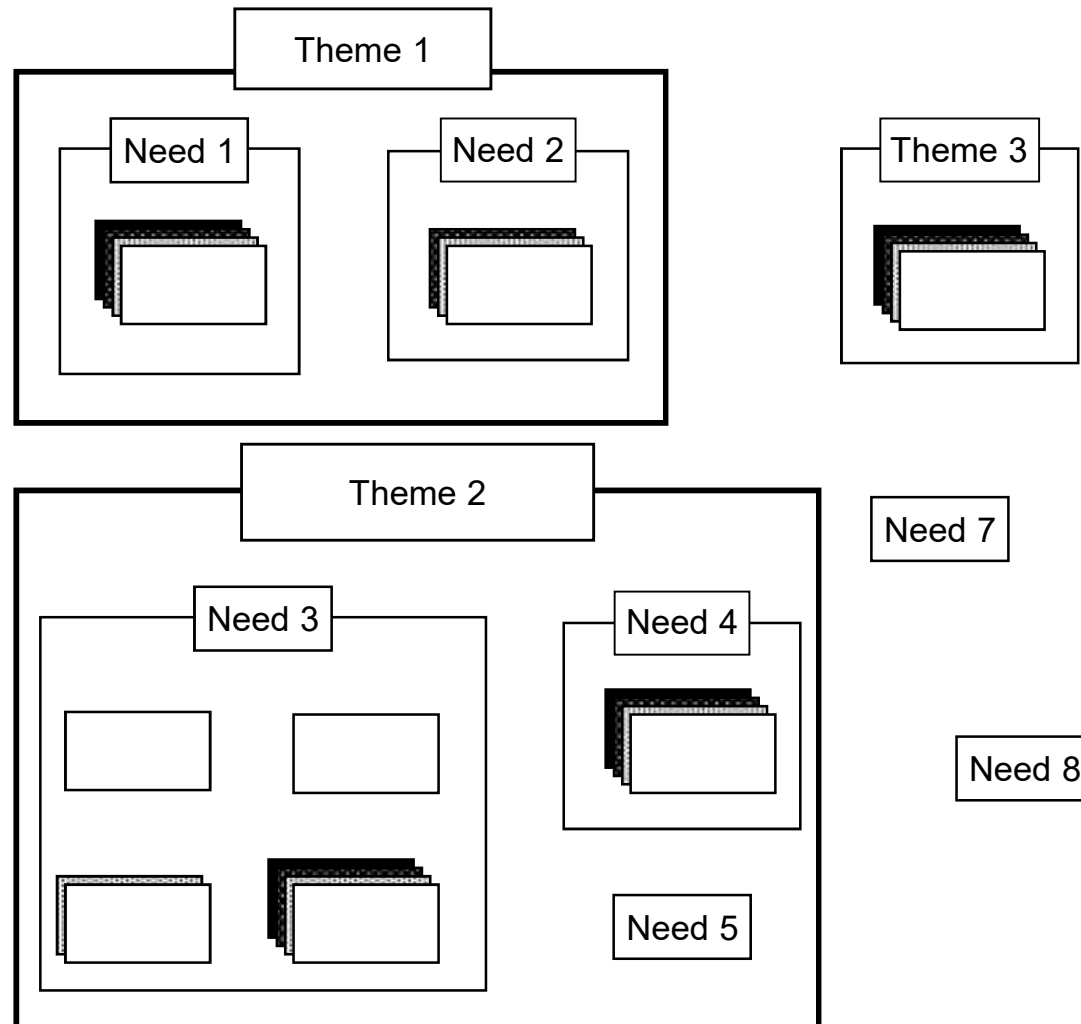
- **Goal**
  - Generate a list of key customer needs in their language
- Much of the data you gathered in the previous steps will be verbal data
  - It is helpful to summarize this information in a meaningful way, perhaps by using an affinity diagram (described next)

# Analyze Needs Data: Process



# Affinity Diagram: Definition

An affinity diagram is a tool that organizes language data into related groups



# Affinity Diagram Uses

- Help grasp very large or complex issues
- Encourage breakthrough thinking
- Find patterns in mountains of data
- Gather large amounts of language data
- Organize ideas, issues, and opinions

# Example: Computerized Training, cont.

<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>	<b>E</b>
System is easy to use	Enjoyable learning experience	System works for all types of students	Students understand skill set	Students master the skill set
System uses appropriate delivery medium	Students enjoy the experience	The system works for different entry skill levels	Students can decide when this skill will work	Students get high-quality feedback on mastery
System uses web technology	Learning isn't a hassle	The system works for varying levels of motivation	Students understand skill set	Students get enough practice
The presentation process feels familiar	Practice doesn't feel embarrassing	The system accommodates different learning styles	Students can do this	Students leave with new skills
System delivery is low cost				

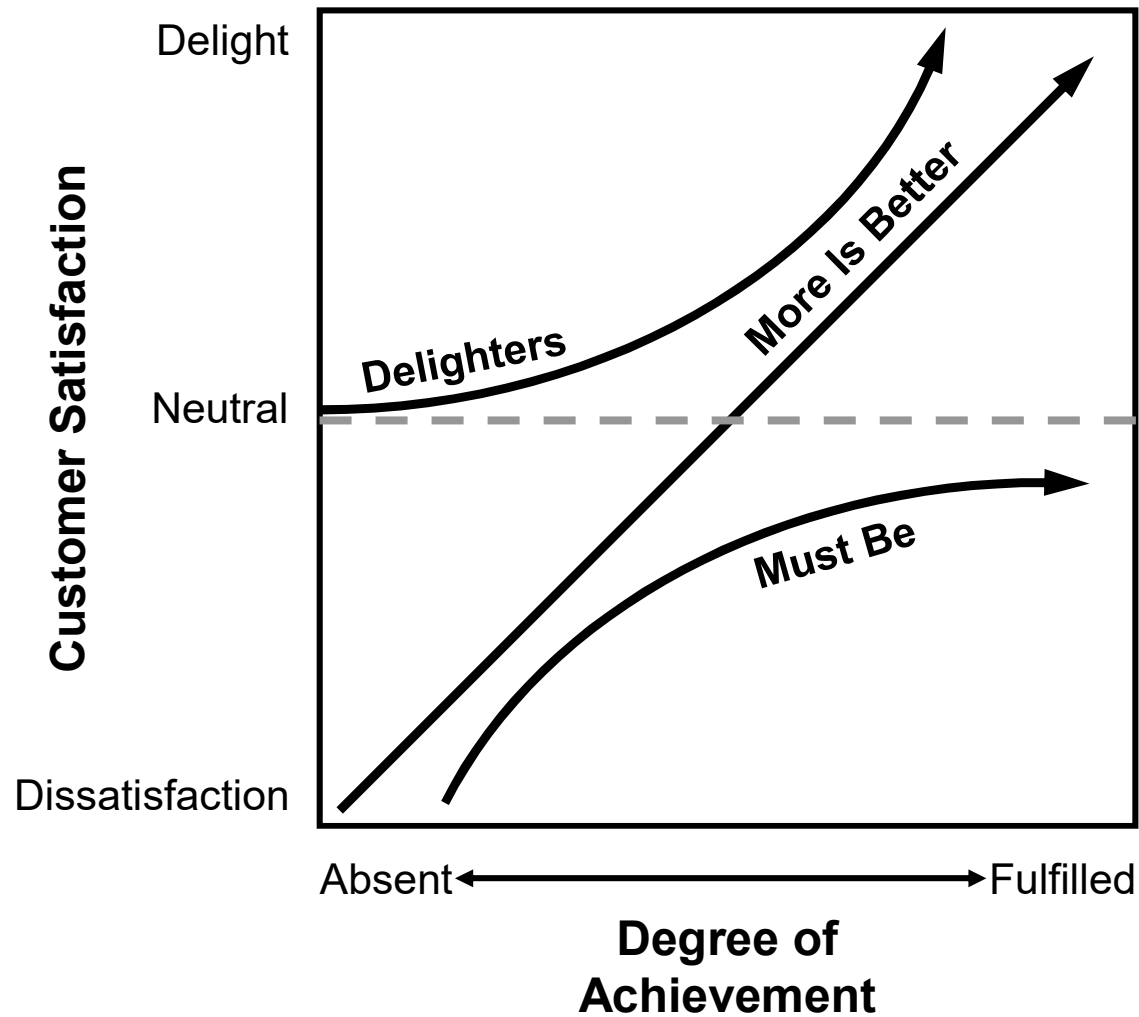
# Tips for Affinity Diagrams

- **Silent Process:** Move cards without talking
  - Encourages unconventional thinking
  - Discourages semantic battles
- **Gut Reaction:** React, do not contemplate
  - If you do not like where a card is, shift it
  - Settles eventually into consensus
  - Duplicate cards that keep shifting and put in both groups
- **Avoid Pigeonholing:** Groups emerge from “the chaos of the cards”
  - Do not sort cards into predetermined categories (pigeonholes)
  - Pigeonholes force-fit everything into existing logic
  - Pigeonholes prevent breakthrough from occurring
- **Group Size:** There can be a “group” of one card, but ask...
  - Should a small group be combined with another?
  - Should a large group be broken down more precisely?

# Generating Customer Need Statements

- The need statements used for an affinity diagram should be in the customer's own language if at all possible
- To get these statements, read through interview notes, survey notes, comment cards, etc., and highlight any statements you think are relevant to your project
- Transcribe the strongest of these statements to cards or self-stick notes you can use for the affinity diagram

# What You Can Learn: The Kano Model





# Exercise: Using the Kano Model

**Objective:** Practice applying the Kano classifications to actual products and services

**Instructions:**

- Identify customer needs related to each of the products or services listed below and divide them based on the three Kano classifications
- Do the first example together with the whole group, then work on your own or in pairs
- Be prepared to discuss your answers with the whole group

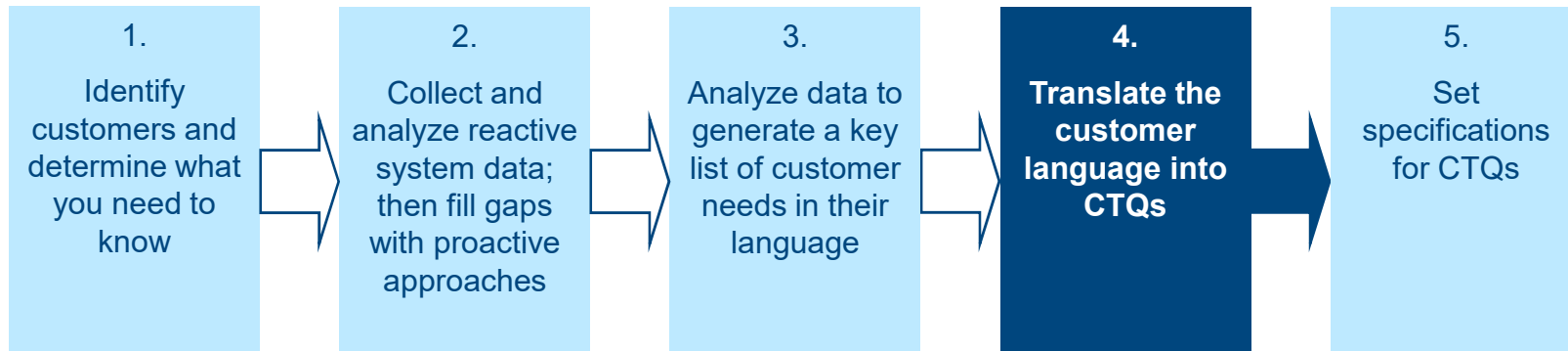
**Time:** 10 minutes

	Must Be	More Is Better	Delighters
Television Set			

# Anexas Consultancy Services

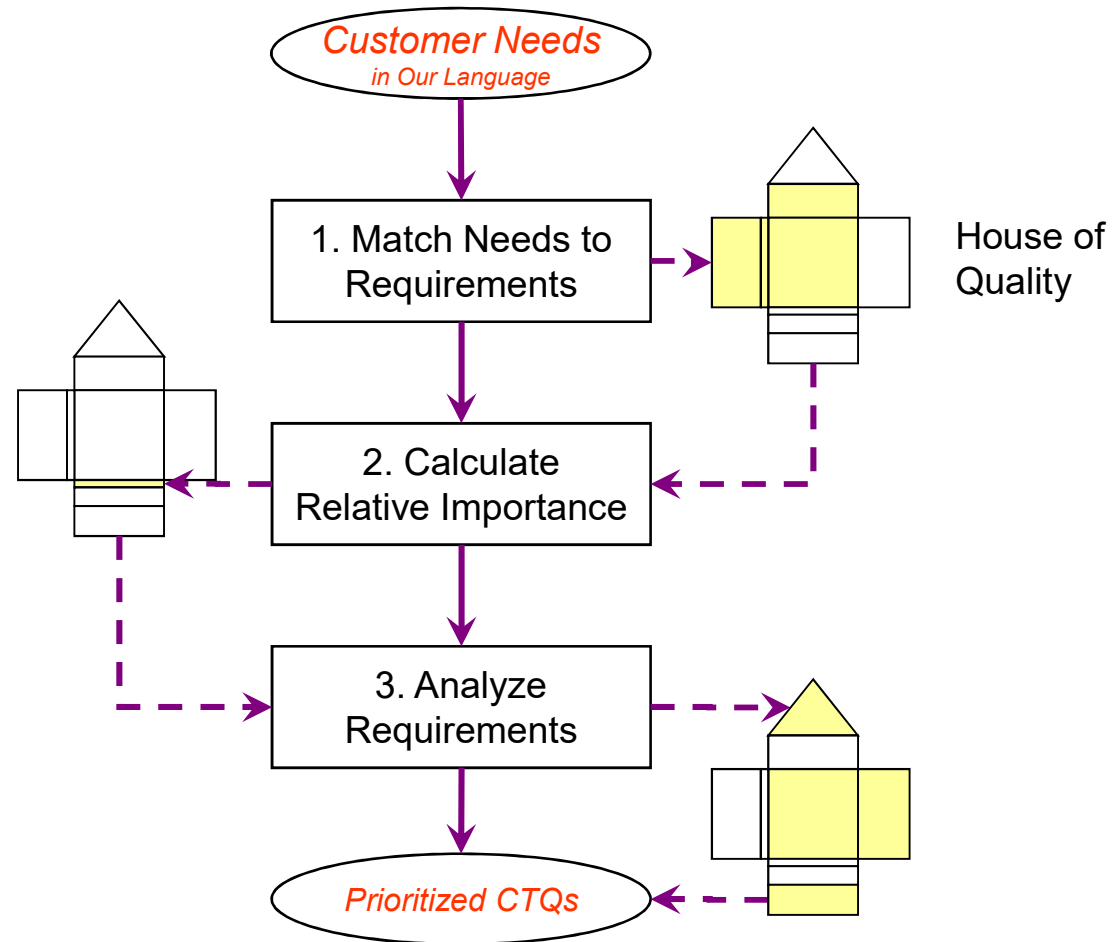
## VOC Data Collection, Step 4

## Step 4: Defining CTQs

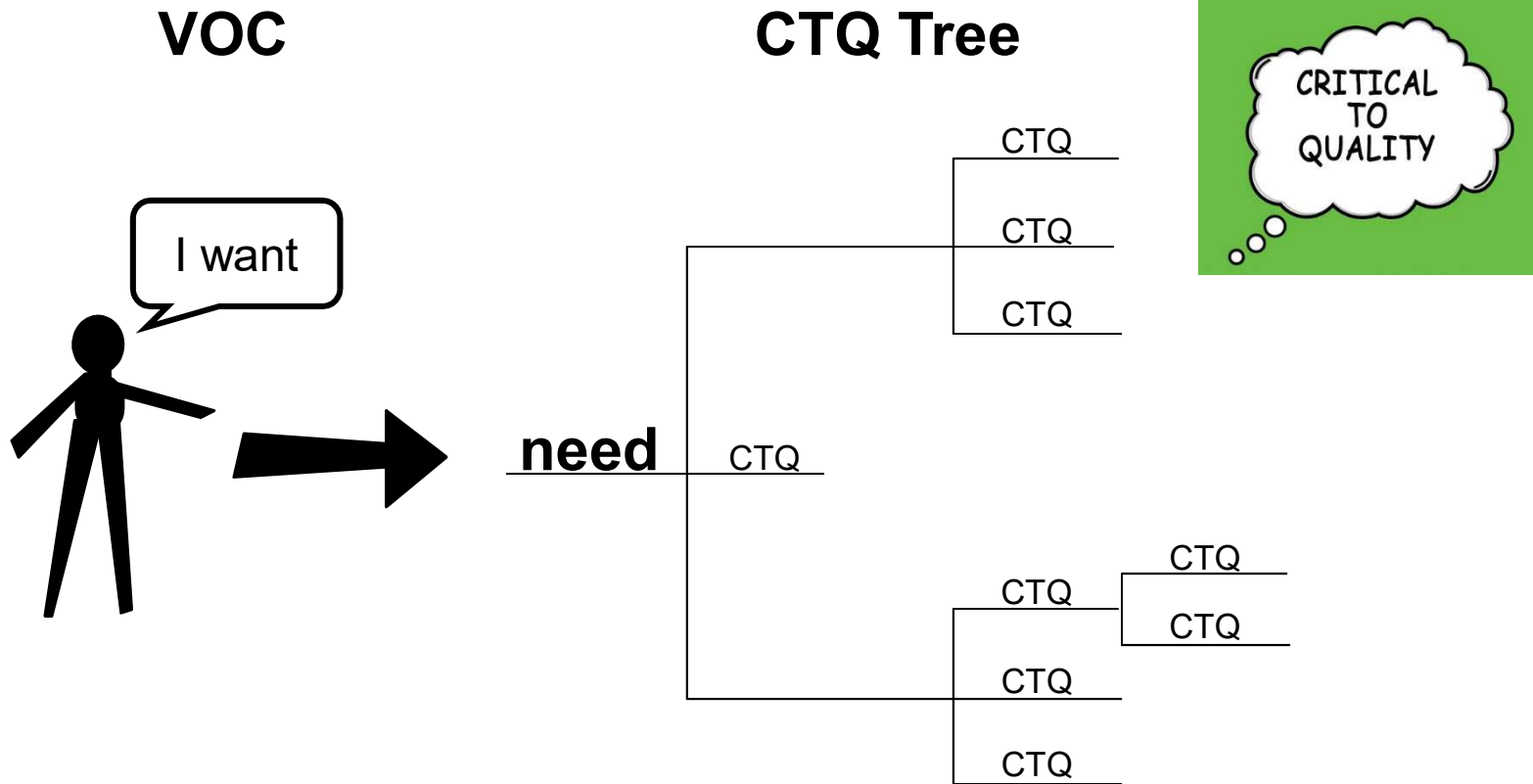


- A useful CTQ will have the following characteristics
  - It is truly critical to the customer’s perception of quality
  - It can be measured
  - A specification can be set by which to determine whether or not the CTQ has been achieved
- Shown in this section are two tools for translating the VOC into CTQs:
  - CTQ tree
  - Quality function deployment (QFD)

# Identify and Prioritize CTQs: Process

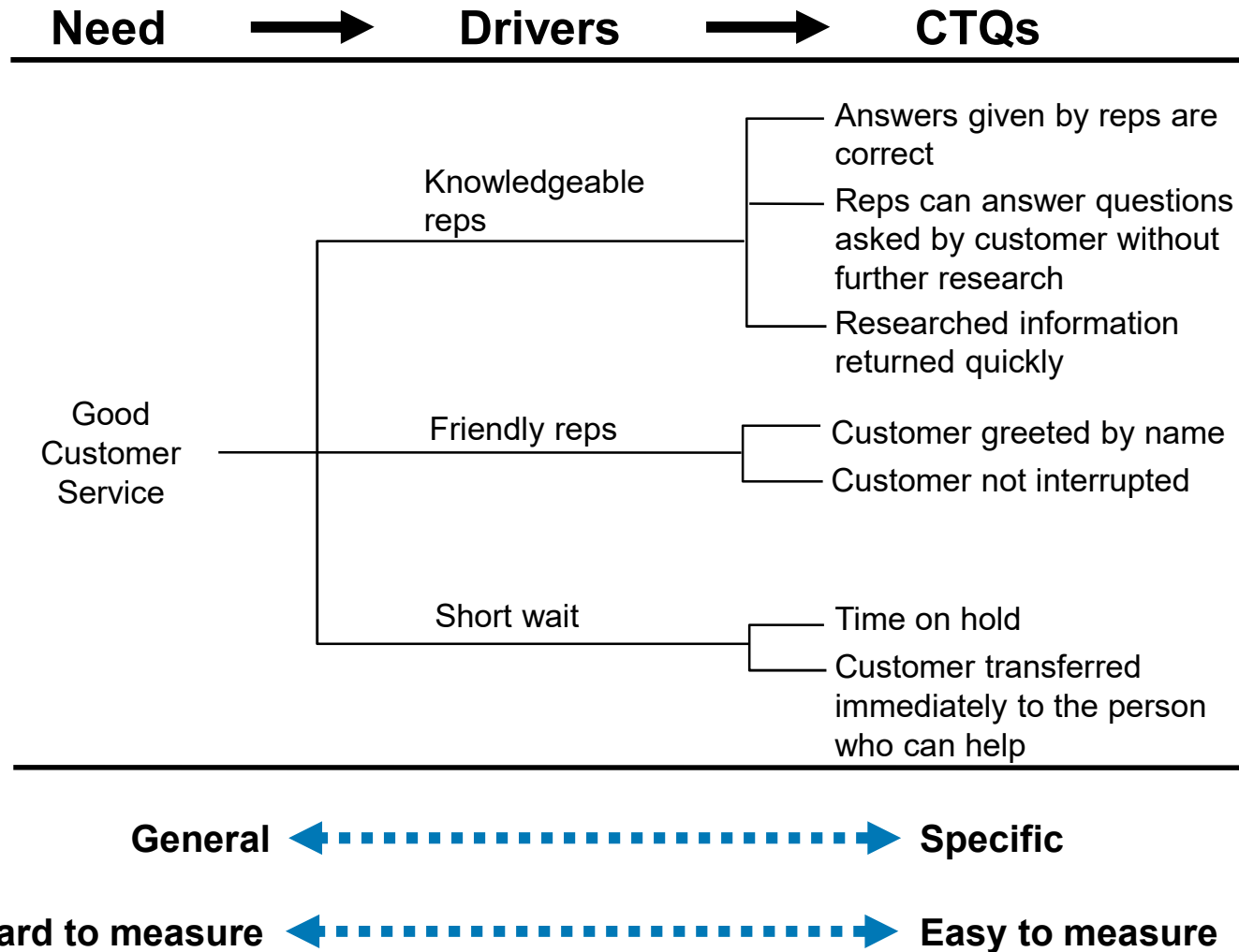


# Translating VOC into CTQs



(CTQ = Critical to Quality)

# Example: CTQ Tree



# Project Worksheet: Generating CTQs

## Instructions:

- Use the blank tree diagram to translate a customer need from your affinity diagram into a CTQ requirement
- Be prepared to discuss your work with the whole group
- You can use the form reprinted here or the copy in the separate booklet of worksheets

**Time:** 15 minutes

# What Is Quality Function Deployment?

- QFD (also called House of Quality) is a graphical tool summarizing information about customer requirements
- QFD is a good way to check the CTQs against customer requirements and use competitive data to help decide how to prioritize the CTQs



# Anexas Consultancy Services

## Quality Function Deployment (QFD)

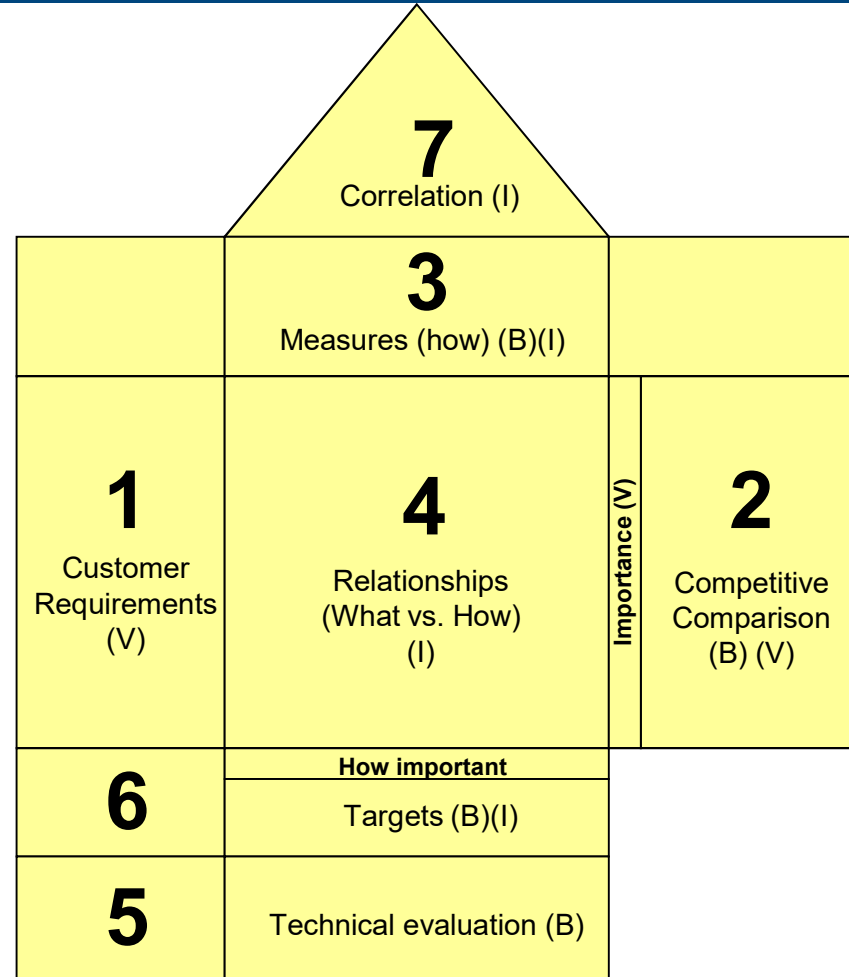
# What Is Quality Function Deployment?

- QFD (also called House of Quality) is a graphical tool summarizing information about customer requirements
- QFD is a good way to check the CTQs against customer requirements and use competitive data to help decide how to prioritize the CTQs

# House of Quality

- Here we break the house into seven rooms
- Just like rooms in a house, there are connections and relationships among the rooms

(V) = Comes from VOC information  
 (B) = Comes from benchmarking information  
 (I) = Comes from internal expertise



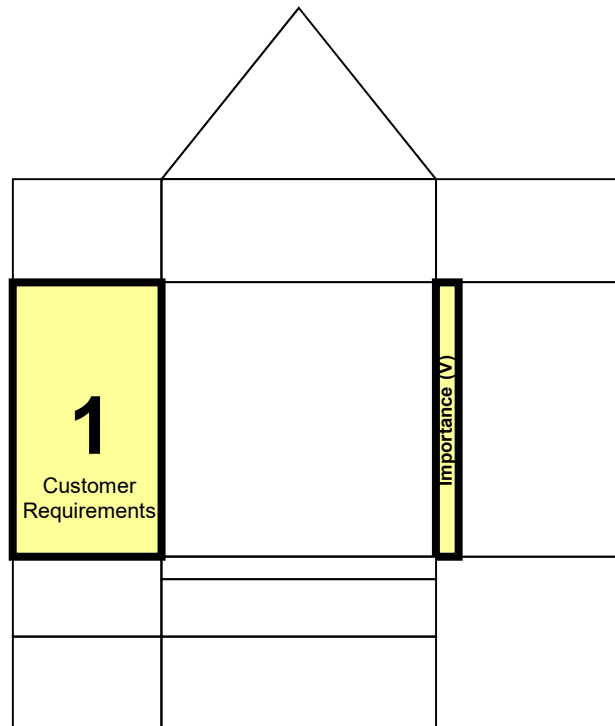
# Example: QFD—Lunch Preparation

## *Lunch Preparation—QFD Example*

	CTQ Measures								Importance of Needs
	Weight of Portion	% Nutrition Requirements	Met Blindfolded Jury Taste Test Rating	Ingredient Cost	Time to Prepare	Number of Ingredient Measurements	Number of Dishes Used	Solubility of Ingredients in Soap and Water	
Fills Us Up	9	1	1						5.00
Is Nutritious	1	9		1					3.40
Tastes Good	0	0	9	1					3.30
Easy to Make				1	9	9			3.70
Easy to Clean Up							9	3	2.00
Sticks with Us	9			1					4.00
Does Not Cost Much	3			9					1.00
Does Not Make a Mess						3	9		2.30
<b>Importance of CTQs</b>	<b>87</b>	<b>36</b>	<b>35</b>	<b>23</b>	<b>33</b>	<b>40</b>	<b>39</b>	<b>6</b>	

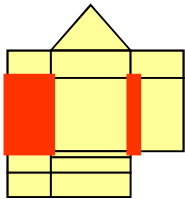
# A: Summarize CTQs (in QFD)

## Room 1: Customer Requirements



- Room 1 lists the key customer needs identified by VOC research
- The needs are defined by customers and are in customer language
- The prioritization of needs is also done by customers

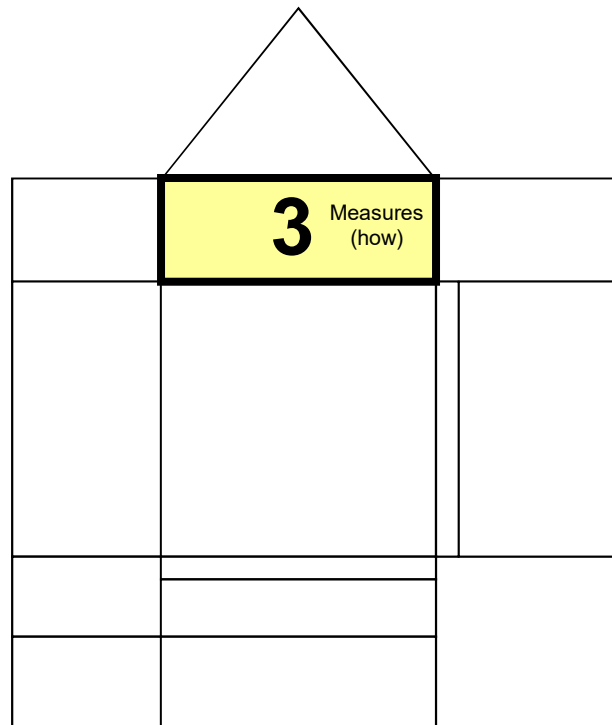
# Example: M&S Inc.—“Easy Order Placement”— QFD Room 1



**Customer Needs**

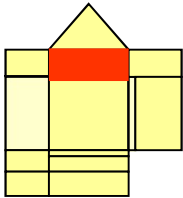
								Importance
<b>I need to place orders when I want</b>								4.30
<b>I need to place orders from any location</b>								3.90
<b>I need to be able to place orders using different technologies</b>								4.80
<b>I want an easily understandable process</b>								4.90
<b>I don't want to go through many steps</b>								4.70
<b>I want to know immediately if an order is confirmed</b>								4.20
<b>I need to be able to get help if I have questions</b>								4.00
<b>How Important</b>								

# Room 3: Measures



- Room 3 lists the measures developed at the end of the VOC analysis
- The ideas for measures came from:
  - The design team’s work translating customer needs into CTQs (which include measures)
  - Benchmarking information on how similar characteristics are measured
  - Measures currently being used for similar designs

# Example: M&S Inc.—“Easy Order Placement”— QFD Room 3

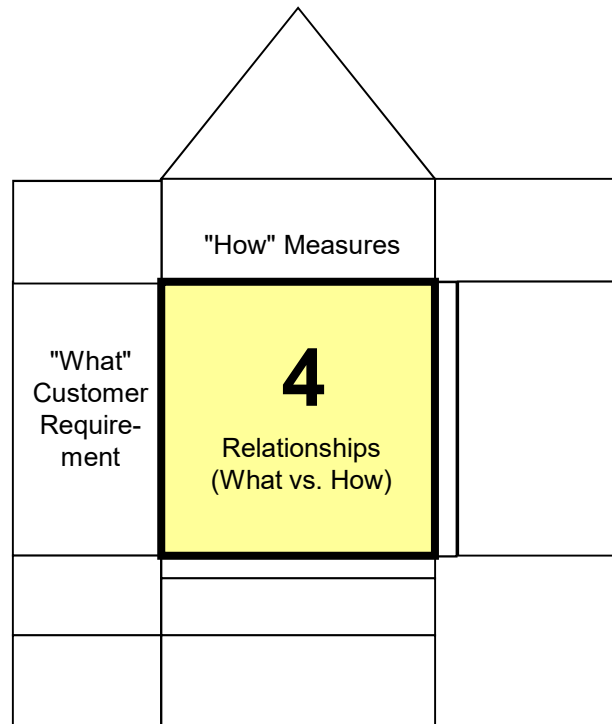


Customer Needs	% of customer-desired technologies supported	# of hours help facility is available	# of process steps	% information about completed order status accessible by customer	% historical information not requiring reentry	# of hours access is available	Importance
I need to place orders when I want							4.30
I need to place orders from any location							3.90
I need to be able to place orders using different technologies							4.80
I want an easily understandable process							4.90
I don't want to go through many steps							4.70
I want to know immediately if an order is confirmed							4.20
I need to be able to get help if I have questions							4.00
<b>How Important</b>							



# B: Prioritize CTQs (with QFD)

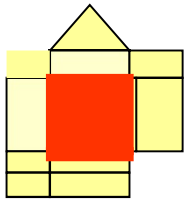
## Room 4: Relationships



- Room 4 summarizes the relationship between potential measures and the customers' needs
- To determine the relationship, compare each measure with each need and ask:

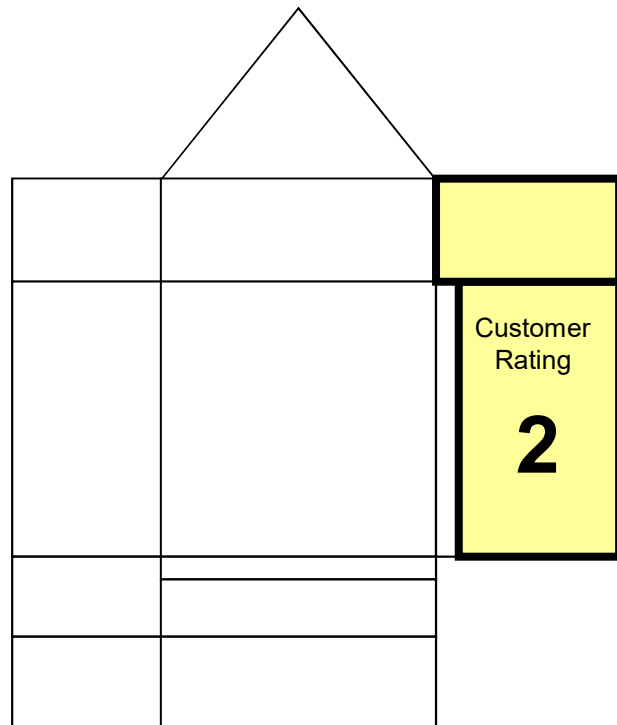
**“If the design meets the target set for this measure, to what extent will the customer need be met?”**

# Example: M&S Inc.—“Easy Order Placement”— QFD Room 4



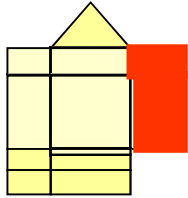
Customer Needs	% of customer-desired technologies supported	# of hours help facility is available	# of process steps	% information about completed order status accessible by customer	% historical information not requiring reentry	# of hours access is available	Importance
I need to place orders when I want	3	3				9	4.30
I need to place orders from any location	3						3.90
I need to be able to place orders using different technologies	9						4.80
I want an easily understandable process			3		3		4.90
I don't want to go through many steps			9		9		4.70
I want to know immediately if an order is confirmed				9			4.20
I need to be able to get help if I have questions		9		3		3	4.00
<b>How Important</b>	68	49	57	50	57	51	

## Room 2: Competitive Comparison

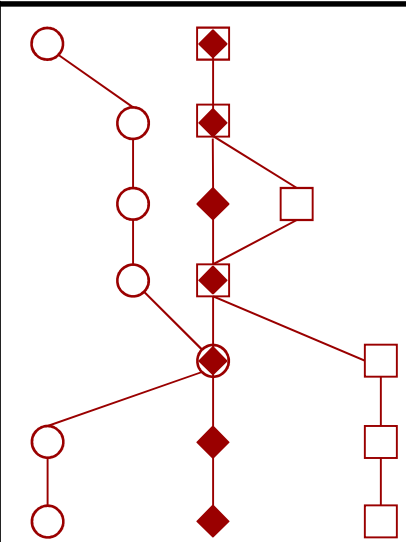
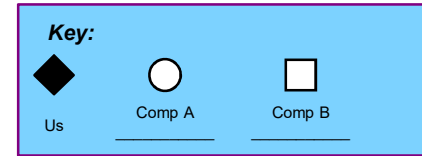
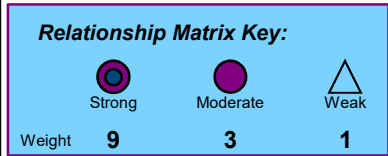


- Room 2 graphically shows how customers perceive your organization compared with competitors with regard to meeting the customer needs listed in Room 1
- Your organization and at least two competitors are compared against customer needs
- The information used in the comparison can come from VOC research and from benchmarking studies

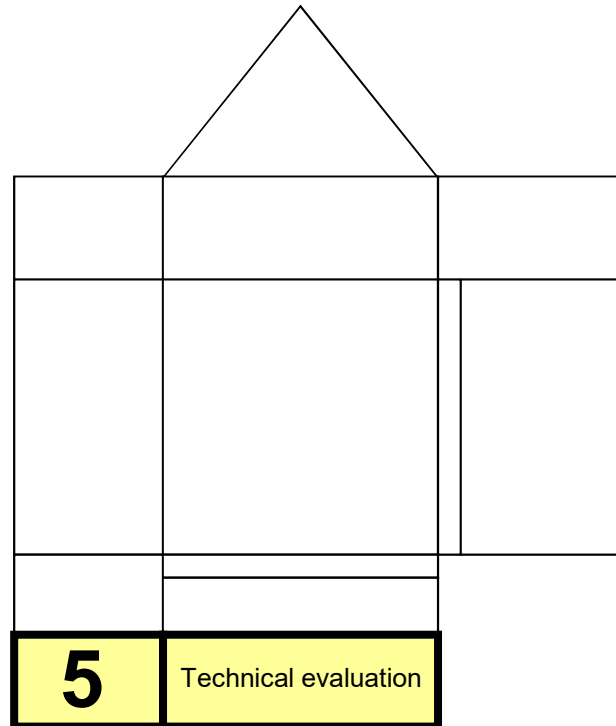
# Example: M&S Inc.—“Easy Order Placement”— QFD Room 2



Customer Needs	CTQ Measures	CTQ Measures						Importance	Competition Comparison					
		% of customer-desired technologies supported	# of hours help facility is available	# of process steps	% information about completed order status accessible by customer	% historical information not requiring reentry	# of hours access is available		1	2	3	4	5	
I need to place orders when I want		3	3				9	4.30						
I need to place orders from any location		3						3.90						
I need to be able to place orders using different technologies		9						4.80						
I want an easily understandable process				3		3		4.90						
I don't want to go through many steps				9		9		4.70						
I want to know immediately if an order is confirmed						9		4.20						
I need to be able to get help if I have questions			9		3		3	4.00						

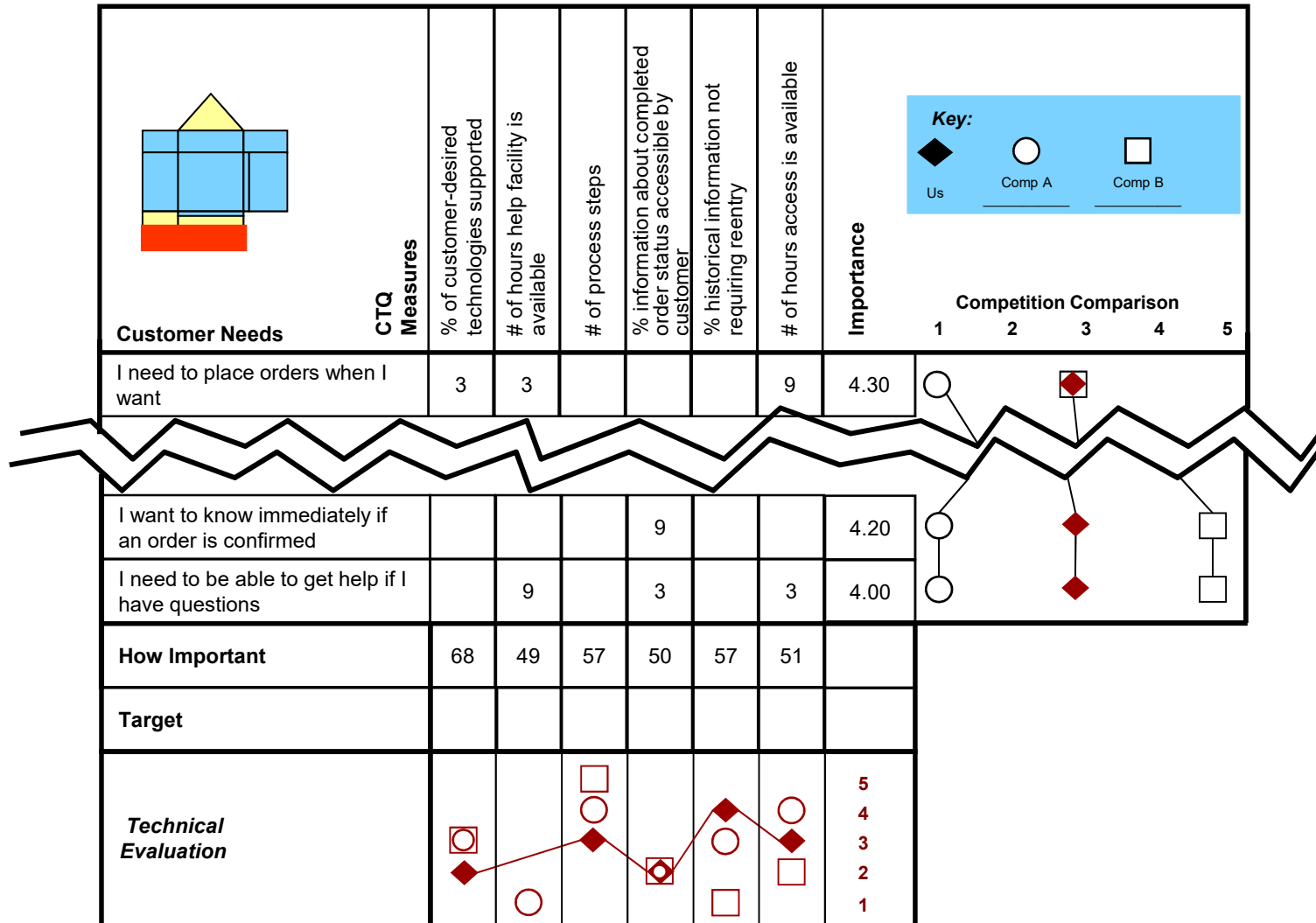


# Room 5: Technical Evaluation

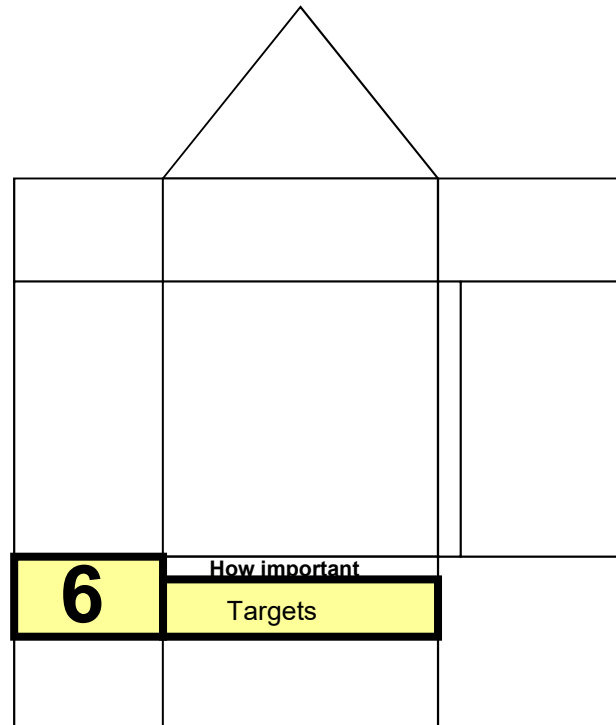


- Room 5 summarizes technical benchmarking data on how your company compares with competitors with respect to performance on key measures/design requirements

# Example: M&S Inc.—“Easy Order Placement”— QFD Room 5

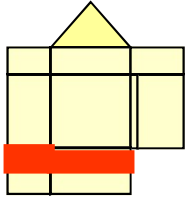


# Room 6: Targets



- Room 6 summarizes the targets established for the measures/ design requirements
- Targets were established in the VOC analysis where the team identified the level of performance required to meet customer needs

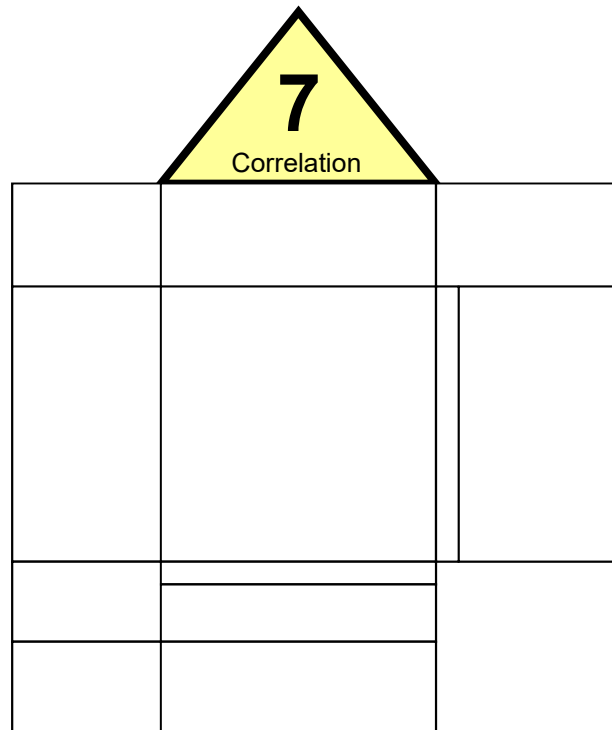
# Example: Order Processing— QFD Room 6



CTQ's/ Measures	CTQ's/ Measures						Importance
	% of customer-desired technologies supported	# of hours help facility is available	# of process steps	% information about completed order status accessible by customer	% historical information not requiring reentry	# of hours access is available	
I need to place orders when I want	3	3				9	4.30
I need to place order from any location	3						3.90
I need to be able to place orders using different technologies	9						4.80
I want an easily understandable process			3		3		4.90
I don't want to go through many steps			9		9		4.70
I want to know immediately if an order is confirmed				9			4.20
I need to be able to get help if I have questions		9		3		3	4.00
<b>How Important</b>	68	49	57	50	57	51	
<b>Target</b>	<b>100</b>	<b>24</b>	<b>4</b>	<b>75</b>	<b>90</b>	<b>24</b>	

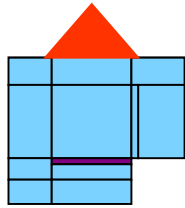


# Room 7: Correlation

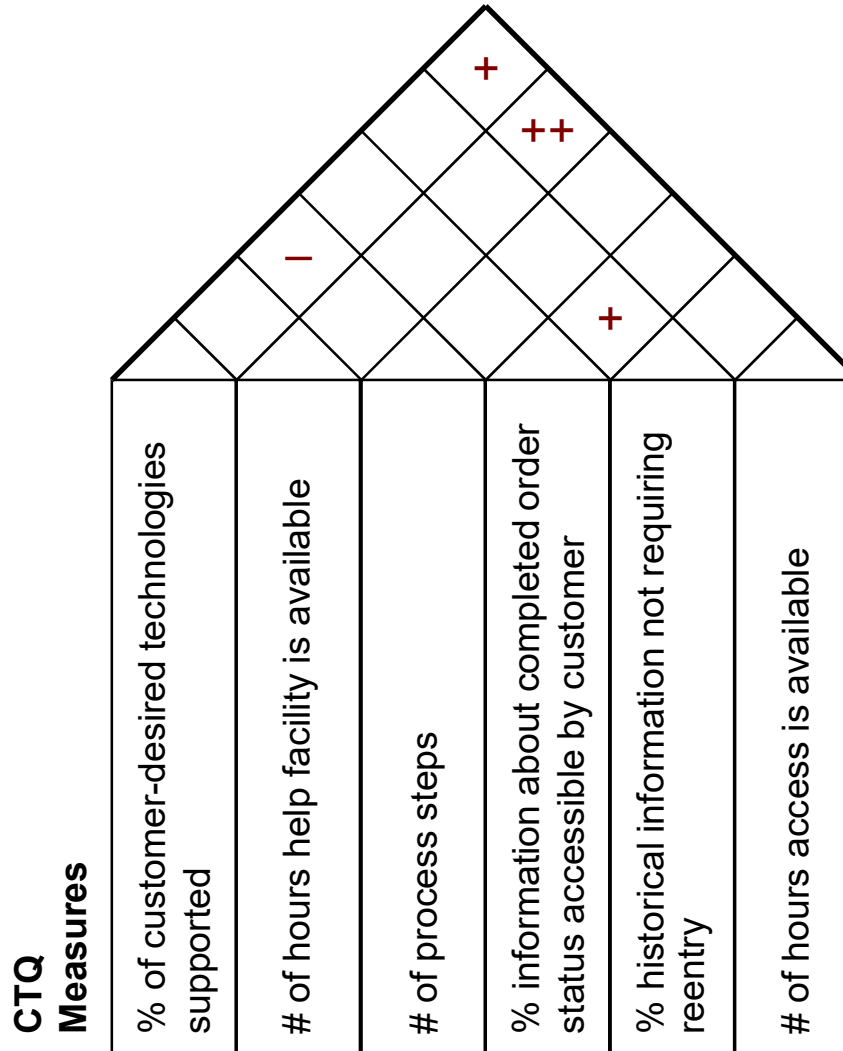


- In Room 7, relationships between measures (the “roof”) are summarized
- Use symbols to represent relationships
- Evaluate the outcome
  - Positive relationships indicate synergy
  - Negative relationships may indicate conflicts
- Key question: If we design to meet CTQ1, to what extent do we satisfy CTQ2?

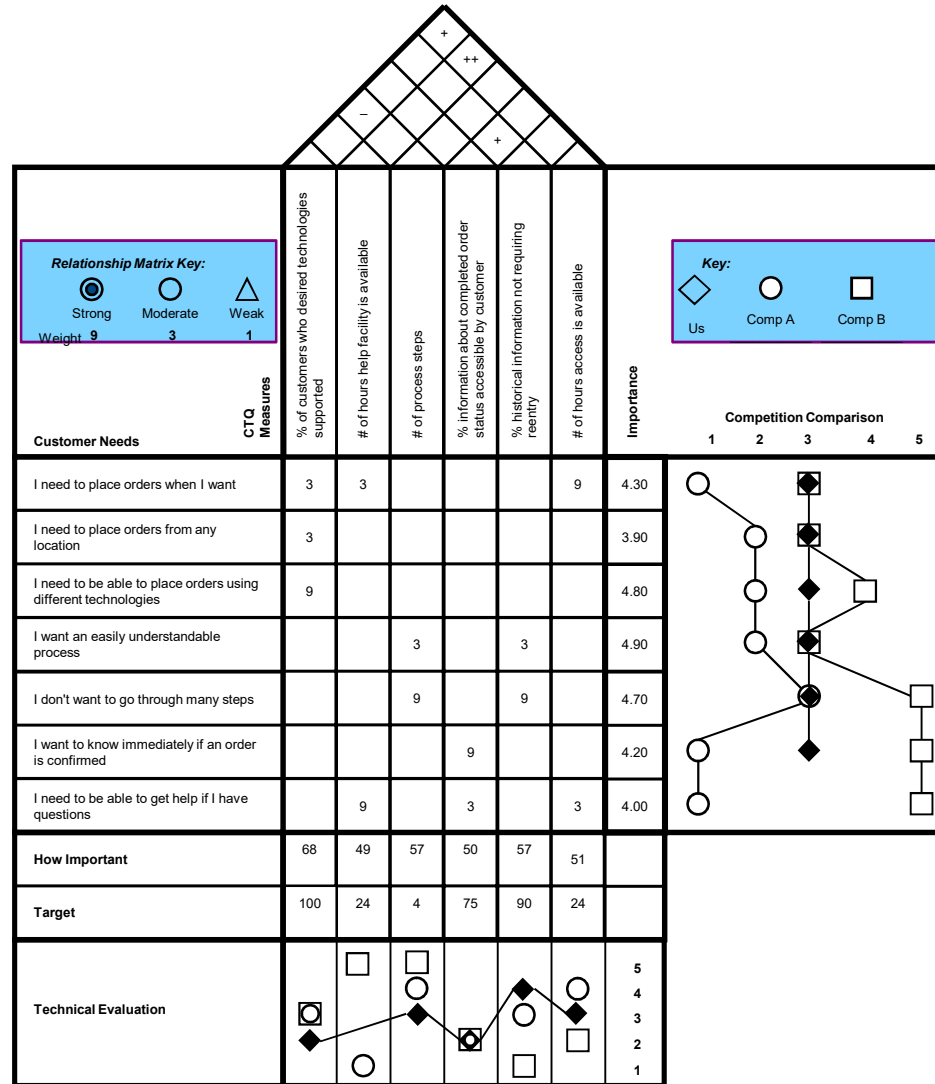
# Example: M&S Inc.—“Easy Order Placement”— QFD Room 7



Relationship	
++	Strong positive
+	Positive
-	Negative
XX	Strong negative



# Example: M&S Inc.—“Easy Order Placement”— Completed HOQ

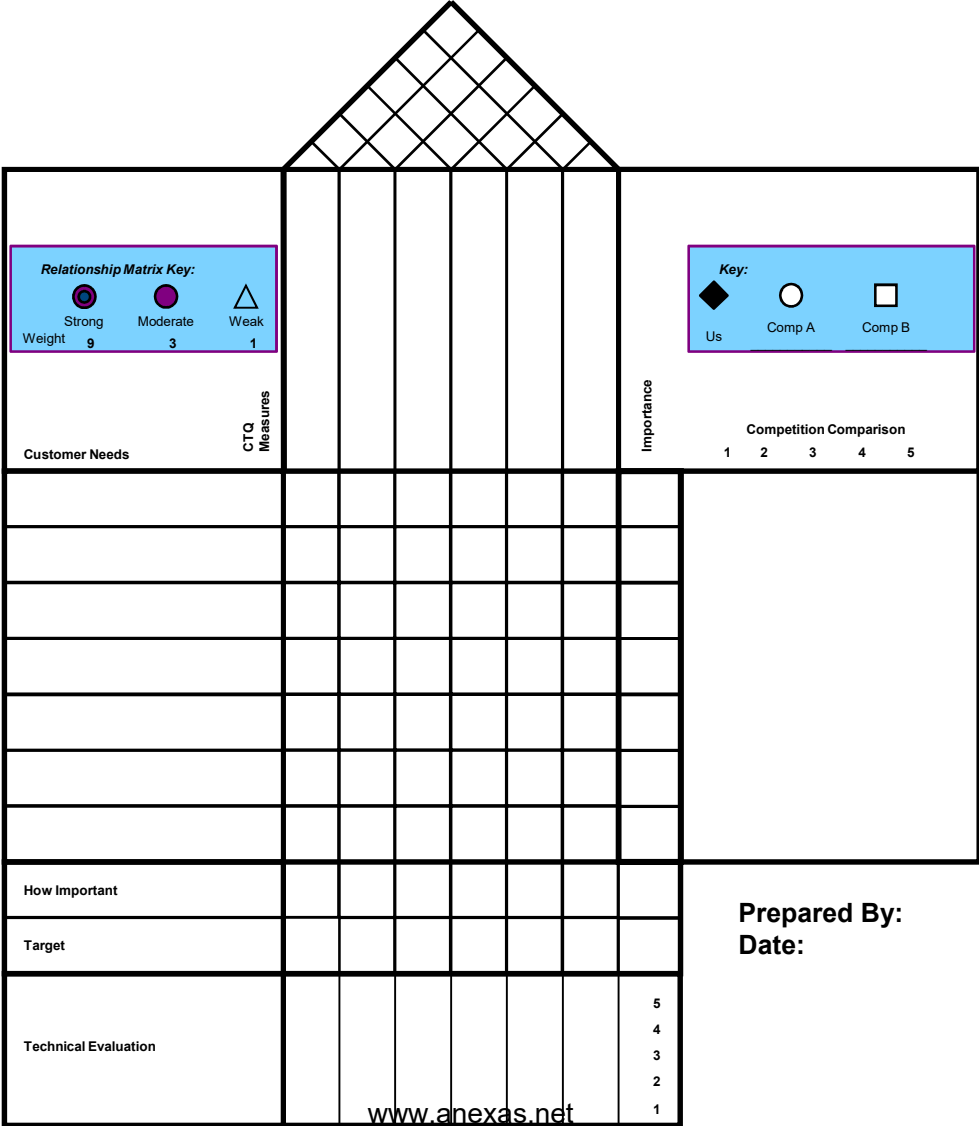


# Example: M&S Inc.—“Easy Order Placement”— QFD Output

Order placement: first-level need

CTQ Measures	Importance	Target	Upper Spec Limit	Lower Spec Limit
% of customer-desired technologies supported	68	100	N/A	75
# of hours help facility is available	49	24	N/A	21
# of process steps	57	4	6	2
% information about completed order status accessible by customer	50	75	N/A	60
% historical information not requiring reentry	57	90	N/A	75
# of hours access is available	51	24	N/A	21

# QFD Template



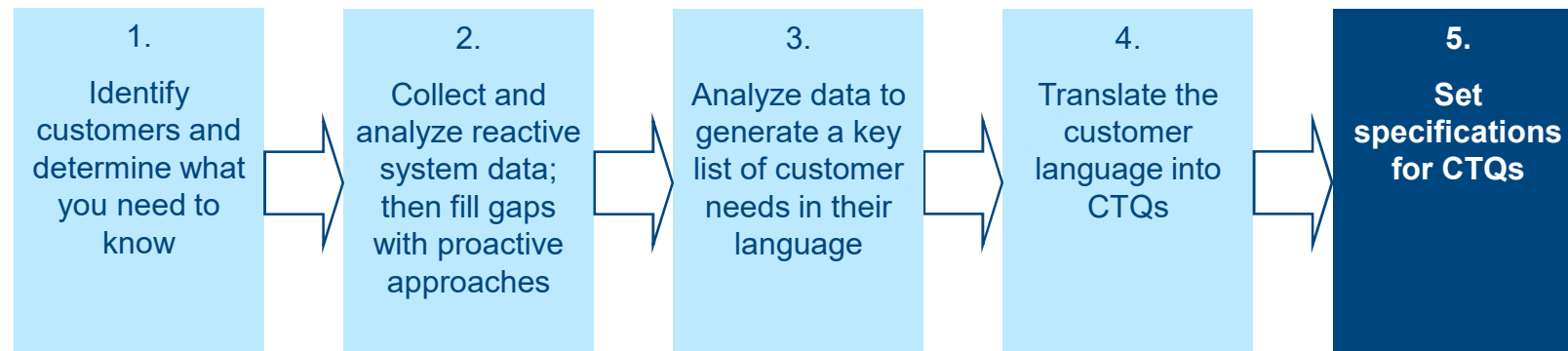
## Selecting CTQs to Focus On

- Which will have the greatest positive impact on the customer?
- Which are within your scope or process area of focus?
- Which of the Kano Must Be characteristics are not addressed?

# Anexas Consultancy Services

## VOC Data Collection, Step 5

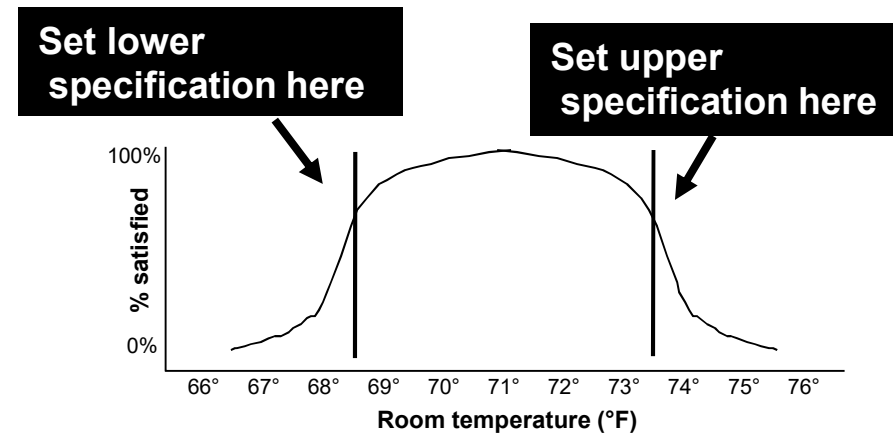
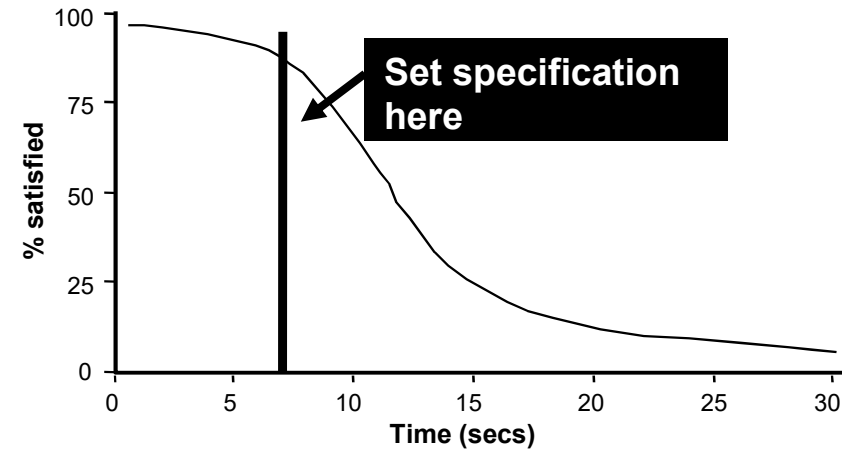
# Step 5: Setting Specifications





# Specifications for CTQs

- In manufacturing, specification limits often come from technical or mechanical requirements
- Otherwise, base specification limits on data about customer needs
- Set specifications at the places where customer satisfaction starts to fall off appreciably



# Exercise: Setting Specifications

**Objective:** Practice establishing specification limits that relate to customer requirements

**Instructions:**

- The sample CTQ tree shown earlier is reprinted below
- Select at least one CTQ for each driver and use the space on the next page to indicate how you would go about setting specifications limits for it
- Be prepared to discuss your answers with the whole group

**Time:** 10 minutes

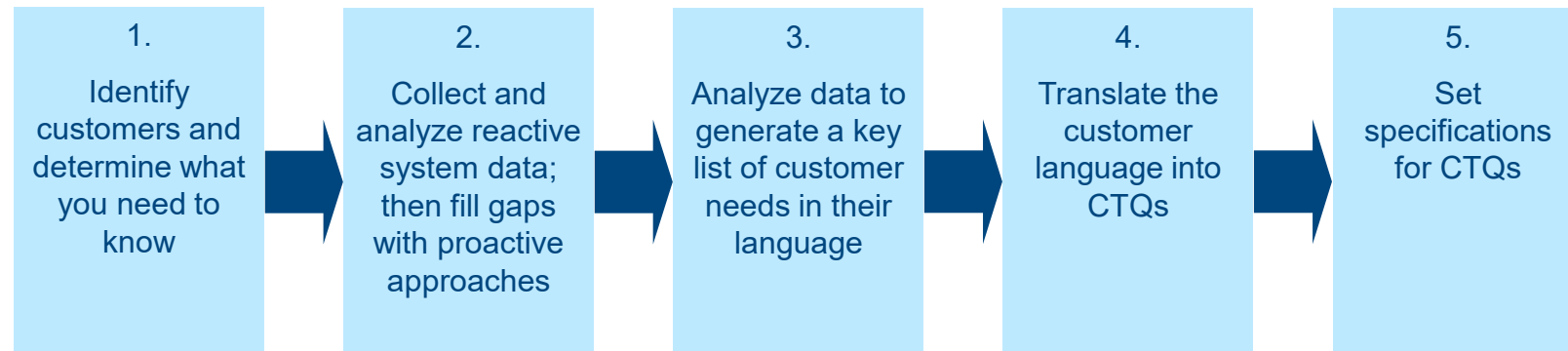
# Exercise: Worksheet

## CTQ

- Answers are correct
- Reps can answer question
- Information returned quickly
- Customer greeted by name
- Customer not interrupted
- Time on hold
- Transfers immediate

## Potential Specification Method

# Summary: The Five-Step VOC Data Collection Process



By following this process, we can help ensure that we have understood the current situation from the customer's perspective

# Revisiting Your Charter

- Do you still think customers would consider the project a high priority?
  - If not, can you modify your focus to make it more relevant to customer needs?
  - If you can't answer the question, what additional information can you collect to determine how the customer might perceive your project?
- Do you have enough data to justify working on this project?
  - If not, can you get timely information so you can keep your project moving forward?
  - Do you need to modify your timeline to include some customer research up front? Whose approval would you need to make that adjustment?

# Anexas Consultancy Services

**DEFINE: Define the Project**

Review

# You Were Here



# DEFINE Phase Deliverables

- Product or process linked to strategic business requirements
- Clear problem and goal statements, and business benefits
- High-level financial benefits approved by finance analyst
- High-level process map of major process steps (SIPOC)
- Customer critical to quality (CTQ) requirements
- Linkage of customer requirements to process outputs



## DEFINE Phase Deliverables, cont.

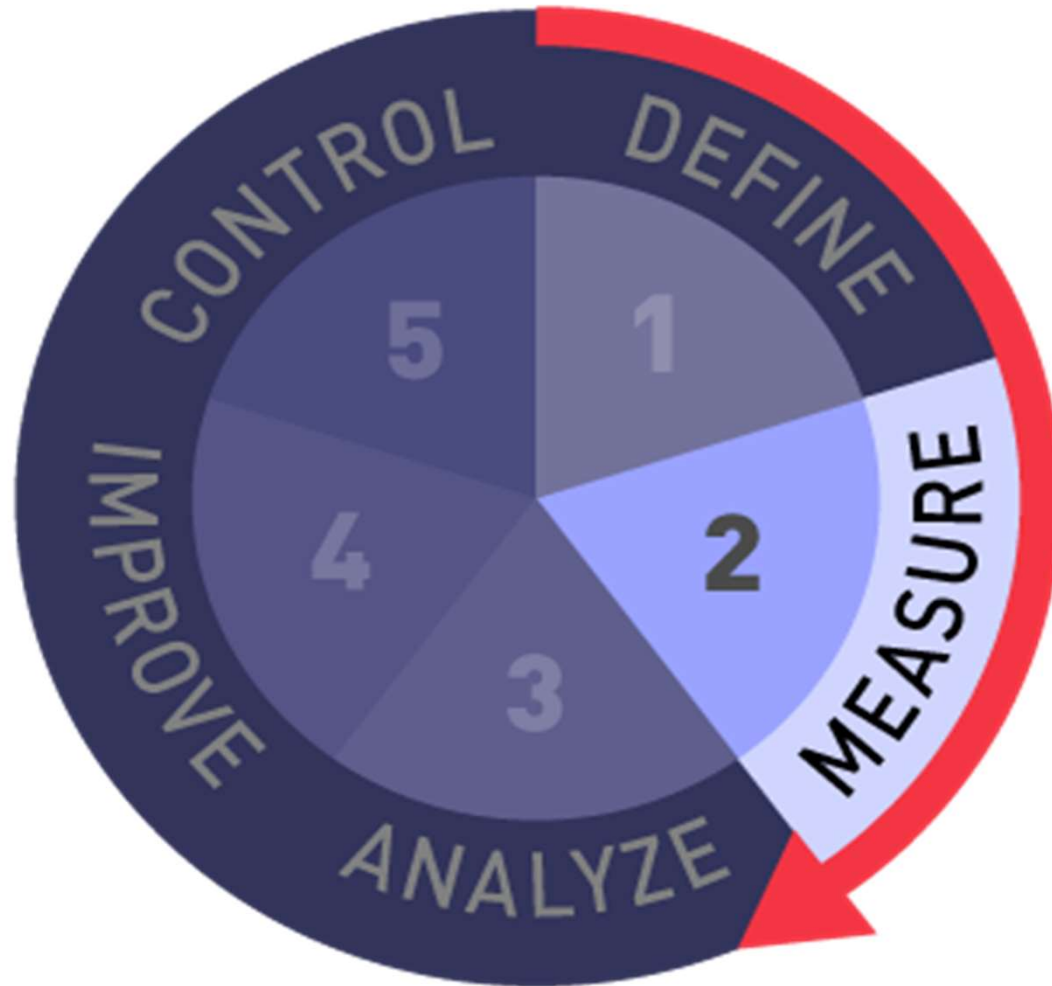
- Project plan with milestones and deliverables
- Stakeholder identification and analysis
- Completed and signed-off project charter
- DEFINE section of storyboard completed
- DEFINE phase discussed and agreed on with sponsor and coach
- DEFINE deliverables completed and phase approved

# Anexas Consultancy Services

## MEASURE: Measure the Current Situation

Overview

# You Are Here — MEASURE



# MEASURE: Measure the Current Situation

## Goal

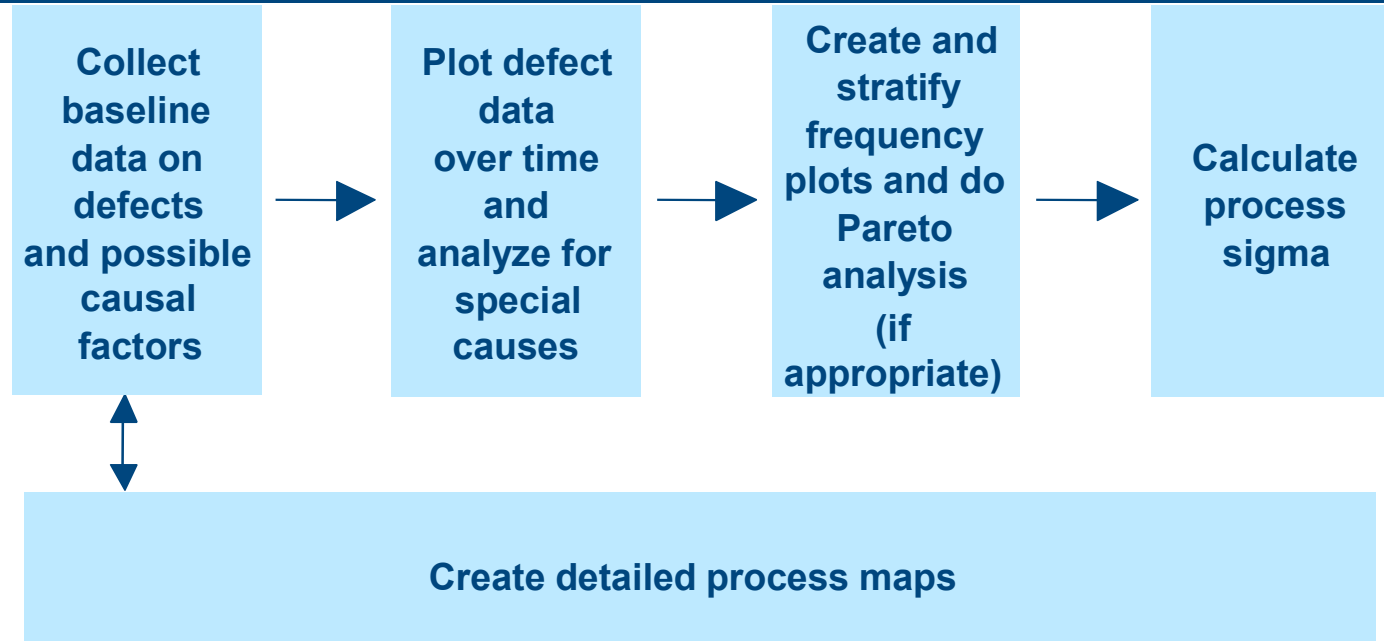
- Focus the improvement effort by gathering information on the current situation



## Output

- Data that pinpoints problem location or occurrence
- Baseline data on how well the process meets customer needs (to determine current process sigma)
- Understanding of how current process operates
- A more focused problem statement

# Approach to MEASURE



- Data methods and process mapping often occur in parallel in the MEASURE step
- Mapping the process may give insight into what data to collect and where
- Collecting and plotting the data may show which process areas need to be mapped in more detail

# Preview of MEASURE Modules

- A factual understanding of MEASURE means your knowledge is based on data—which means you have to know what data to collect, what to do with them, and how to interpret the patterns or clues you uncover
- The modules we'll cover will help you:
  - Understand the various types of data and how to collect appropriate data (Data Collection)
  - Identify patterns related to variation over time (Data Analysis I: Identifying Patterns over Time)

## Preview of MEASURE Modules, cont.

- The modules we'll cover will help you: (cont.)
  - Identify other kinds of patterns in data (Data Analysis II: Looking for Patterns Not Related to Time)
  - Define your baseline process sigma level (Determining Process Sigma)\*
  - Identify waste in the way the process is organized (Process Analysis)

# Anexas Consultancy Services

## Basic Statistics Theory

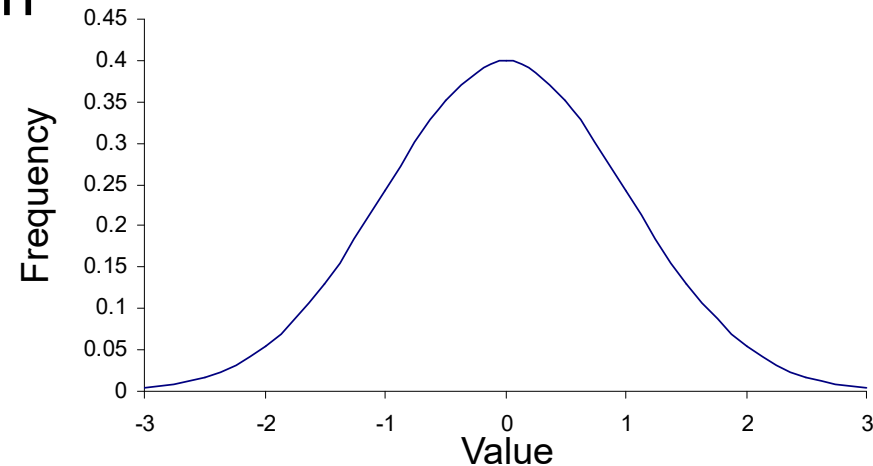


# Anexas Consultancy Services

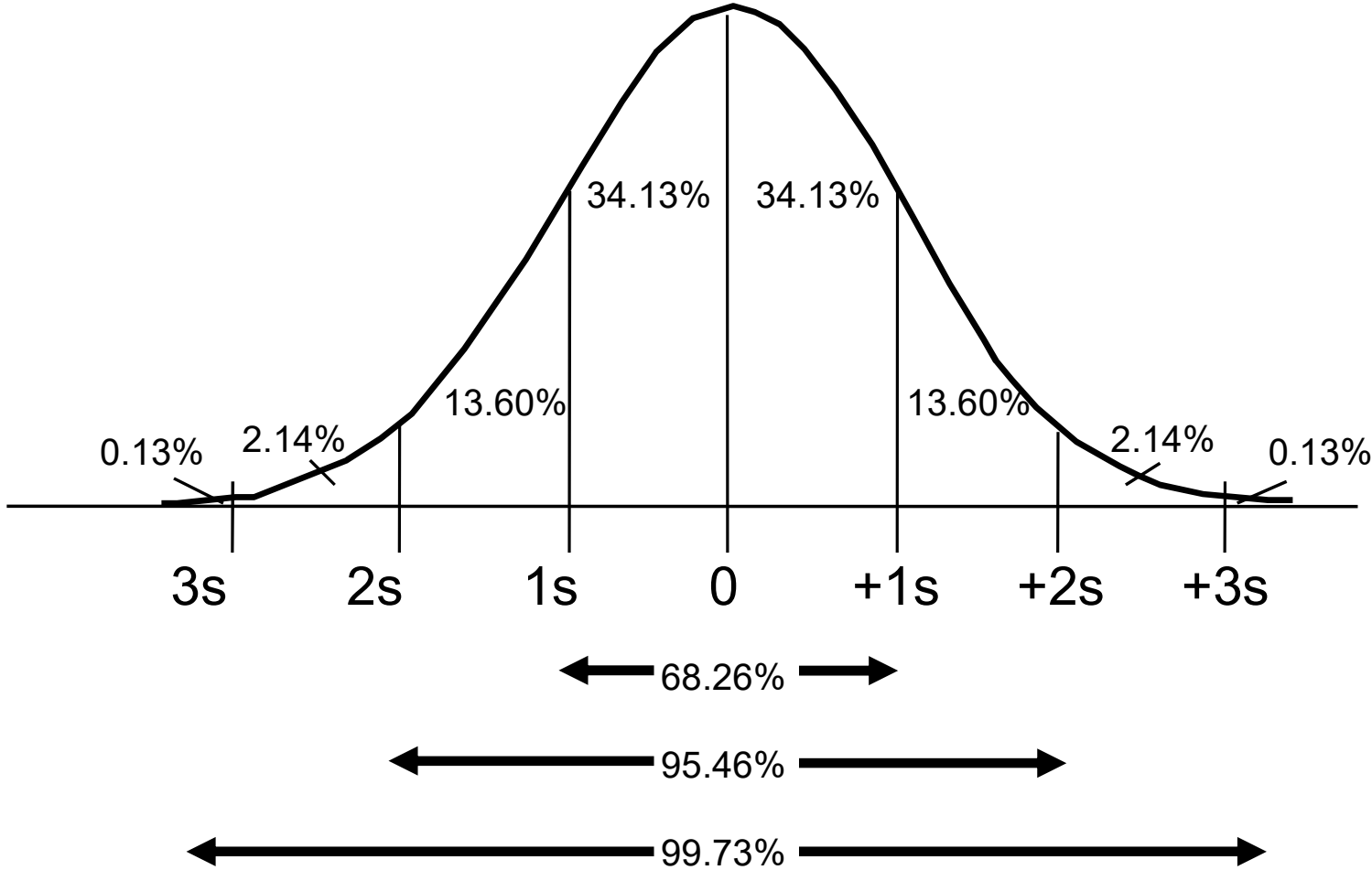
## The Normal Distribution

# Data and Statistics

- Data is the way we measure process performance
- Two main types of data
  - Continuous—measured
    - Example: Temperature
  - Discrete—categorized and counted
    - Example: Acceptable yes or no
- Continuous data are often treated as if they represented a sample from a distribution
- As shown, distribution is represented by a curve that shows all the possible values of the data and their relative frequency



# Normal Distribution



# Equations for Mean and Standard Deviation

**Sample Mean**

$$\bar{X} = \frac{\sum_{i=1}^n X_i}{n}$$

**Sample Standard Deviation **S****

$$= \sqrt{\frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n-1}}$$

# Calculating Mean and Standard Deviation

**1. List the data (order doesn't matter)**

**2. Sum all values, then divide by the number of values to determine the mean**

**3. Subtract the mean from each individual data value**

**4. Square the differences between the values and the mean**

**5. Sum the square of the differences**

**6. Divide the sum from Step 5 by (n-1)—the number of data values minus 1**

**7. Take the square root of the answer from Step 6**

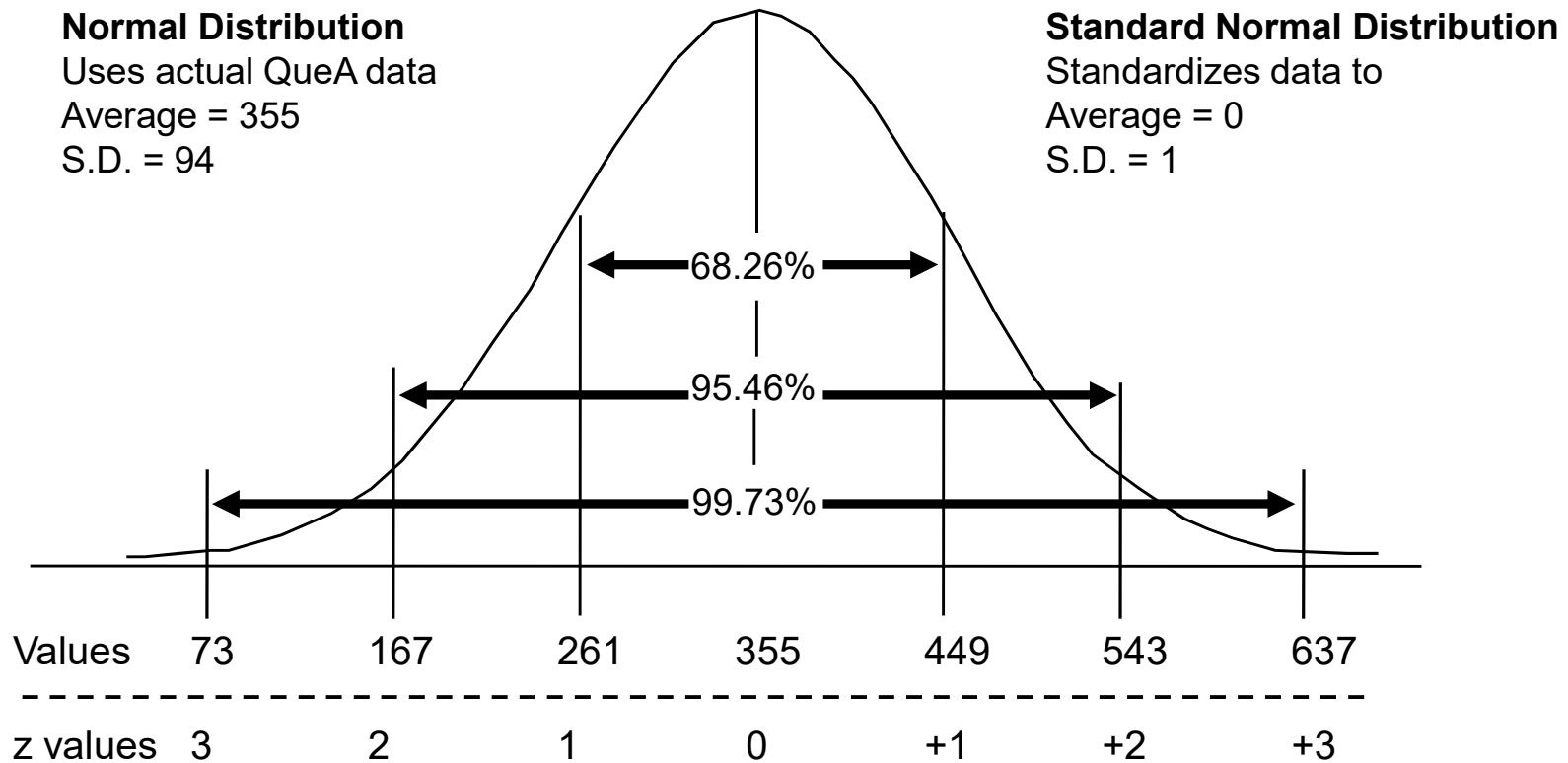
	$X$	$(X - \bar{X})$	$(X_i - \bar{X})^2$
1			
2			
3			
4			
5			
6			
7			
8			
9			
.			
.			
.			
.			
.			
n			
Sum			
Mean			
$s^2$			
s			

# Anexas Consultancy Services

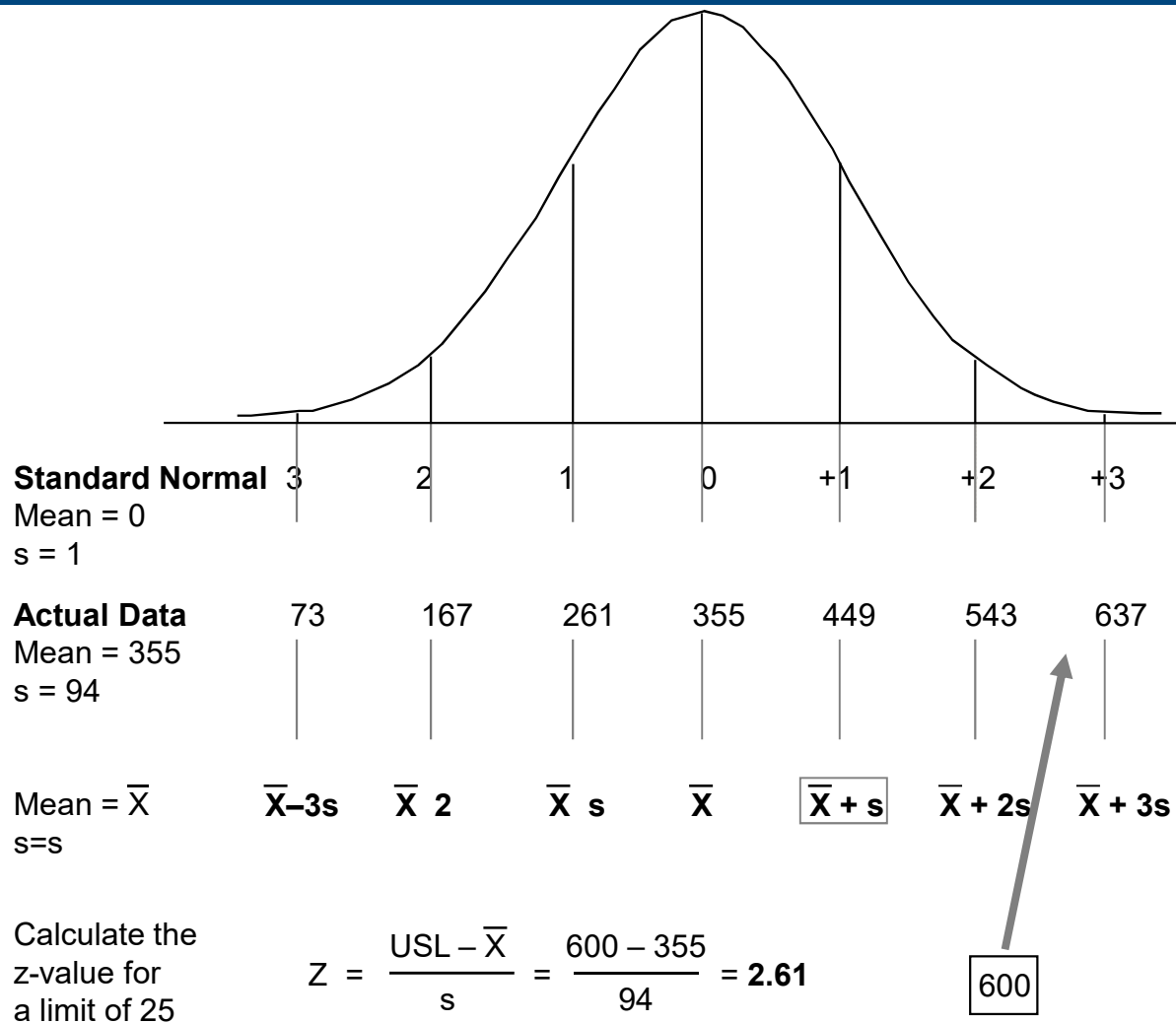
## Areas Under Normal Curves

# Normal Curve Probabilities

What's the probability that a caller will wait less than 600 seconds?

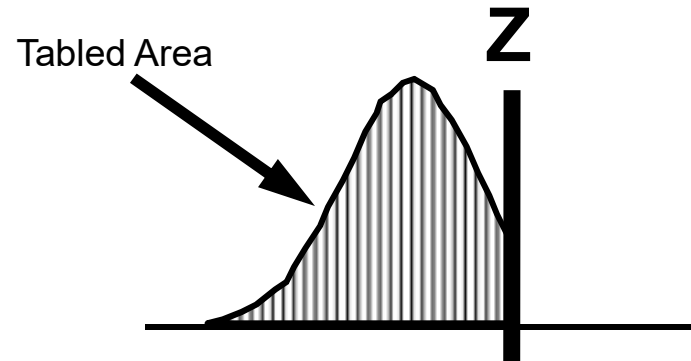


# Converting to Standard Normal





# Z Table



Z Decimal

		0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Z Whole Number	4	0.000032	0.000021	0.000013	0.000009	0.000005	0.000003	0.000002	0.000001	0.000001	0.000000
	3	0.001350	0.000968	0.000687	0.000483	0.000337	0.000233	0.000159	0.000108	0.000072	0.000048
	2	0.022750	0.017864	0.013903	0.010724	0.008198	0.006210	0.004661	0.003467	0.002555	0.001866
	1	0.158655	0.135666	0.115070	0.096801	0.080757	0.066807	0.054799	0.044565	0.035930	0.028716
	0	0.500000	0.460172	0.420740	0.382089	0.344578	0.308538	0.274253	0.241964	0.211855	0.184060
	0	0.500000	0.539828	0.579260	0.617911	0.655422	0.691462	0.725747	0.758036	0.788145	0.815940
	1	0.841345	0.864334	0.884930	0.903199	0.919243	0.933193	0.945201	0.955435	0.964070	0.971284
	2	0.977250	0.982136	0.986097	0.989276	0.991802	0.993790	0.995339	0.996533	0.997445	0.998134
	3	0.998650	0.999032	0.999313	0.999517	0.999663	0.999767	0.999841	0.999892	0.999928	0.999952
	4	0.999968	0.999979	0.999987	0.999991	0.999995	0.999997	0.999998	0.999999	0.999999	1.000000

## Discussion: Is Something Fishy?

- You manage the shipping department for a manufacturer of medical devices
- You get a charge for \$642 for expedited parts that you think usually comes in closer to \$500
- You obtain 250 recent charges for that same item to make comparisons
- You want to know if there is something fishy about this charge or if it is within the usual range for that item
- How can you use the data to answer the question?
  - 
  -

## Discussion Answers: Is Something Fishy?

### **Option 1: Create a control chart of the data**

- Use a control chart of claims amounts to see if the fishy charge is outside the limits

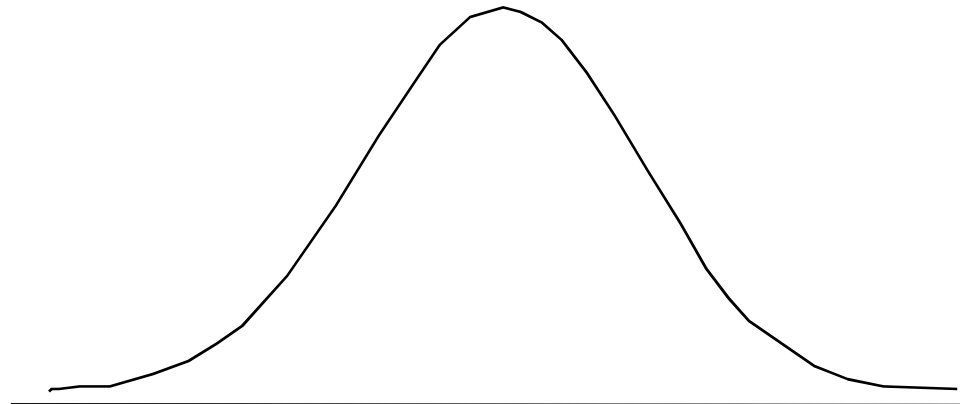
### **Option 2: Apply Normal theory**

- Use Normal theory, a body of statistics built around the properties of the Normal distribution
- See where the fishy value falls in the distribution of these data

# Recap: The Normal Curve

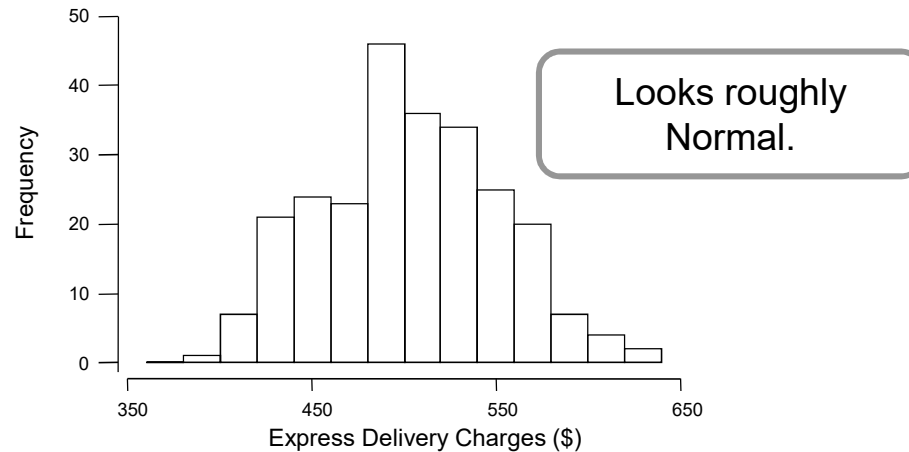
## Definition

- A probability distribution in which the most frequently occurring value is in the middle and other probabilities tail off symmetrically in both directions
  - This shape is sometimes called a **bell shaped curve**



# Overview: Using Normal Theory to Answer “Fishy” Questions

1. Make a histogram to see if (continuous) data are Normal



2. Get the mean and standard deviation of data

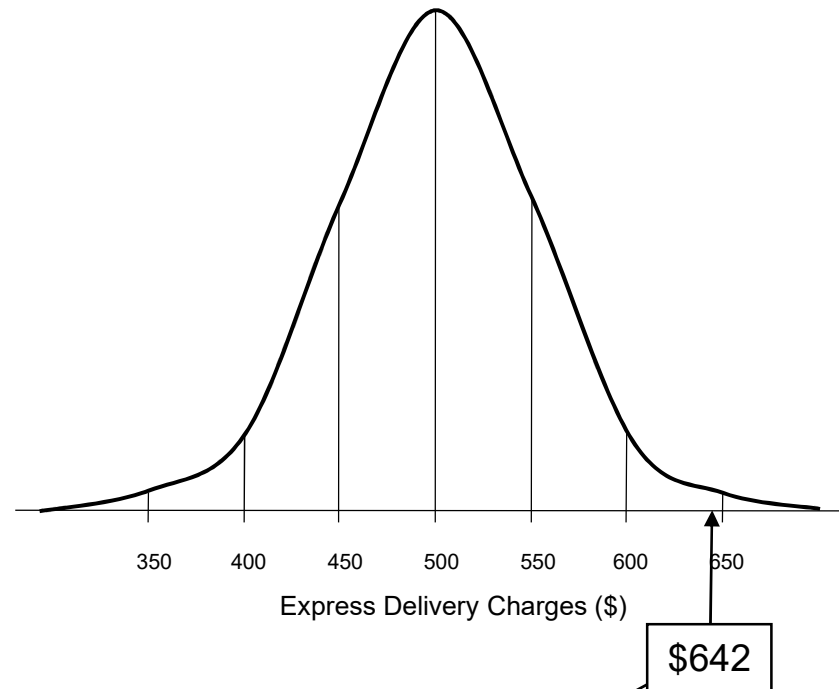
$$\bar{X} = \frac{\sum X}{n} = \$500$$

$$S = \sqrt{\frac{\sum (X - \bar{X})^2}{n - 1}} = \$50$$

A standard deviation is the typical amount an observation varies from the mean.

# Overview: Using Normal Theory to Answer “Fishy” Questions, cont.

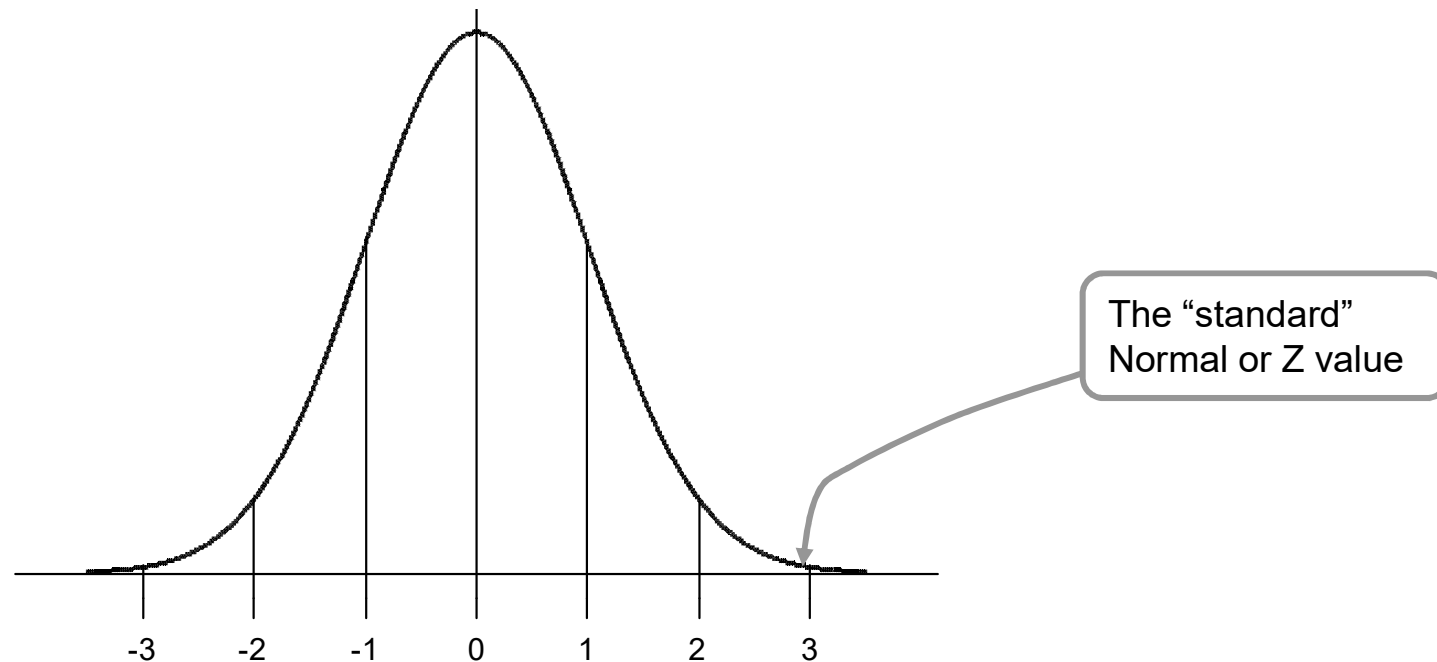
## 3. Draw Normal curve for data



## 4. Compare the value of interest (\$642) with the Normal curve

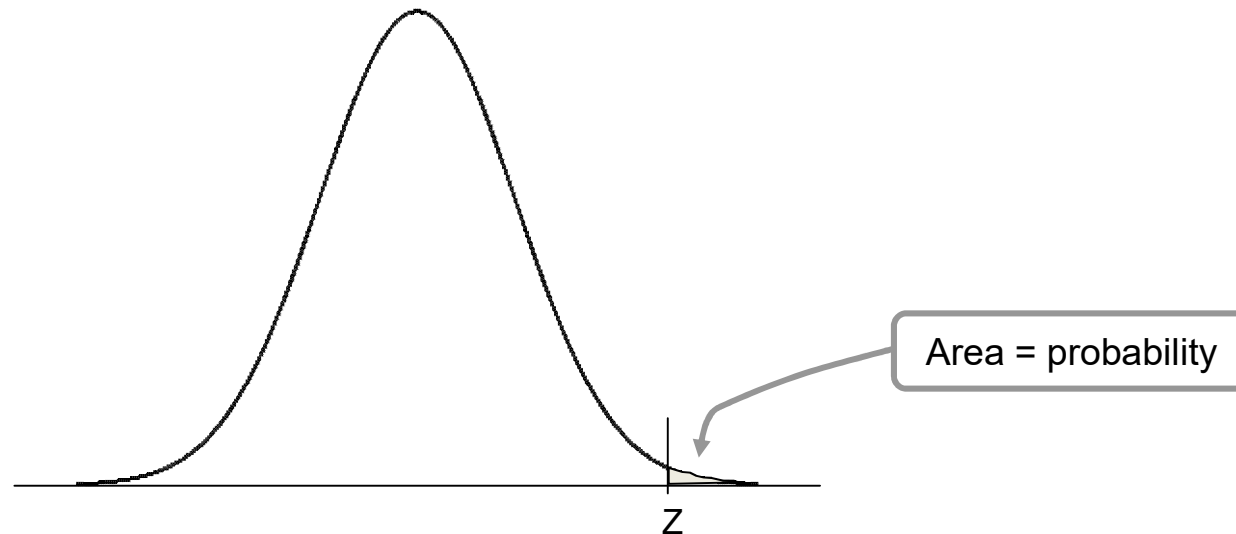
# Overview: Using Normal Theory to Answer “Fishy” Questions, cont.

5. How many standard deviations away from the mean is the value of interest?



# Overview: Using Normal Theory to Answer “Fishy” Questions, cont.

6. Look up the probability of a claim being that value or higher in the Normal table of z values

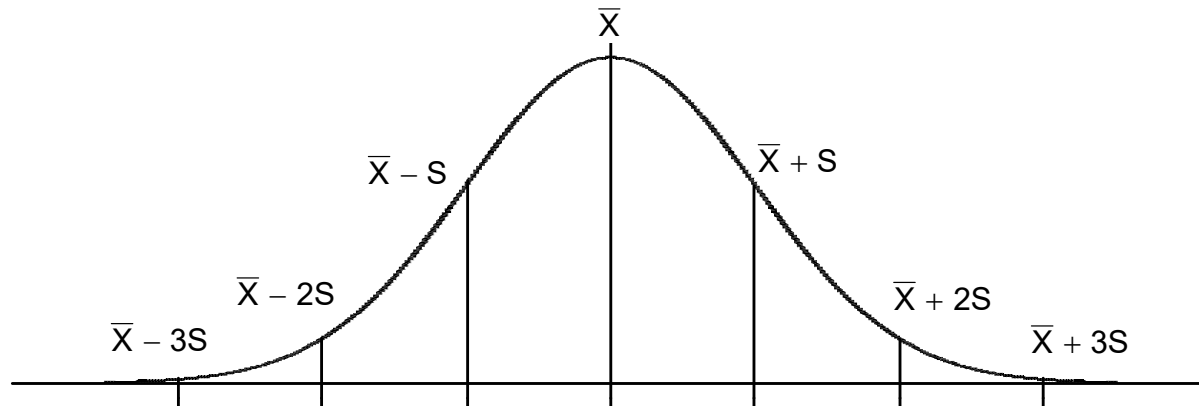


7. If probability is low, something is probably fishy

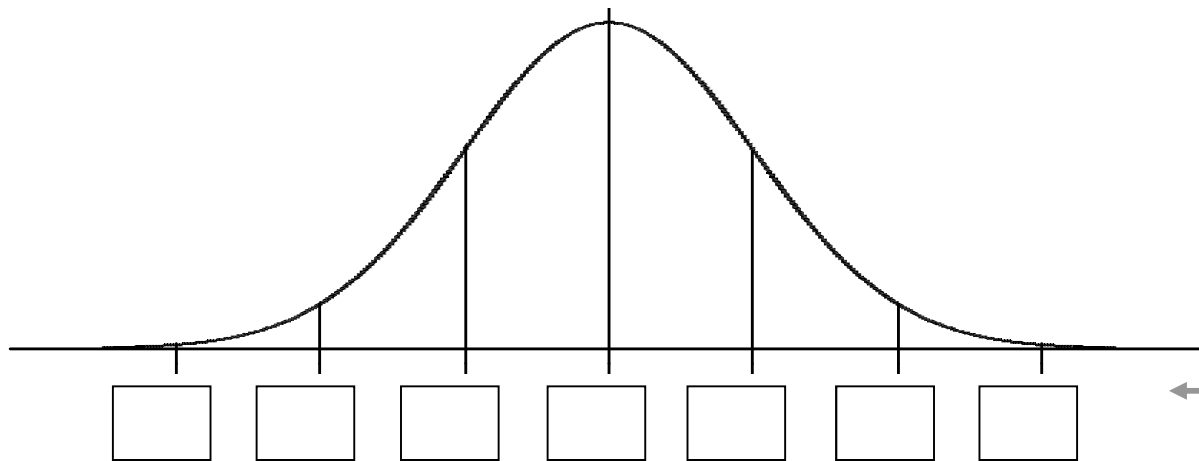
“Low” usually means  $< 0.05$



# Drawing Normal Curves

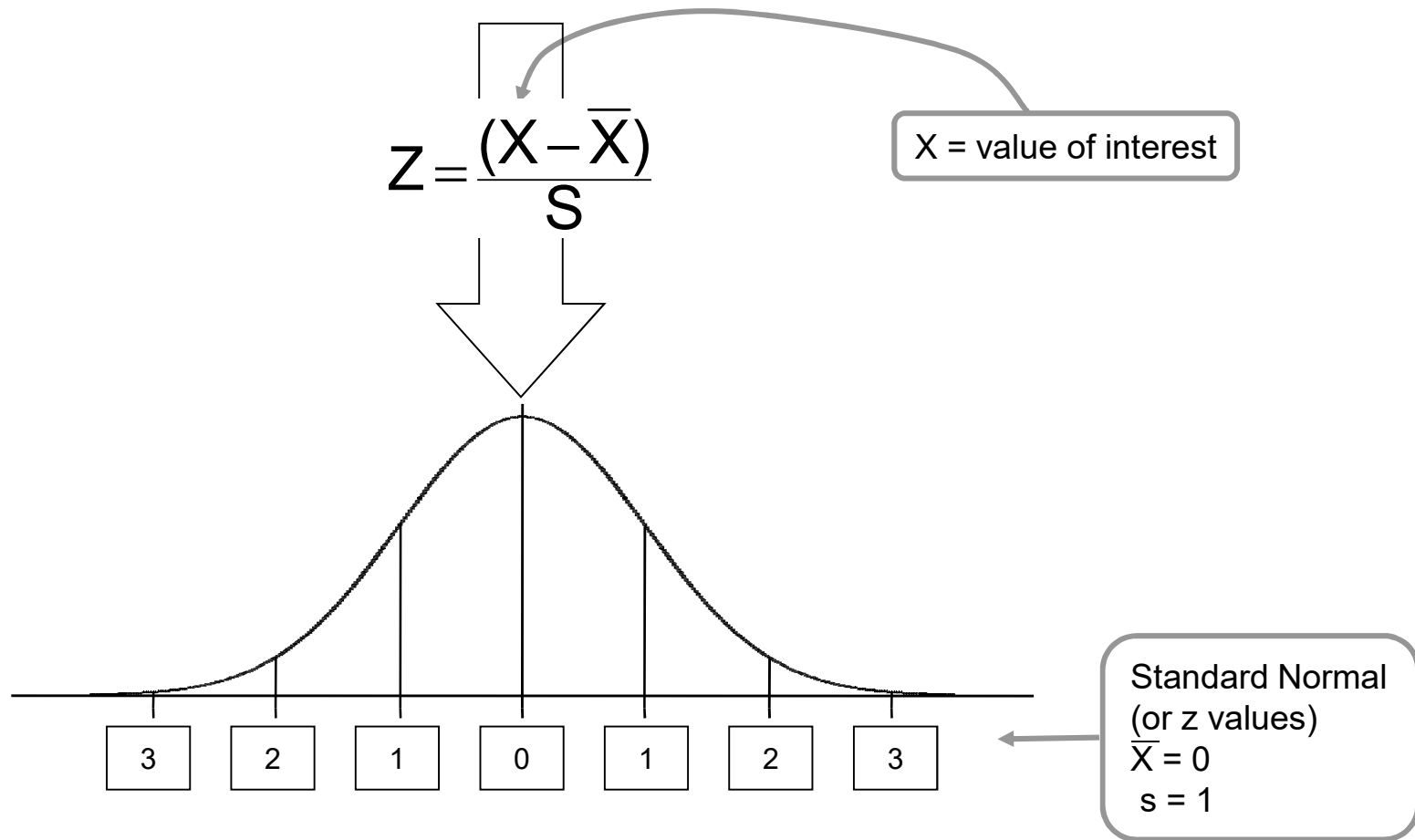


Normal data;  
"any" scale



Fill in the blanks  
for data with  
 $\bar{X} = 500$   
 $s = 50$

# Drawing Normal Curves, cont.



# Z value: Same as a “Standard” Normal Value

## Z value

How many standard deviations is the value of interest away from the mean?

$$Z = \frac{(\text{value of interest}) - \bar{X}}{S}$$

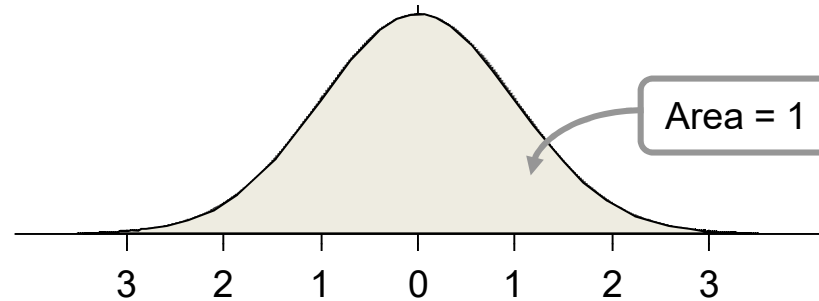
## Claims example

How many standard deviations is \$642 away from \$500? (Or “What is the Z value for 642?”)

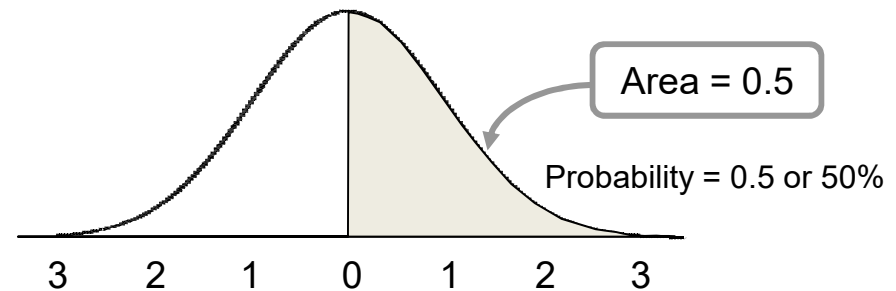
$$Z = \frac{642 - 500}{50} = \frac{142}{50} = 2.84$$

# Probabilities of z values

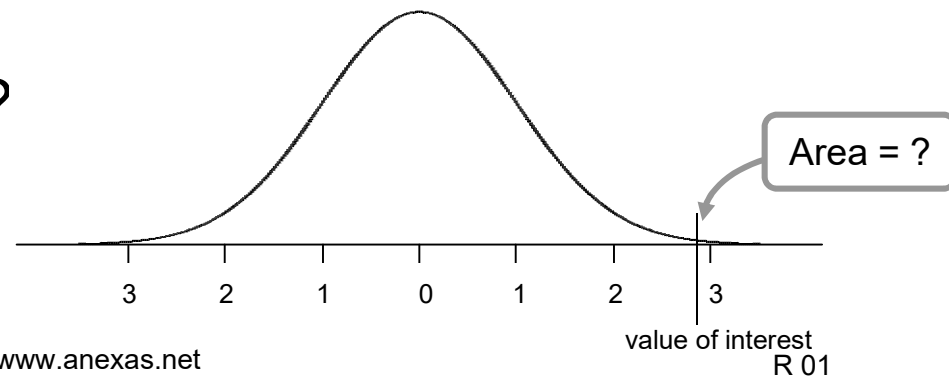
- Area under the standard Normal curve = probability



- What is the probability a Z value will be  $\geq$  zero?



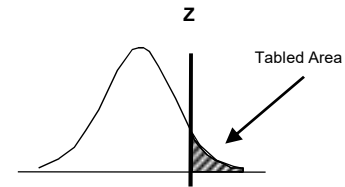
- What is the probability a Z value will be  $\geq$  2.84?



# Normal Tables: Looking up Probabilities

## Two types of tables

1. Reports area  $\geq$  (to the right of) the value of interest



Standard Normal Table															
Z	Area	Z	Area	Z	Area	Z	Area	Z	Area	Z	Area	Z	Area	Z	Area
0.00	0.500000	0.60	0.274253	1.20	0.115070	1.80	0.035930	2.40	0.008198	3.00	0.001350	3.60	0.000159	4.20	0.000013
0.01	0.496011	0.61	0.270931	1.21	0.111319	1.81	0.035148	2.41	0.007976	3.01	0.001306	3.61	0.000153	4.21	0.000013
0.02	0.492022	0.62	0.267629	1.22	0.111232	1.82	0.034380	2.42	0.007760	3.02	0.001264	3.62	0.000147	4.22	0.000012
0.03	0.488034	0.63	0.264347	1.23	0.109349	1.83	0.033625	2.43	0.007549	3.03	0.001223	3.63	0.000142	4.23	0.000012
0.04	0.484047	0.64	0.261086	1.24	0.107488	1.84	0.032884	2.44	0.007344	3.04	0.001183	3.64	0.000136	4.24	0.000011
0.05	0.480061	0.65	0.257846	1.25	0.105650	1.85	0.032157	2.45	0.007143	3.05	0.001144	3.65	0.000131	4.25	0.000011
0.06	0.476078	0.66	0.254627	1.26	0.103835	1.86	0.031443	2.46	0.006947	3.06	0.001107	3.66	0.000126	4.26	0.000010
0.07	0.472097	0.67	0.251429	1.27	0.102042	1.87	0.030742	2.47	0.006756	3.07	0.001070	3.67	0.000121	4.27	0.000010
0.08	0.468119	0.68	0.248252	1.28	0.100273	1.88	0.030054	2.48	0.006569	3.08	0.001035	3.68	0.000117	4.28	0.000009
0.09	0.464144	0.69	0.245097	1.29	0.098525	1.89	0.029379	2.49	0.006387	3.09	0.001001	3.69	0.000112	4.29	0.000009
0.10	0.460172	0.70	0.241964	1.30	0.096800	1.90	0.028717	2.50	0.006210	3.10	0.000968	3.70	0.000108	4.30	0.000009
0.11	0.456205	0.71	0.238852	1.31	0.095098	1.91	0.028067	2.51	0.006037	3.11	0.000935	3.71	0.000104	4.31	0.000008
0.12	0.452242	0.72	0.235763	1.32	0.093418	1.92	0.027429	2.52	0.005868	3.12	0.000904	3.72	0.000100	4.32	0.000008
0.13	0.448283	0.73	0.232695	1.33	0.091759	1.93	0.026803	2.53	0.005703	3.13	0.000874	3.73	0.000096	4.33	0.000007
0.14	0.444330	0.74	0.229650	1.34	0.090123	1.94	0.026190	2.54	0.005543	3.14	0.000845	3.74	0.000092	4.34	0.000007
0.15	0.440382	0.75	0.226627	1.35	0.088508	1.95	0.025588	2.55	0.005386	3.15	0.000816	3.75	0.000088	4.35	0.000007

A larger version of this table is on the next page.



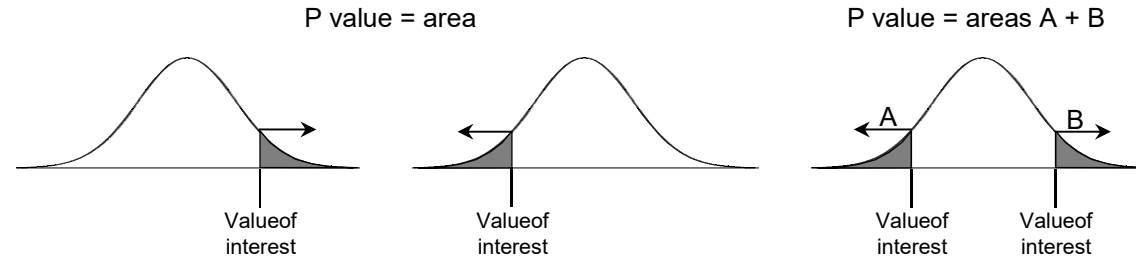
# Anexas Consultancy Services

Looking  
Up Probabilities

# P values Are Probabilities for Values of Interest

P value =

- Tail area
- Area under curve beyond value of interest
- Probability of being at the value of interest or beyond



- A small P value (0 to 0.05) means:
  - The probability is small that the value of interest did, indeed, come from that distribution; it is likely to have come from some other distribution
  - Something's fishy!



# P values Are Probabilities for Values of Interest, cont.

## Claims example

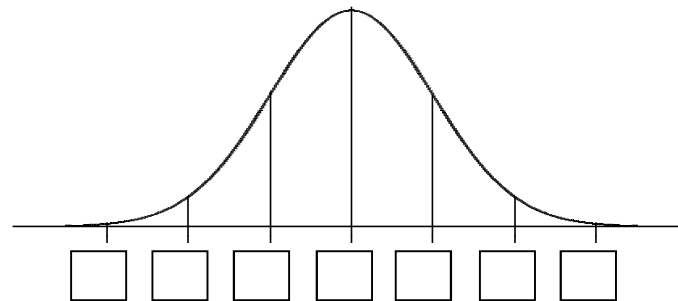
- Charge of interest = \$642
- Z value = 2.84
- P value = 0.0023 (very small)

## Conclusion

- The likelihood is small that the charge of \$642 came from the same distribution as the other charges
- Something's probably fishy; investigate the charge further

## Exercise: Using Normal Theory and P values

- You receive another charge of \$970 for a different item
- Find its P value given that 250 recent charges of the same type are roughly Normal with an average of \$800 and a standard deviation of \$200
  - Are the data Normal (and continuous)?
  - Get mean and standard deviation of the data
  - Label the Normal curve for these data



- Mark on the curve where the value of interest falls in the distribution

## Exercise: Using Normal Theory and P values, cont.

- How many standard deviations is the value of interest from the mean? (What is the Z value?)

$$Z = \frac{(\text{value-of-interest}) - \bar{X}}{S} = \boxed{\phantom{000000}}$$

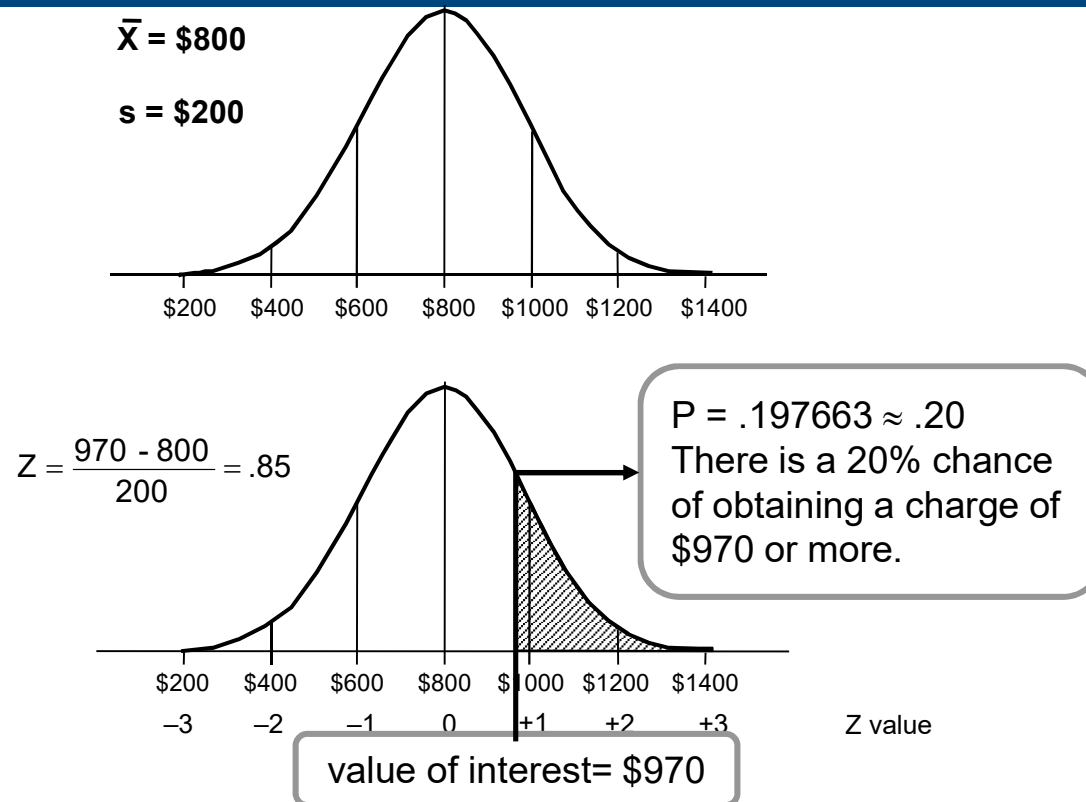
- Look up the probability of the claim being that Z value or beyond in the Standard Normal table

P value =

- P value means:

Will you investigate the claim or not?

# Answer: Using Normal Theory and P values



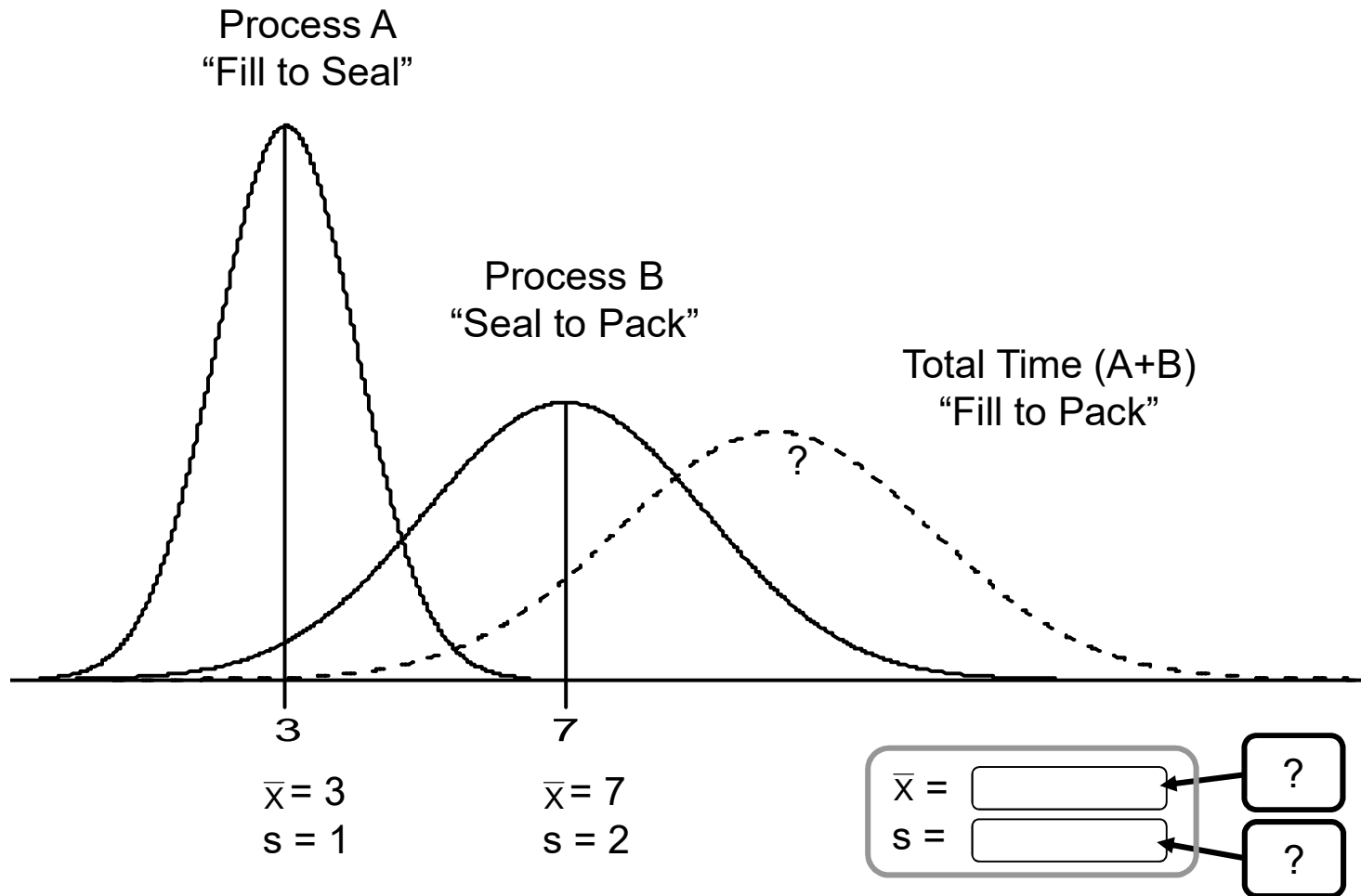
## Conclusion

- It is likely enough that \$970 came from the same distribution as the other charges
- Do not investigate; pay the charge

# Anexas Consultancy Services

## Combining Distributions

# Part 1: Adding Distributions



# How to Add Normal Distributions

**Means are additive      Are standard deviations additive?**

$$\begin{aligned}\bar{X}_{A+B} &= \bar{X}_A + \bar{X}_B \\ &= 3 + 7 \\ &= 10\end{aligned}$$

$$S_{A+B} \stackrel{?}{=} S_A + S_B$$

**NO! Incorrect**

- When you add two variables, the variation increases, but by how much?
- The combined distribution will be wider than either curve alone

# Variances Are Additive; Standard Deviations Are Not

- A standard deviation is the typical amount an observation varies from the mean
- Standard deviations are derived from a statistical term called **variance**

$$\text{St. Dev.} = \sqrt{\text{Variance}}$$

- Variances are additive, but standard deviations are not
- To combine two standard deviations, square each to get the variances, add the variances, and then take a square root of the combined variance



# Variations Are Additive; Standard Deviations Are Not, cont.

## Process Time Example

To add distributions:

- Add means to get new mean
- Add variances and then take square root to get standard deviation of combined distribution

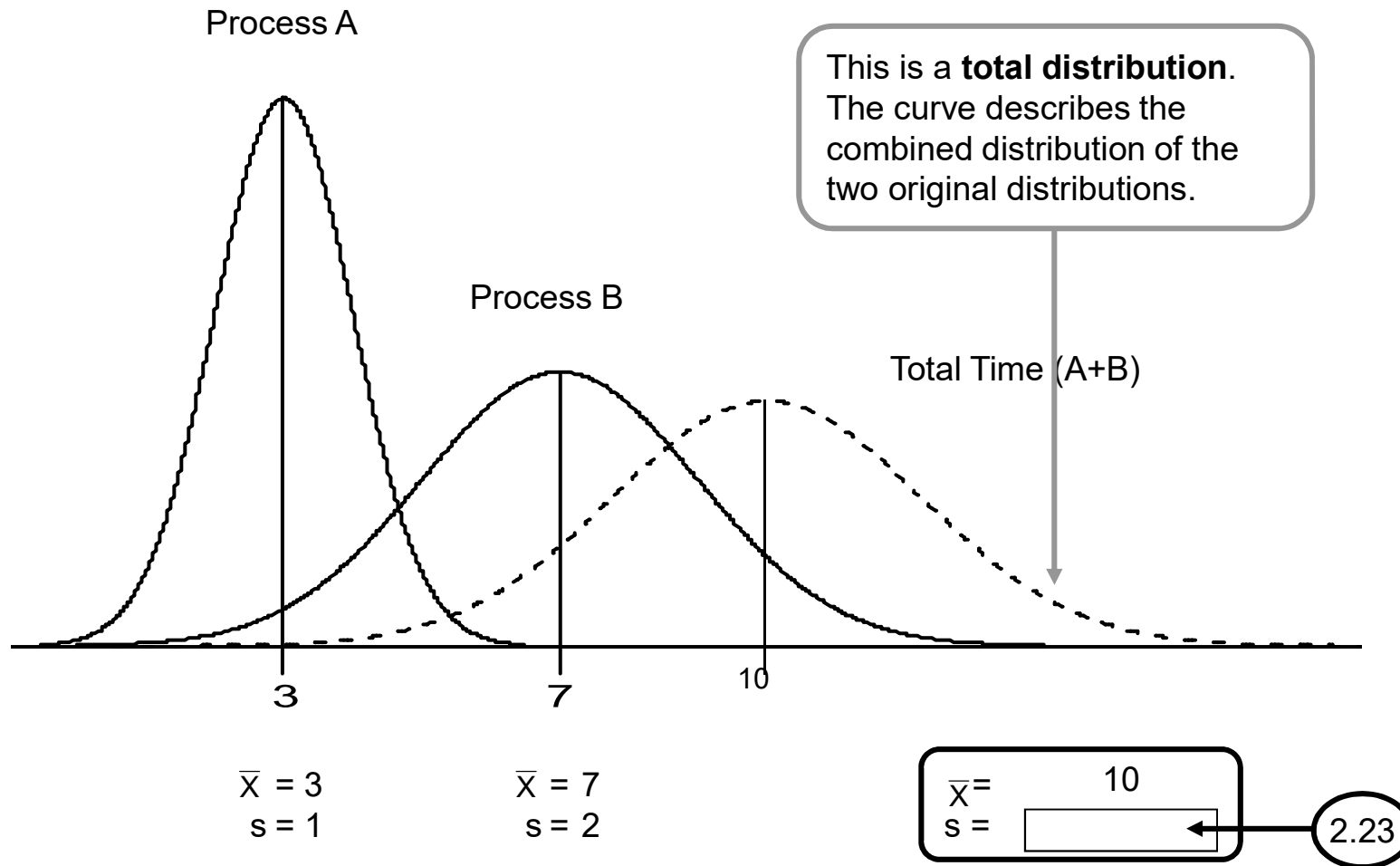
$$S_{A+B} = \sqrt{S_A^2 + S_B^2} = \sqrt{(1)^2 + (2)^2} = \sqrt{5} = 2.23$$

$$\neq 1 + 2 = 3$$

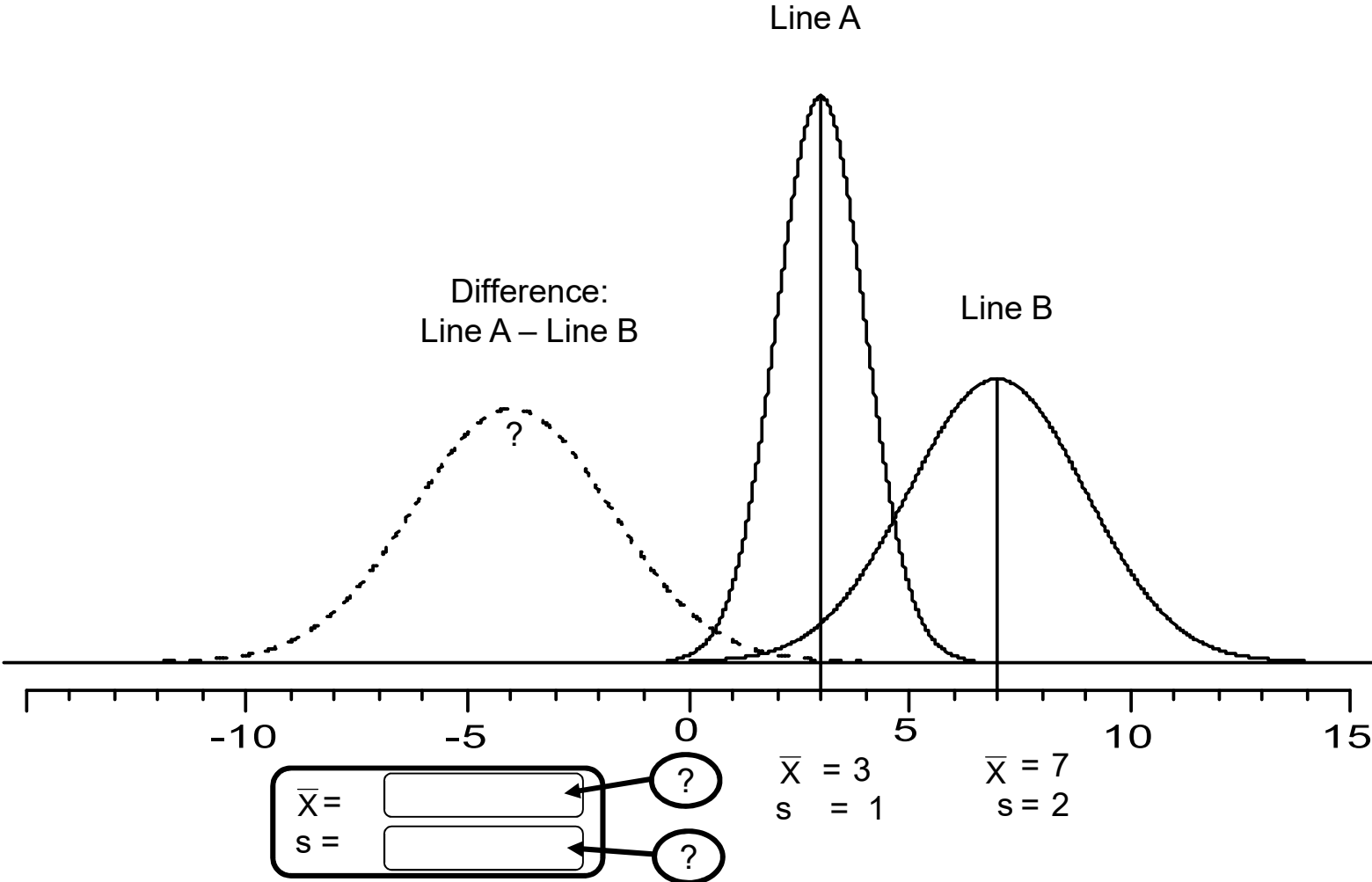
Incorrect; do not add standard deviation

Correct; add variances and then take square root.

# Variations Are Additive; Standard Deviations Are Not, cont.



# Part 2: Subtracting Distributions



# How to Subtract Normal Distributions

## For differences, subtract means, but add variances

- It may seem counterintuitive at first, but any time you combine distributions, variation always increases
- So even when you subtract means, you add variances

Mean  $\bar{X}_{A+B} = \bar{X}_A + \bar{X}_B$

Variance  $S_{A-B}^2 = S_A^2 + S_B^2$

Standard Deviation  $S_{A-B} = \sqrt{S_A^2 + S_B^2}$

# Example: Subtracting Distributions

## Process Time Example

$$\bar{X}_{A-B} = \bar{X}_A - \bar{X}_B = 3 - 7 = -4$$

It's negative because the process time for Line A is on average 4 minutes shorter than the process time for Line B.

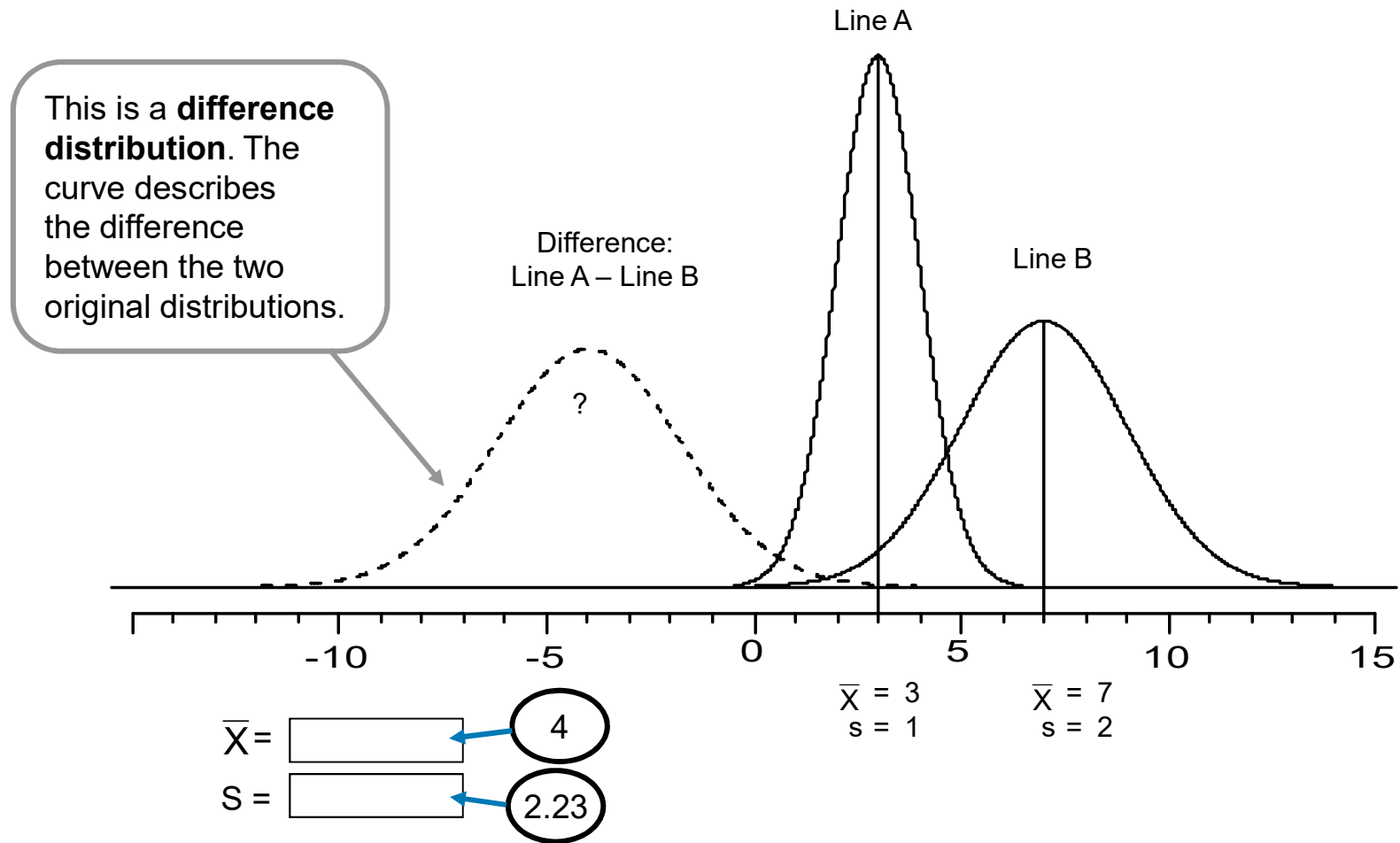
$$S_{A-B} = \sqrt{S_A^2 + S_B^2} = \sqrt{(1)^2 + (2)^2} = \sqrt{5} = 2.23$$

Correct

$$\neq 1 - 2 = -1$$

Incorrect

# Example: Subtracting Distributions, cont.



# Additivity of Variances

## 1. Adding distributions to get a total

$$\bar{X}_{A+B} = \bar{X}_A + \bar{X}_B$$

$$S_{A+B}^2 = S_A^2 + S_B^2$$

$$S_{A+B} = \sqrt{S_A^2 + S_B^2}$$

Same

## 2. Subtracting distributions to get a difference

$$\bar{X}_{A-B} = \bar{X}_A + \bar{X}_B$$

$$S_{A-B}^2 = S_A^2 + S_B^2$$

$$S_{A-B} = \sqrt{S_A^2 + S_B^2}$$

When adding or subtracting distributions, the procedure is the same for getting a standard deviation

- Add variances
- Take a square root

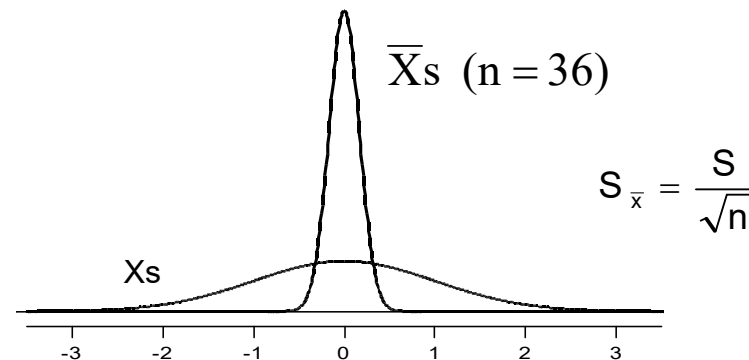
# Anexas Consultancy Services

## Central Limit Theorem and Confidence Intervals



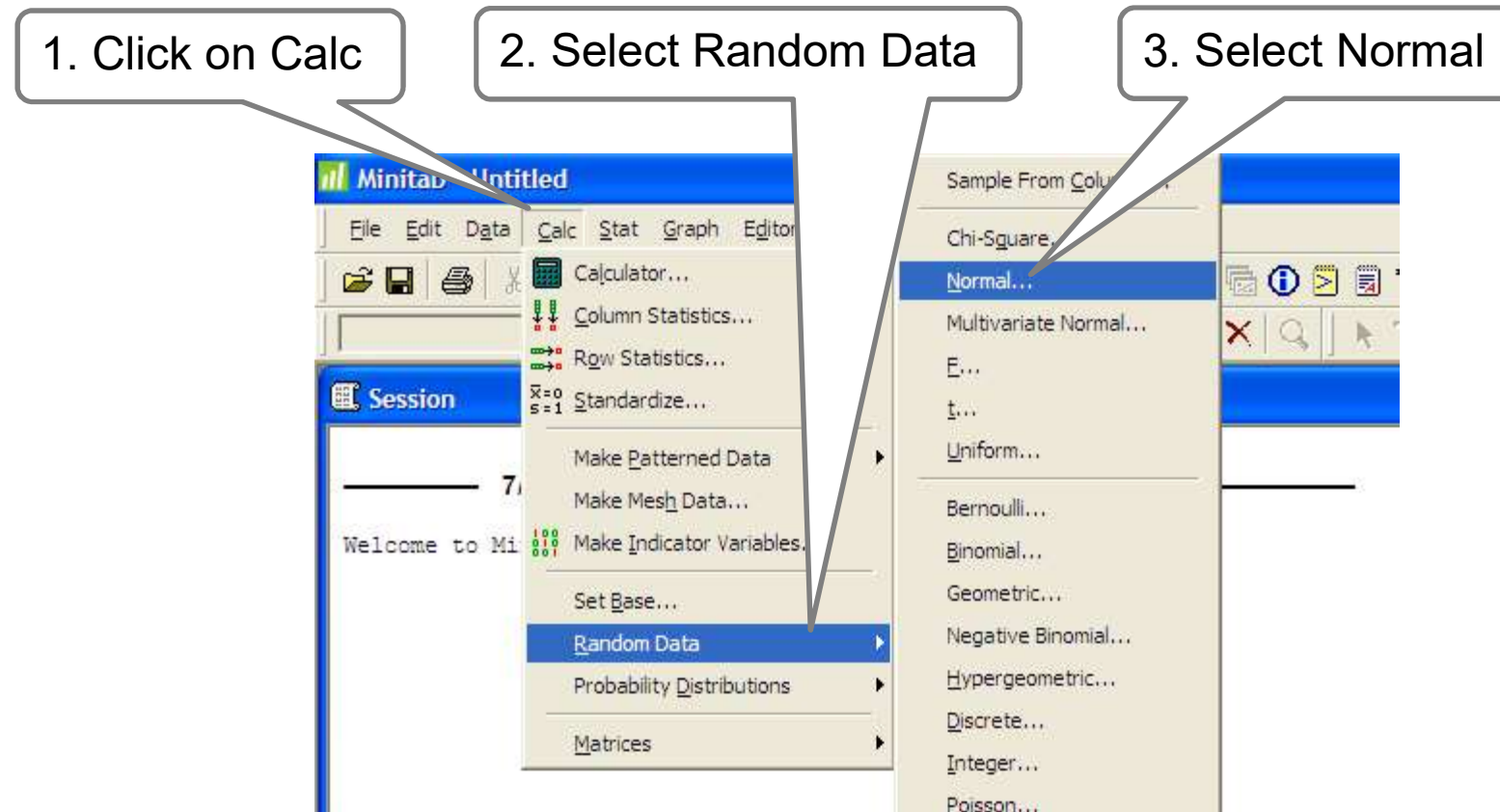
# The Central Limit Theorem, Part 1

- The Central Limit Theorem is an important statistical concept that applies to a large number of statistical tools within the Six Sigma Body of Knowledge
- Understanding it is indispensable to interpreting results of various statistical tools (e.g., regression analysis or hypothesis testing)
- We will use Minitab to generate “random” data to illustrate how the Central Limit Theorem works
- Part 1 of the Central Limit Theorem says the standard deviation of averages is less than the standard deviation of individual observations by a factor of  $\frac{1}{\sqrt{n}}$



# The Central Limit Theorem, Part 1

- Open a blank Minitab worksheet



# The Central Limit Theorem, Part 1, cont.

- Have Minitab generate random normal data as shown below:

1. Type 1000 here

2. Type c1-c9 here

3. Enter 80 for Mean

4. Enter 3 for the Std. Dev.

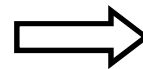
5. Click OK

# The Central Limit Theorem, Part 1, cont.

**Output:** your numbers will be different

	C1	C2	C3	C4	C5	C6	C7	C8	C9
1	83.6550	79.5600	78.4506	82.2106	75.9103	76.6537	79.4996	74.8787	78.4519
2	79.8536	79.8202	77.4575	87.0574	75.8258	81.7003	83.0624	78.5713	81.1571
3	84.9487	87.0001	74.2243	78.3876	78.7760	76.1865	84.8863	78.4661	81.9700
4	81.1384	78.7630	80.6872	86.3217	79.0401	80.3601	83.0348	84.7032	79.0495
5	79.4711	79.5336	79.8951	83.2326	82.3533	83.3356	78.7162	87.2212	72.8625

**Next:** Type X1...X9 as shown

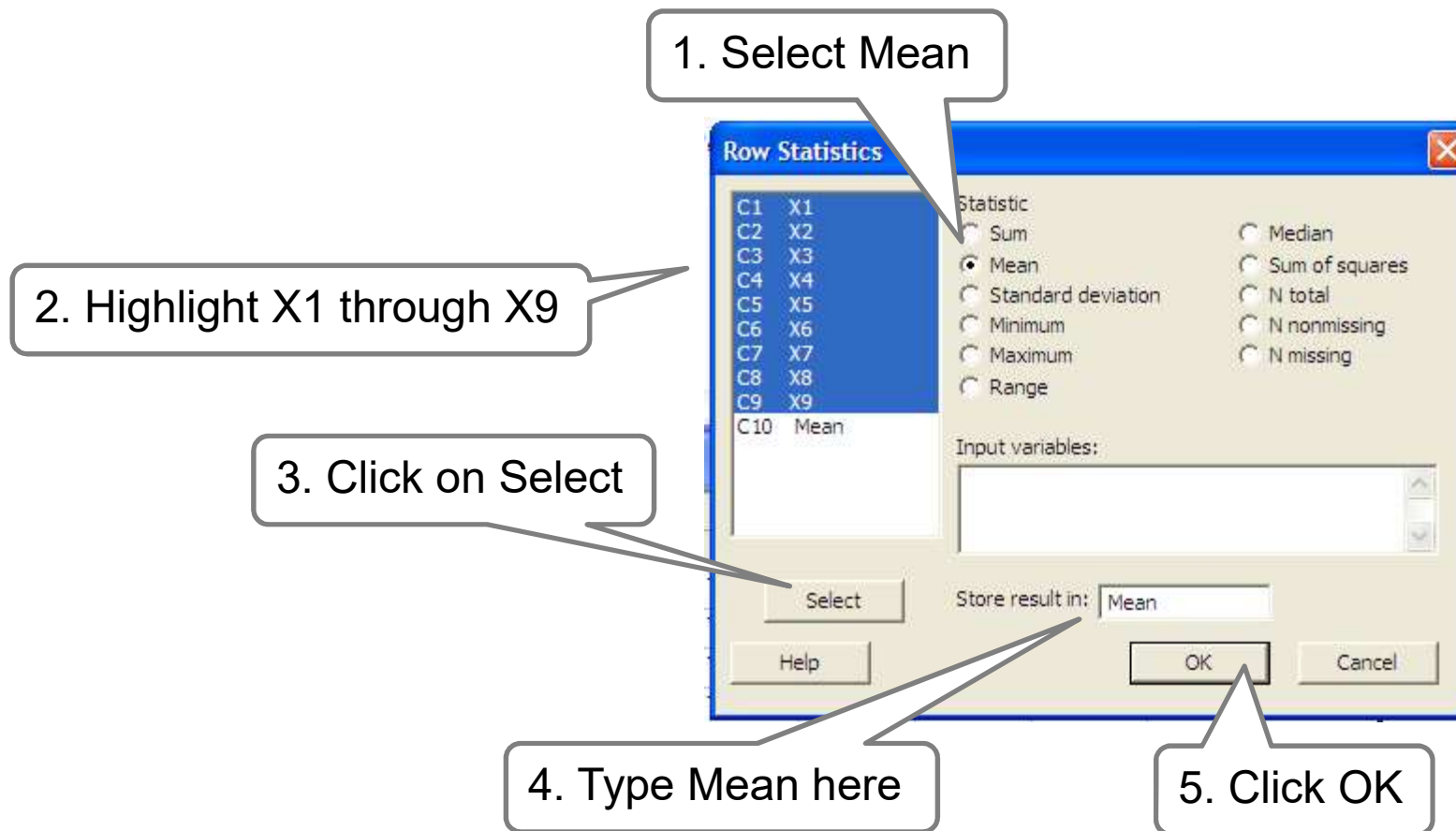


Type Mean as C10's name

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
	X1	X2	X3	X4	X5	X6	X7	X8	X9	Mean
1	83.6550	79.5600	78.4506	82.2106	75.9103	76.6537	79.4996	74.8787	78.4519	
2	79.8536	79.8202	77.4575	87.0574	75.8258	81.7003	83.0624	78.5713	81.1571	
3	84.9487	87.0001	74.2243	78.3876	78.7760	76.1865	84.8863	78.4661	81.9700	
4	81.1384	78.7630	80.6872	86.3217	79.0401	80.3601	83.0348	84.7032	79.0495	
5	79.4711	79.5336	79.8951	83.2326	82.3533	83.3356	78.7162	87.2212	72.8625	

# The Central Limit Theorem, Part 1, cont.

- From the main menu, click on Calc > Row Statistics



# The Central Limit Theorem, Part 1, cont.

**Output:** your numbers will be different

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
	X1	X2	X3	X4	X5	X6	X7	X8	X9	Mean
1	83.6550	79.5600	78.4506	82.2106	75.9103	76.6537	79.4996	74.8787	78.4519	78.8078
2	79.8536	79.8202	77.4575	87.0574	75.8258	81.7003	83.0624	78.5713	81.1571	80.5006
3	84.9487	87.0001	74.2243	78.3876	78.7760	76.1865	84.8863	78.4661	81.9700	80.5384
4	81.1384	78.7630	80.6872	86.3217	79.0401	80.3601	83.0348	84.7032	79.0495	81.4553
5	79.4711	79.5336	79.8951	83.2326	82.3533	83.3356	78.7162	87.2212	72.8625	80.7357
6	81.5827	81.9029	83.6519	77.9239	81.5689	76.1915	78.4964	81.9043	81.7659	80.5543
7	82.8922	81.9180	77.7750	78.2163	75.8942	77.3754	78.7867	74.3572	77.2389	78.2727
8	80.1942	79.0787	79.5300	74.8835	80.9407	79.3496	76.6232	72.2527	80.5293	78.1535
9	77.5850	78.1653	79.4201	81.8047	82.4503	83.8673	76.5159	77.9000	78.6011	79.5900
10	73.6482	81.4310	78.5088	76.5771	72.6084	80.5710	81.4985	78.5333	76.9386	77.8128

- **Next:** Compare the distribution of the X's with the distribution of the Mean



# The Central Limit Theorem, Part 1, cont.

- From the main menu, click on Graph > Dotplot...

1. Select Multiple Y's Simple

2. Click OK

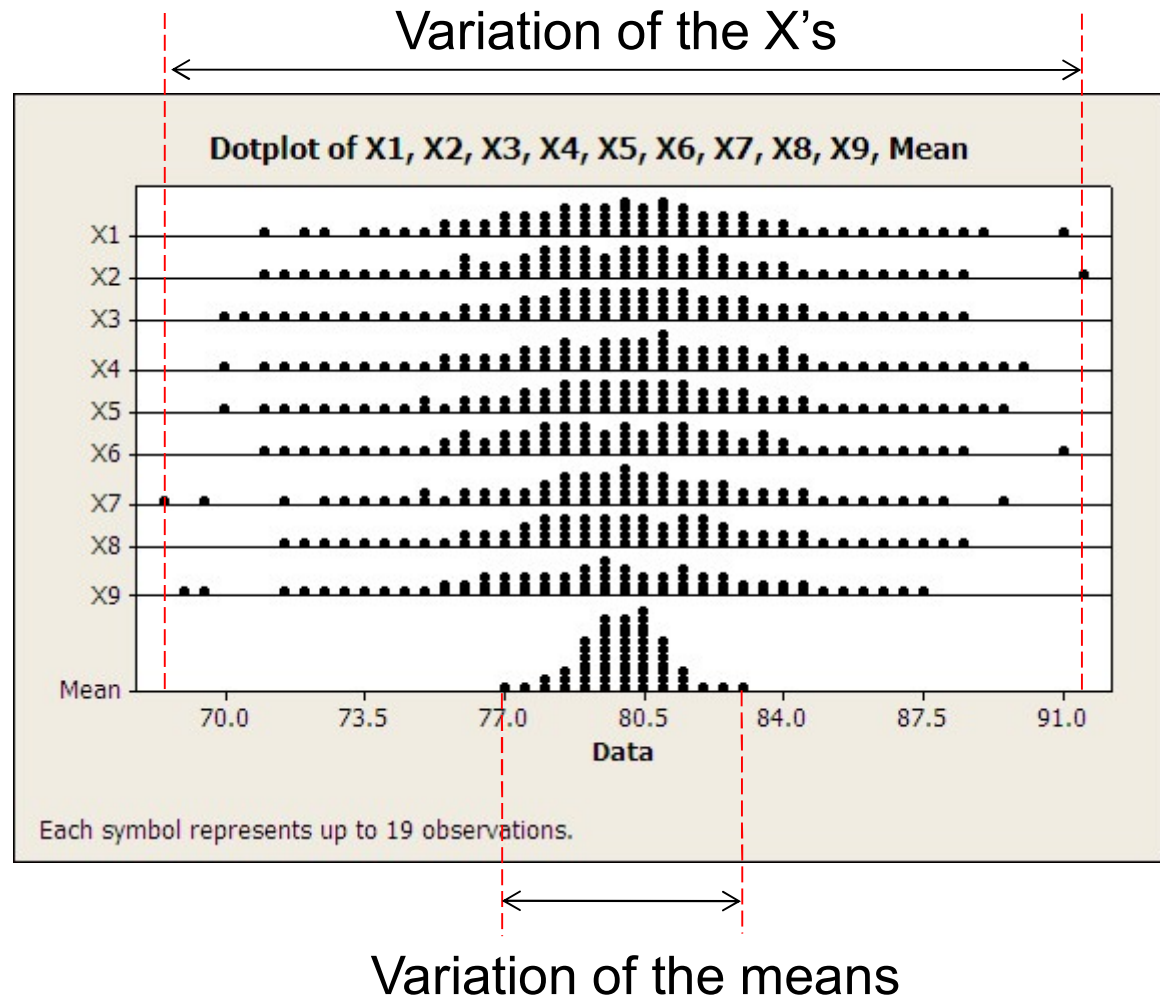
3. Highlight X1 through Mean

4. Click Select

5. Click OK

# The Central Limit Theorem, Part 1, cont.

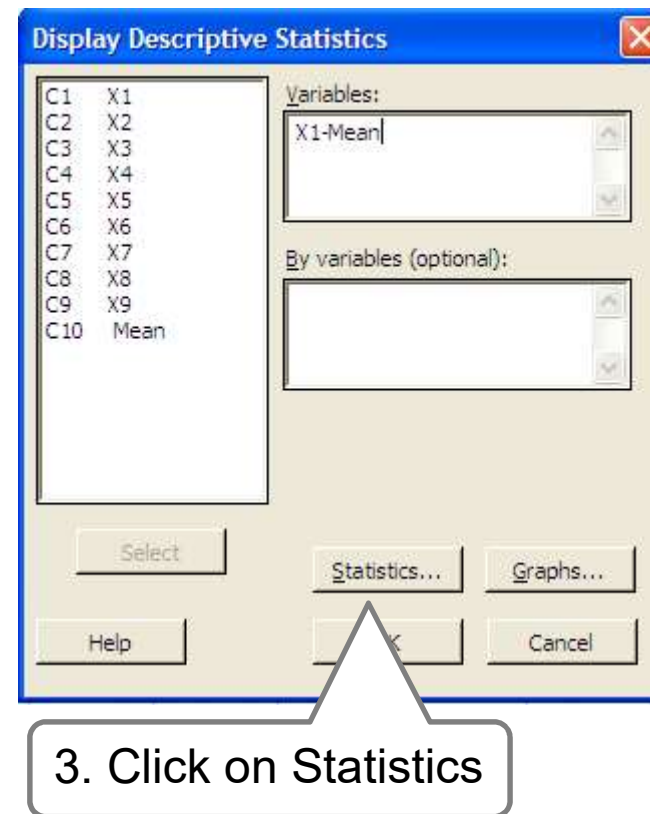
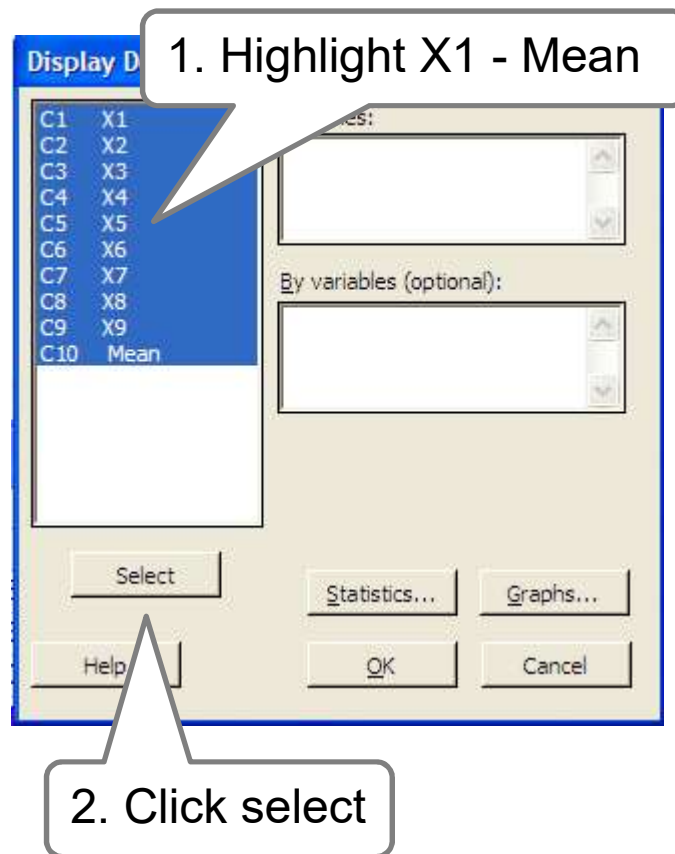
- The dot plot shows that the variation of the means is less than the variation of the X's
- **By how much?**





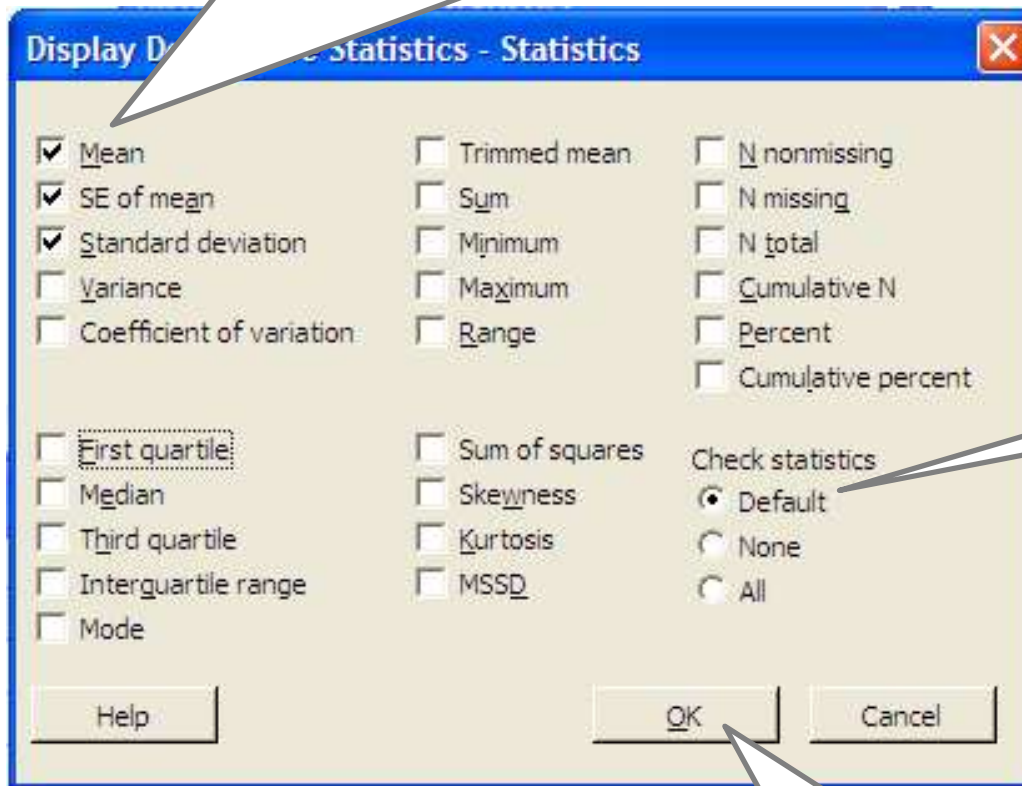
# The Central Limit Theorem, Part 1, cont.

- Click on Stat > Basic Statistics > Display Descriptive Statistics...



# The Central Limit Theorem, Part 1, cont.

1. Select only the first 3 statistics. Unselect all others



2. Select default

3. Click OK

# The Central Limit Theorem, Part 1, cont.

We will explain SE Mean later

The formula

Variable	Mean	SE Mean	StDev
X1	80.006	0.0929	2.937
X2	79.775	0.0962	3.043
X3	80.181	0.0962	3.042
X4	80.140	0.0968	3.062
X5	79.998	0.0952	3.011
X6	79.912	0.0957	3.027
X7	79.987	0.0939	2.970
X8	80.059	0.0927	2.930
X9	79.860	0.0959	3.031
Mean	79.991	0.0308	0.974

$$\text{Std. Dev. Means} = \frac{\text{Std. Dev } Xs}{\sqrt{n}}$$

⇒ Std. Dev. Xs ≈ 3.0

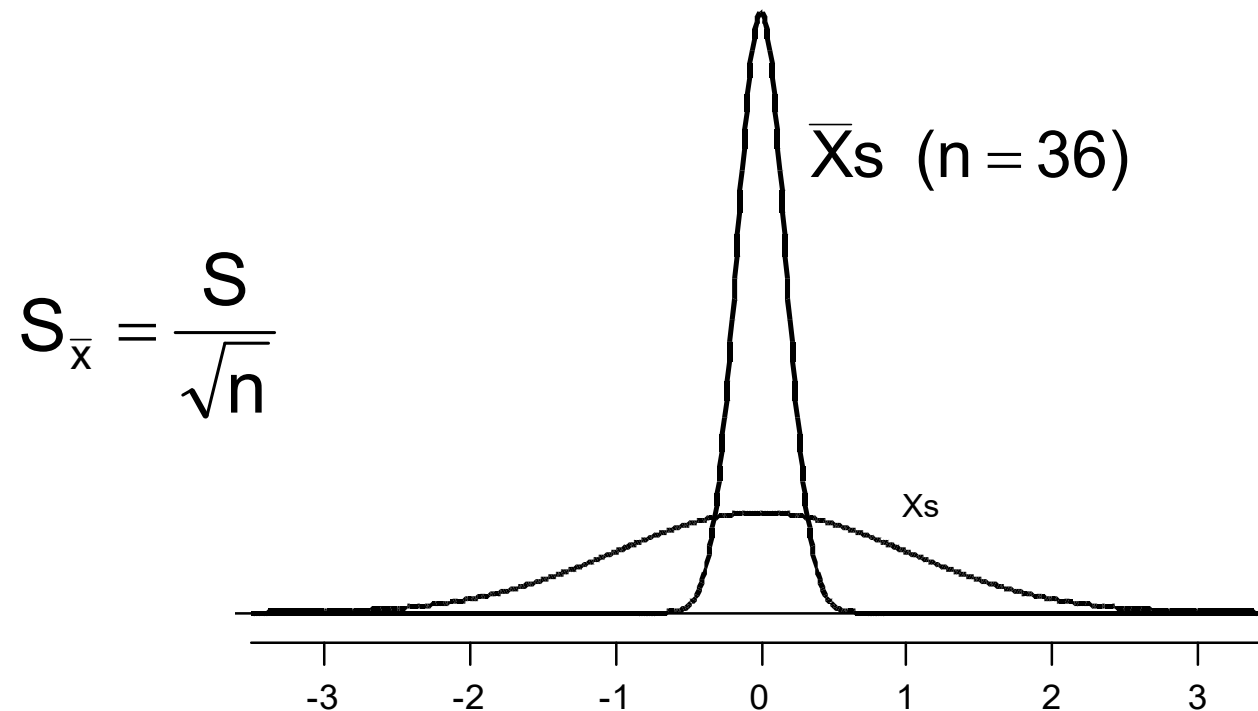
⇒ Std. Dev. Means ≈ 1.0

The formula holds

$$\text{Std. Dev. Means} = 3.0 / \sqrt{9} = 1$$

# The Central Limit Theorem, Part 1: Conclusion

- The standard deviation of averages is less than the standard deviation of individual observations by a factor of  $\frac{1}{\sqrt{n}}$



# The “Standard Error” of the Mean

SE Mean = Standard Deviation of the Mean

Variable	Mean	SE Mean	StDev
X1	80.006	0.0929	2.937
X2	79.775	0.0962	3.043
X3	80.181	0.0962	3.042
X4	80.140	0.0968	3.062
X5	79.998	0.0952	3.011
X6	79.912	0.0957	3.027
X7	79.987	0.0939	2.970
X8	80.059	0.0927	2.930
X9	79.860	0.0959	3.031
Mean	79.991	0.0308	0.974

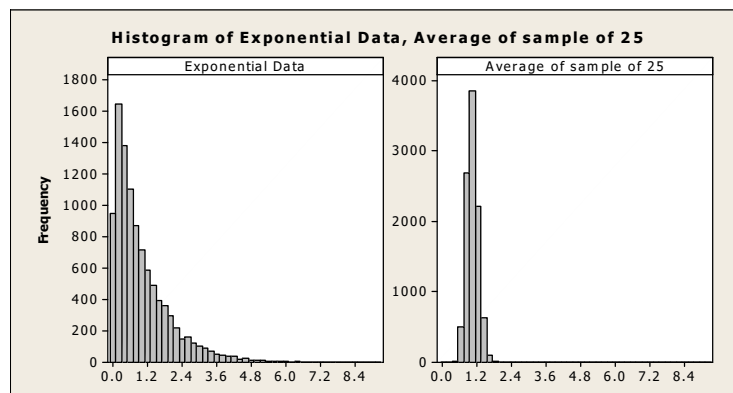
$$\text{Std. Dev. Means} = \frac{\text{Std. Dev X's}}{\sqrt{n}}$$

$$\text{Std. Dev. X1} = 2.937$$

$$\text{SE Mean X1} = \frac{2.937}{\sqrt{1000}} = 0.0929$$

# The Central Limit Theorem, Part 2

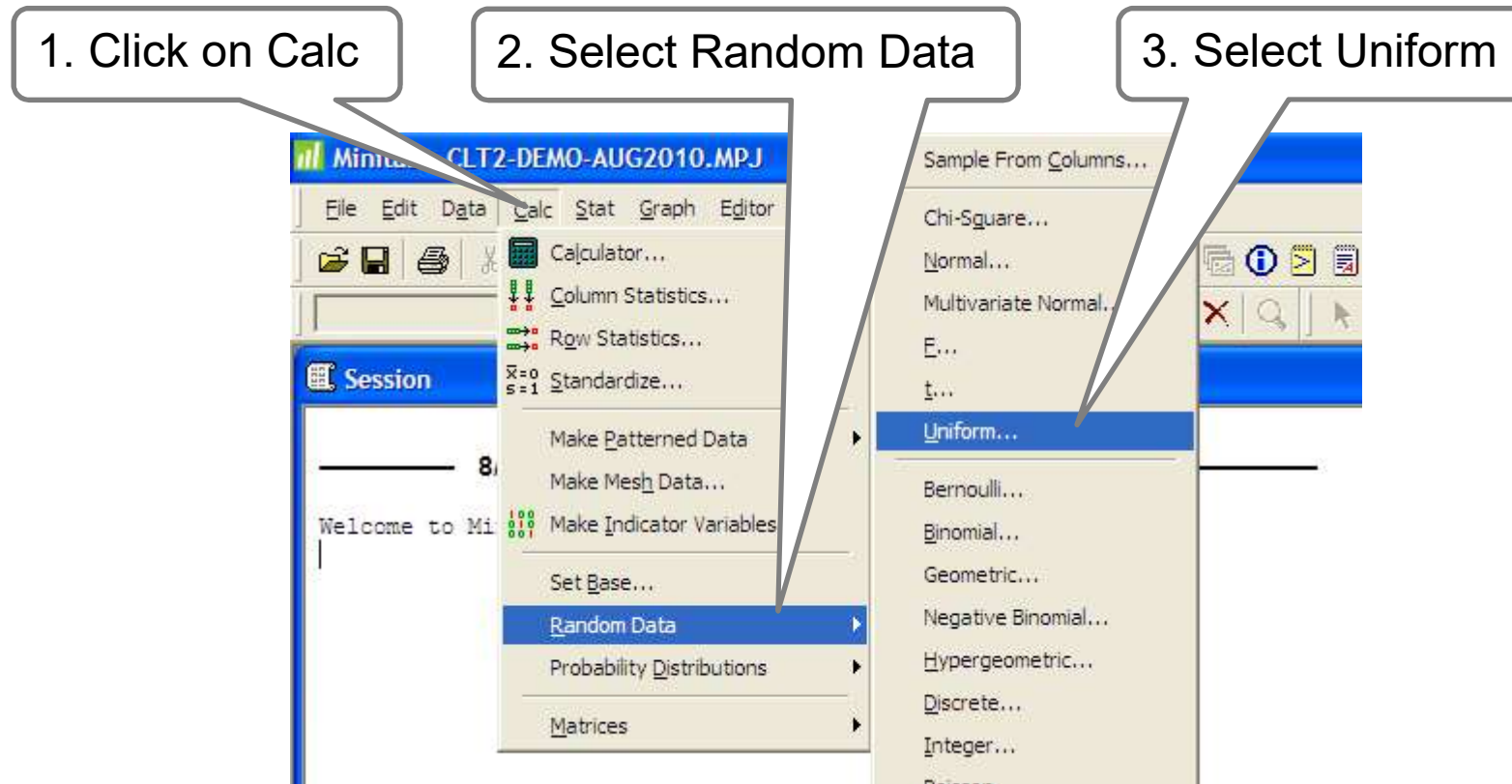
- Part 2 of the Central Limit Theorem states that the averages of independent random data tend to a normal distribution even if the raw data themselves are not normally distributed
- The important role of the normal distribution in statistical theory derives, in large part, from this remarkable property
- Here's a graphical example below. Again, we will use Minitab to illustrate how Part 2 works:



- On the left is a histogram of 10,000 pieces of exponential data
- On the right is the average of 10,000 samples of size 25 taken from the distribution on the left
- The averages are normal even though the values being averaged are far from normal

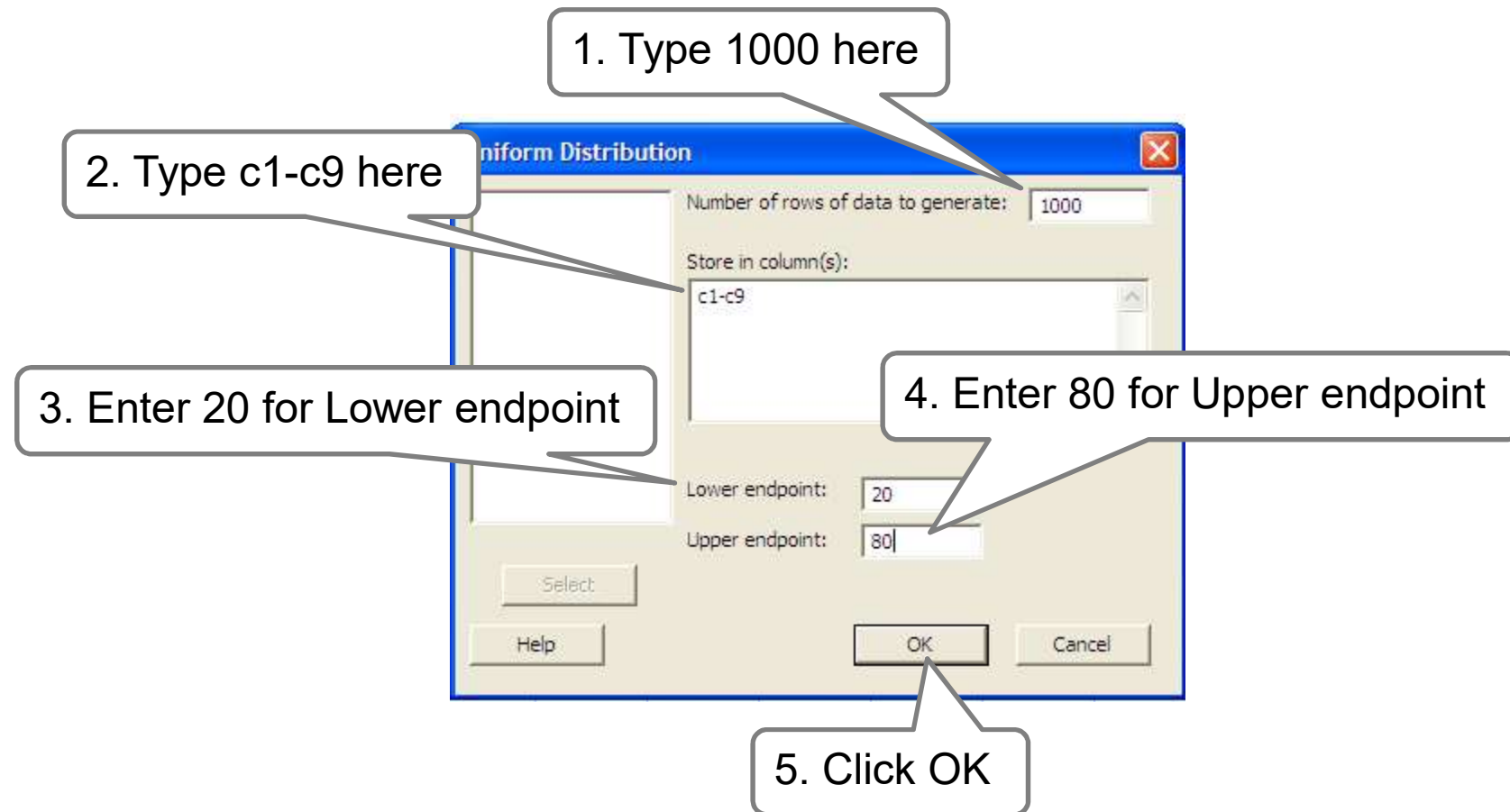
# The Central Limit Theorem, Part 2

- Open a blank Minitab worksheet



# The Central Limit Theorem, Part 2, cont.

- Have Minitab generate random normal data as shown below:



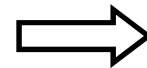


# The Central Limit Theorem, Part 2, cont.

**Output:** your numbers will be different

	C1	C2	C3	C4	C5	C6	C7	C8	C9
1	25.1980	55.9567	55.2737	78.1289	54.9771	45.0978	51.2510	28.0212	77.8197
2	68.9381	66.6212	29.1924	44.3467	47.7908	40.9916	73.9495	70.4063	31.2044
3	32.2529	56.7699	41.1999	31.1444	47.1724	45.3103	65.2176	37.1441	60.9313
4	36.4569	46.5723	39.7521	55.9497	66.7198	61.6150	53.0317	64.9298	35.4842
5	49.6973	43.3836	54.2835	61.6928	79.3652	25.5048	75.4674	66.3605	30.0810

**Next:** Type X1...X9 as shown



Type Mean as C10's name

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
	X1	X2	X3	X4	X5	X6	X7	X8	X9	Mean
1	25.1980	55.9567	55.2737	78.1289	54.9771	45.0978	51.2510	28.0212	77.8197	
2	68.9381	66.6212	29.1924	44.3467	47.7908	40.9916	73.9495	70.4063	31.2044	
3	32.2529	56.7699	41.1999	31.1444	47.1724	45.3103	65.2176	37.1441	60.9313	
4	36.4569	46.5723	39.7521	55.9497	66.7198	61.6150	53.0317	64.9298	35.4842	
5	49.6973	43.3836	54.2835	61.6928	79.3652	25.5048	75.4674	66.3605	30.0810	

# The Central Limit Theorem, Part 2, cont.

- From the main menu, click on Calc > Row Statistics

The image shows the 'Row Statistics' dialog box in Minitab. The 'Statistic' section has 'Mean' selected. The 'Input variables' field contains 'X1-X9'. The 'Store result in:' field contains 'Mean'. The 'Select' button is highlighted. Five callout boxes provide instructions: 1. Select Mean (pointing to the Mean radio button), 2. Highlight X1 through X9 (pointing to the variable list), 3. Click on Select (pointing to the Select button), 4. Type Mean here (pointing to the Store result in: field), and 5. Click OK (pointing to the OK button).

# The Central Limit Theorem, Part 2, cont.

**Output:** your numbers will be different

↓	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
	X1	X2	X3	X4	X5	X6	X7	X8	X9	Mean
1	25.1980	55.9567	55.2737	78.1289	54.9771	45.0978	51.2510	28.0212	77.8197	52.4138
2	68.9381	66.6212	29.1924	44.3467	47.7908	40.9916	73.9495	70.4063	31.2044	52.6046
3	32.2529	56.7699	41.1999	31.1444	47.1724	45.3103	65.2176	37.1441	60.9313	46.3492
4	36.4569	46.5723	39.7521	55.9497	66.7198	61.6150	53.0317	64.9298	35.4842	51.1679
5	49.6973	43.3836	54.2835	61.6928	79.3652	25.5048	75.4674	66.3605	30.0810	53.9818
6	51.6288	46.4577	45.5643	35.4001	35.3005	49.4366	54.5525	77.2835	78.1380	52.6402
7	53.3256	27.4623	59.7192	76.4494	52.5922	71.7285	20.5115	47.9774	70.1768	53.3270
8	49.7997	75.0673	65.6376	61.5809	21.2702	64.0382	76.9825	51.5027	74.3060	60.0206
9	42.8136	71.2806	43.1861	74.0209	76.3645	20.6063	75.5673	66.1525	58.3835	58.7084
10	64.8112	57.6858	69.5942	35.4901	75.3026	71.5510	58.6209	77.9676	69.0517	64.4528

- **Next:** Compare the distribution of the X's with the distribution of the Mean

# The Central Limit Theorem, Part 2, cont.

- From the main menu, click on Graph > Dotplot...

1. Select Multiple Y's Simple

2. Click OK

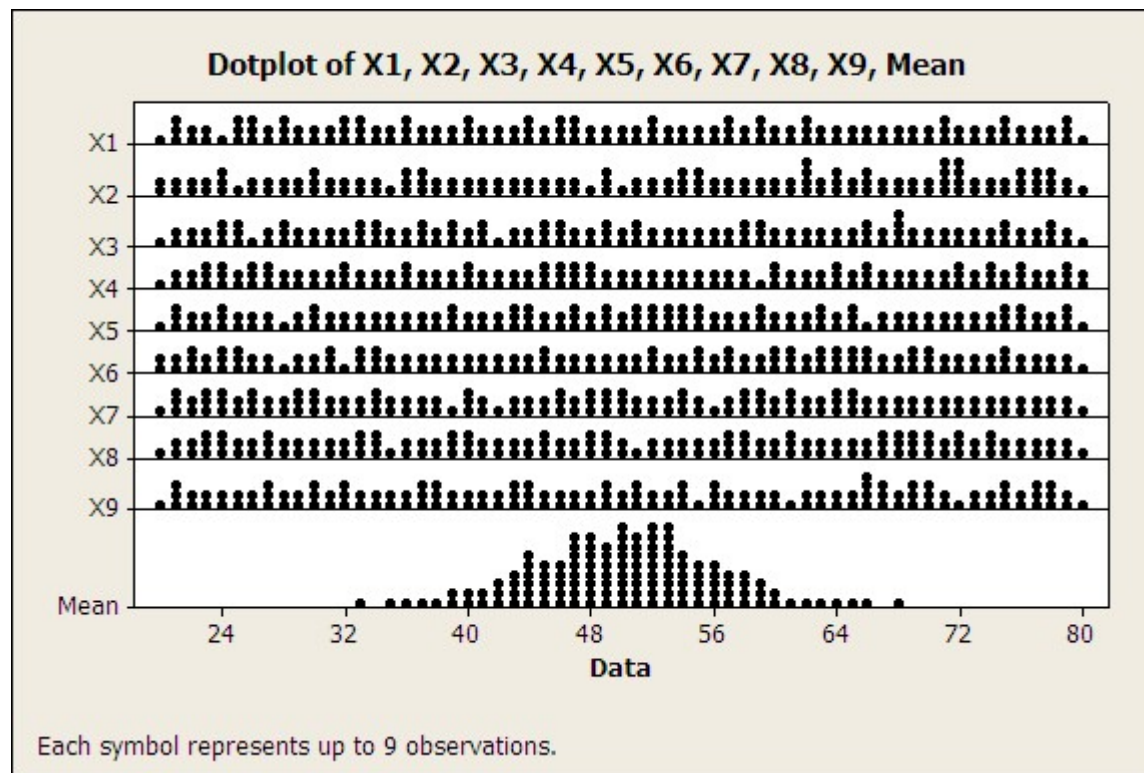
3. Highlight X1 through Mean

4. Click Select

5. Click OK

# The Central Limit Theorem, Part 2: Conclusion

- The dot plot shows that even though the distribution of the X columns is uniform, the distribution of the Mean column approximates the normal distribution



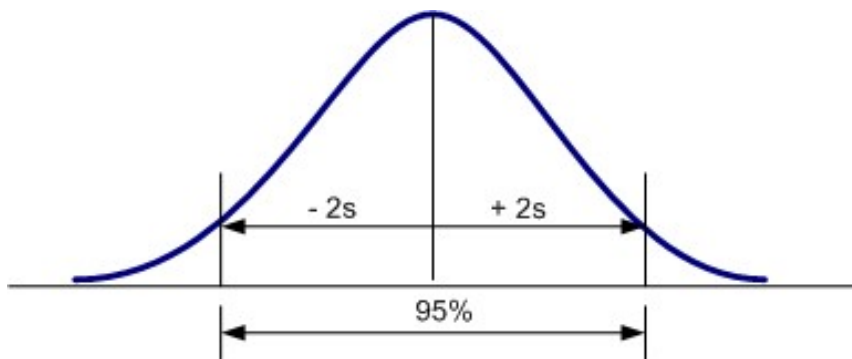
# Confidence Intervals

- An important application of the Central Limit Theorem and the normal distribution is the concept of confidence interval
- The issue: the need to know the mean of a “population” to make business decisions
- The question: how much confidence do we have when using a representative sample?
- The answer: calculate a confidence interval from the sample that is likely to contain the mean of the population



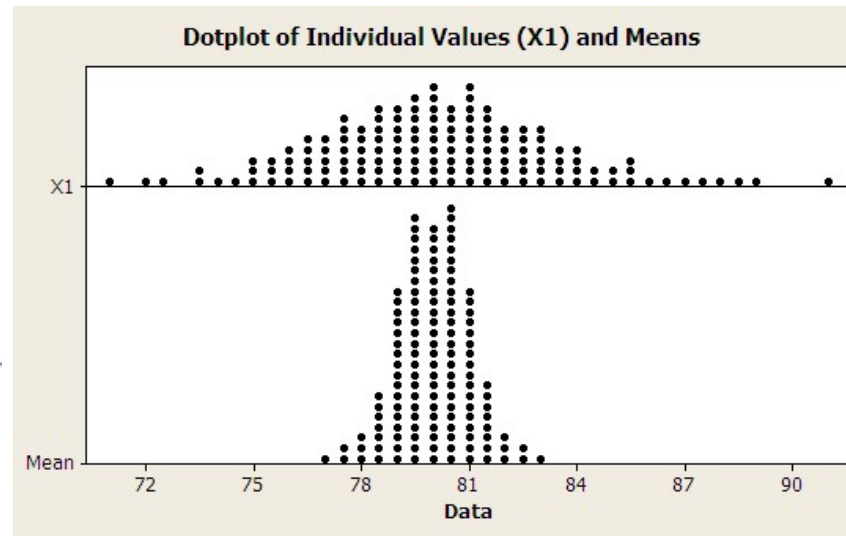
# Confidence Intervals, cont.

From the normal distribution



95% of the data within  $\pm 2$  std. dev.

From the Central Limit Theorem



$$\text{Std. Dev. Means} = \frac{\text{Std. Dev } Xs}{\sqrt{n}}$$

## Confidence Intervals, cont.

- A 95% confidence interval on an average is given by the formula:

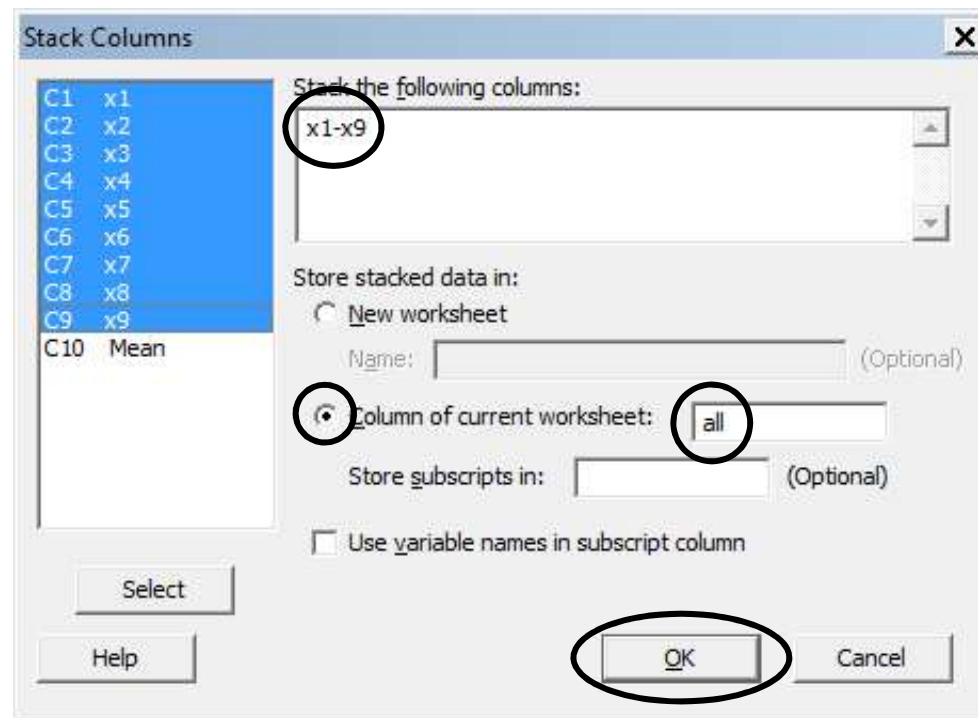
$$\bar{X} \pm 2 \frac{S}{\sqrt{n}}$$

- Because roughly 95% of normal data falls within 2 standard deviations of the mean, we can use the above formula to compute the 95% confidence interval on a mean or average



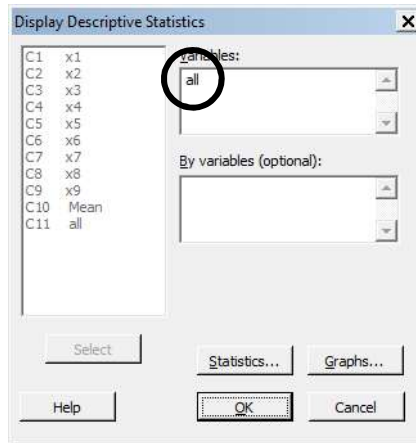
# Calculating the 95% Confidence Interval

- Stack columns 1-9 and compute the standard deviation:  
Data > Stack > Columns



# Calculating the 95% Confidence Interval, cont.

- Display the basic statistic for column all  
Stat > Basic Statistics > Display Basic Statistics



## Descriptive Statistics: all

Variable	N	N*	Mean	SE Mean	StDev	Minimum	Q1	Median	Q3
all	9000	0	49.711	0.183	17.382	20.002	34.601	49.829	64.743

Variable	Maximum
all	79.992

- The true mean of all the data we generated was 50.
- 95% of the time the confidence interval around our overall mean (in the above example, 49.711) will contain the true mean, 50. Here, the CI is  $49.711 \pm 2 \times 0.183 = (49.345 \text{ to } 50.08)$  and contains the true mean. Does your CI contain the mean?

# Summary

- Many statistical tools are built based on the assumption of normality (e.g., process capability)
- Understanding the properties and characteristics of the normal distribution is indispensable to Black Belts
- The Central Limit Theorem (CLT) impacts many statistical tools (e.g., control charts, hypothesis testing)
- Confidence intervals measure the “uncertainty” there is in our conclusions

# Anexas Consultancy Services

## Sample Size Calculations

# Sample Size

Purpose of Sample	Formula
<p><b>Estimate average</b> (e.g., determine baseline cycle time) CONTINUOUS / VARIABLE DATA</p>	$n = \left( \frac{2s}{d} \right)^2$ <p>(Where d = precision: ± __ units) S = estimate of standard deviation</p>
<p><b>Estimate proportion</b> (e.g., determine baseline % defective) ATTRIBUTE / DISCRETE DATA</p>	$n = \left( \frac{2}{d} \right)^2 (p)(1-p)$ <p>(Where d = precision: ± __ units) P = proportion of defective in the output</p>

## Thumb Rule

Variable Data = Optimum Sample Size is 30 to 50  
 Attribute Data = Optimum sample size is 200 to 1100

# Precision (d)

- Precision is how narrow you want the range to be for an estimate of a characteristic
  - Estimate cycle time within 2 days
- Use the symbol  $d$  to represent precision
- Precision is equal to half the width of a confidence interval
  - A 95% CI = (48, 52) for cycle time (in days) means we are 95% confident the interval from 48 days to 52 days contains the average cycle time
  - Width of the CI = 4 days
  - Precision =  $d$  = 2 days (= estimate is within  $\pm 2$  days)
- You decide what precision you want
- Precision is inversely proportional to the square root of sample size  $\frac{1}{\sqrt{n}}$

# Precision and Sample Size

- To improve precision, you need to increase sample size (which incurs more cost)
- There is no clear-cut answer about how much precision you need; the answer depends on the business impact of using the estimate
- Each situation is unique; don't pattern your decisions after someone else's decision
- As a thumb rule 3% to 4% of the mean (for Continuous data) or 3% of uncertainty in proportion estimate (for Attribute data) can be taken as precision in absence of any guideline

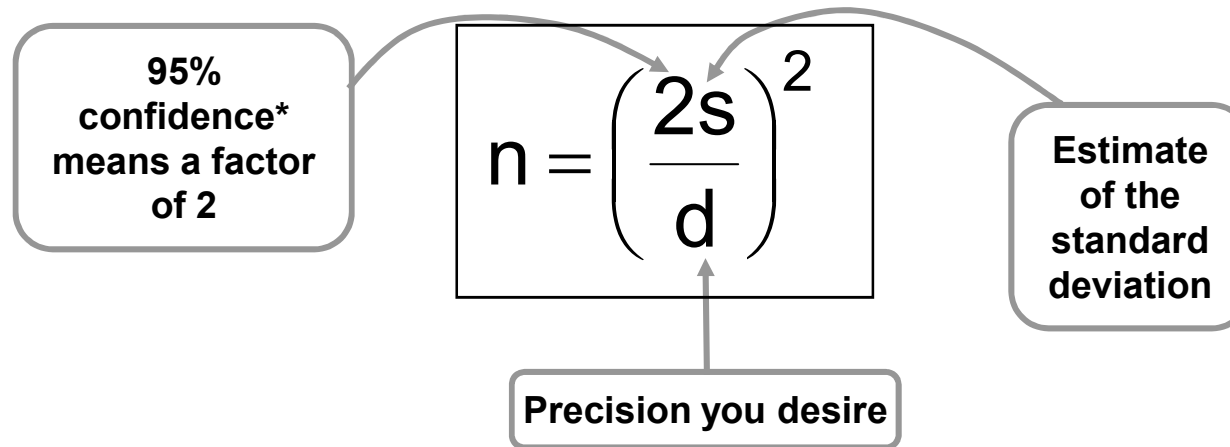
# Example: Choosing a Precision Level

## Background:

- Suppose you work in a telephone call center that usually answers 2,000 incoming calls/day
  - You want to time a sample of calls to find average call length
- Before advising you on how many calls to sample, a quality consultant asks what level of precision you want in the estimate: “Does your estimate have to be within 5 minutes? 1 minute? 30 seconds?”
- You intend to use the average call length to help you make staffing decisions
  - You figure that if average call length changes by 1/4 minute, you will need to add (or drop) a staff person to answer incoming calls
- Half of 1/4 minute is 1/8 minute, so the quality consultant assigns the value of 1/8 minute to  $d$ , the precision
  - The range of your estimate will therefore be  $\pm 1/8$  minute



# Sample Size for Estimating an Average



# Sample Size for Estimating an Average, cont.

## Example

- What is the sample size required to estimate the average cycle time (in days) of a new process?

- Historical data from similar processes show a typical standard deviation = 5 days and you want the estimate to be within 1 day

$$n = \left( \frac{(2)(5)}{1} \right)^2 = 10^2 = 100$$

- How does the sample size change if you want the estimate to be within 2 days?

$$n = \left( \frac{(2)(5)}{2} \right)^2 = 5^2 = 25$$

## Exercise: Sample Size for Estimating Averages

Objective: Practice using sample size formula for averages

Instructions:

- Use the sample size formula for averages to answer the following questions 
$$n = \left( \frac{2s}{d} \right)^2$$
- 1. **Supposing you want to estimate** the average length of incoming phone calls within 1 minute, what sample size do you need?  
(Historical data show a typical standard deviation = 3 minutes)
- 2. **How many calls do you need** to sample to get an estimate within 1/8 minute?

Time: 3 minutes

# Exercise Answers: Sample Size for Estimating Averages

Question 1

$$n = \left( \frac{(2)(3)}{1} \right)^2 = 6^2 = 36$$

Question 2

$$n = \left( \frac{(2)(3)}{1/8} \right)^2 = 48^2 = 2304$$

# How to Estimate the Standard Deviation

## The Catch-22

- To estimate sample size, you need to know the standard deviation
- You need to have some idea of the amount of variation in the data because as the variability increases, the necessary sample size increases
- But if you haven't sampled anything yet, how can you know the standard deviation?

## How to Estimate the Standard Deviation, cont.

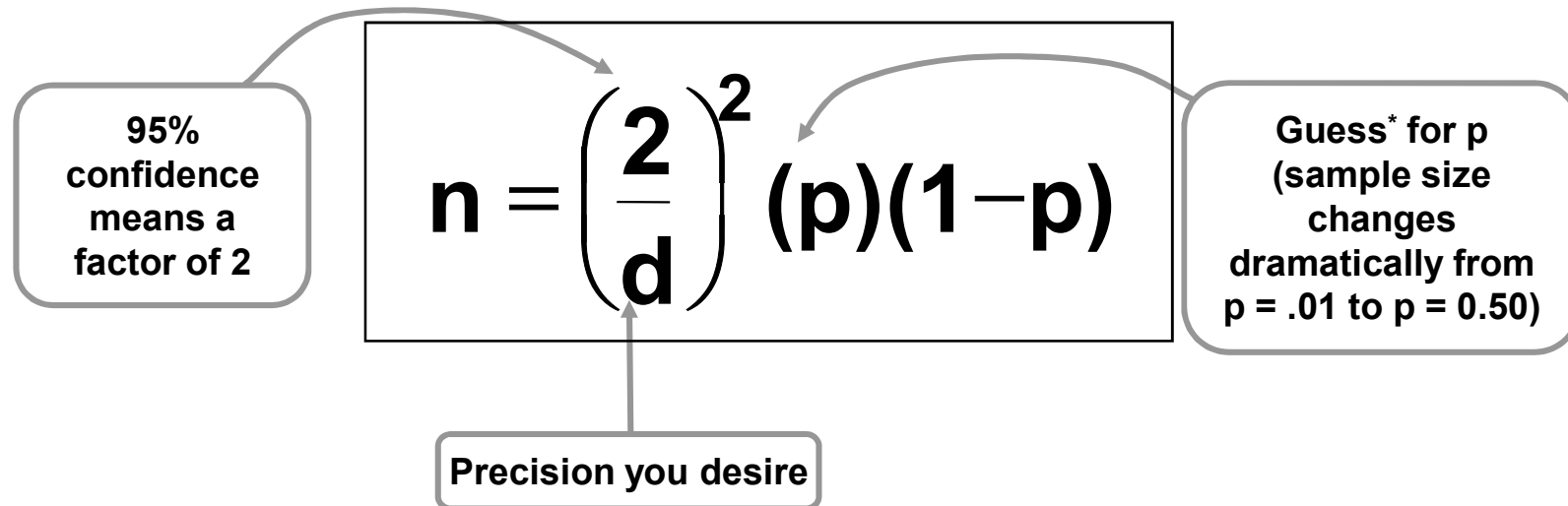
Options for estimating standard deviation:

- Find existing data and calculate  $s$
- Use a control chart (for individuals) from a similar process

$$s = \frac{UCL - LCL}{6} \quad \text{or} \quad s = \frac{UCL - \text{Average}}{3}$$

- Collect a small sample and calculate  $s$
- Take an educated guess based on your process knowledge and memory of similar data (most people are not too good at this)

# Sample Size for Estimating a Proportion



# Sample Size for Estimating a Proportion, cont.

## Example

- What is the sample size required to estimate the defect rate, with 95% confidence, when you expect it to be around “10% defective” and you want the estimate to be within 3%?
- How does the sample size change if you want the estimate to be within  $\pm 1\%$ ?

$$\begin{aligned}n &= \left(\frac{2}{.03}\right)^2 (.10)(1-.10) \\ &= (4444.4)(.10)(.90) \\ &= 400\end{aligned}$$

$$\begin{aligned}n &= \left(\frac{2}{.01}\right)^2 (.10)(1-.10) \\ &= 3600\end{aligned}$$



# Exercise: Sample Size for Estimating Proportions

Objective: Practice using the sample size formula for proportions

Instructions:

$$n = \left( \frac{2}{d} \right)^2 (p)(1-p)$$

■ Use the sample size formula for proportions to answer the following questions

**1. Suppose you want to estimate** (within 2%) the proportion of customers who will buy a new service

- You're guessing that 50% of them will buy
- What sample size do you need?

**2. How many customers do you need** to sample for your estimate to be within 4%?

Time: 3 minutes

# Exercise Answers : Sample Size for Estimating Proportions

Question 1

$$n = \left( \frac{2}{.02} \right)^2 (.5)(1-.5)$$
$$= 2500$$

Question 2

$$n = \left( \frac{2}{.04} \right)^2 (.5)(1-.5)$$
$$= 625$$

# Using Precision to Justify Sample Sizes

1. Determine how many samples you can afford (n)
2. Then ask, what will that sample provide in terms of precision?

– That is, an average within  $\pm d$  units  $d = \frac{2s}{\sqrt{n}}$

– Or a proportion within  $\pm d\%$   $d = 2\sqrt{\frac{(p)(1-p)}{n}}$

3. Is that precise enough?

4. If not precise enough:

- Make a table of precision and cost for various sample sizes to determine the gain in precision per dollar spent on samples
- Then choose a sample size that you can justify based on the gain in precision or the needed precision for your situation

# Sampling from a Limited (Finite) Population

- The sample size formulas assume the sample size (n) is small relative to the population (N)
- If  $\frac{n}{N}$  is  $> .05$ 
  - You are sampling more than 5% of the population
  - You can adjust the sample size with the finite population formula:

$$n_{\text{finite}} = \frac{n}{1 + \frac{n}{N}}$$

# Sampling from a Limited (Finite) Population, cont.

## Example

- You want to know the average assembly cycle time, within  $\pm 1.5$  days, of a complex device your facility assembles
- Given a st. dev. of 12 days (from another study of cycle time), the number of files to sample is 256

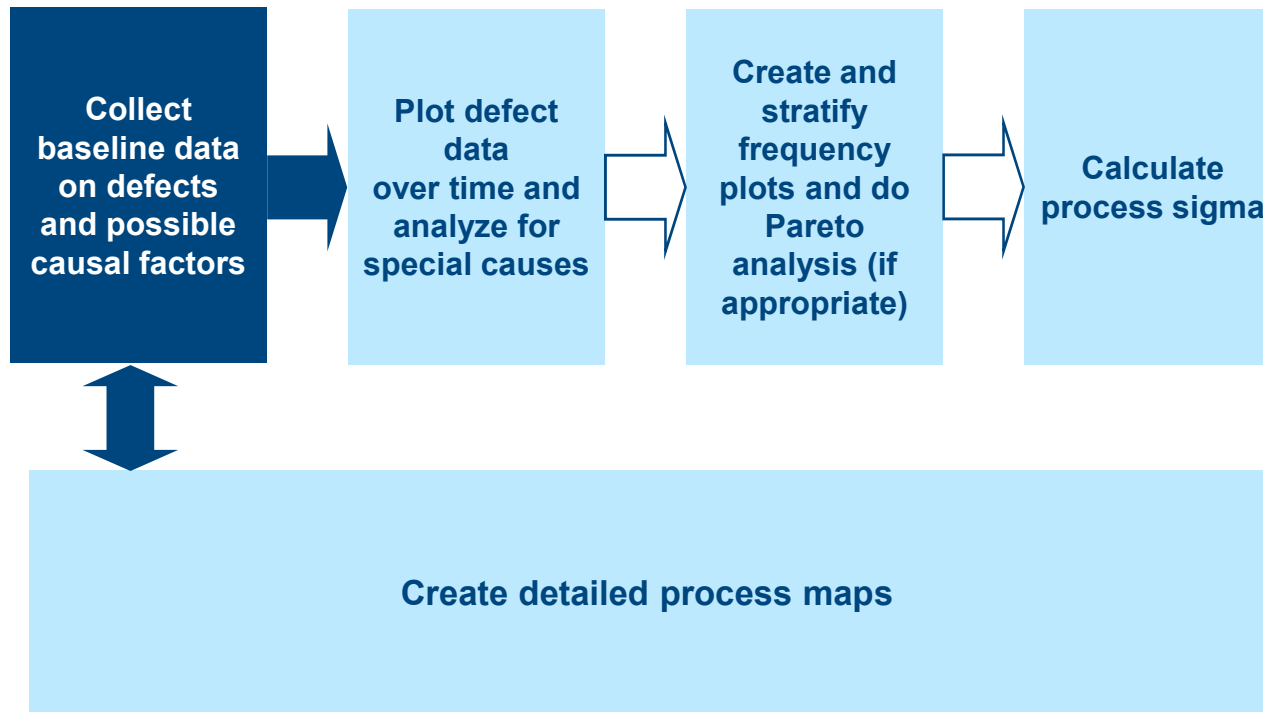
$$n = \left( \frac{24}{1.5} \right)^2 = 256$$

- Suppose that only 300 units were made in the last year, since the cycle time data started to be collected
  - Do you need to sample nearly all 300 units?
  - No, you can use the finite population formula
  - Applying finite population formula, the sample size would be 140

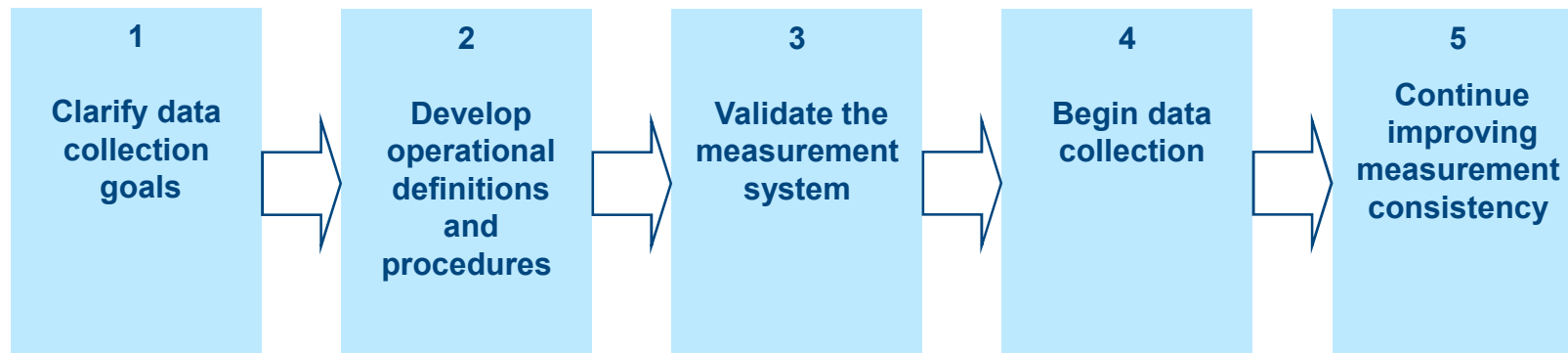
# Anexas Consultancy Services

## Data Collection

# You Are Here



# Five-Step Data Collection Process





# Project Worksheet: Data Collection Plan (Preview)

**Data Collection Plan**      **Project** \_\_\_\_\_

What questions do you want to answer?

Data		Operational Definition and Procedures			
What	Measure type/ Data type	How measured <sup>1</sup>	Related conditions to record <sup>2</sup>	Sampling notes	How/where recorded (attach form)

How will you ensure consistency and stability?

What is your plan for starting data collection? (attach details if necessary)

How will the data be displayed? (Sketch below)

**Notes**

1) Be sure to test and monitor any measurement procedures/instruments.  
 2) "Related factors" are stratification factors or potential causes you want to monitor as you collect data.

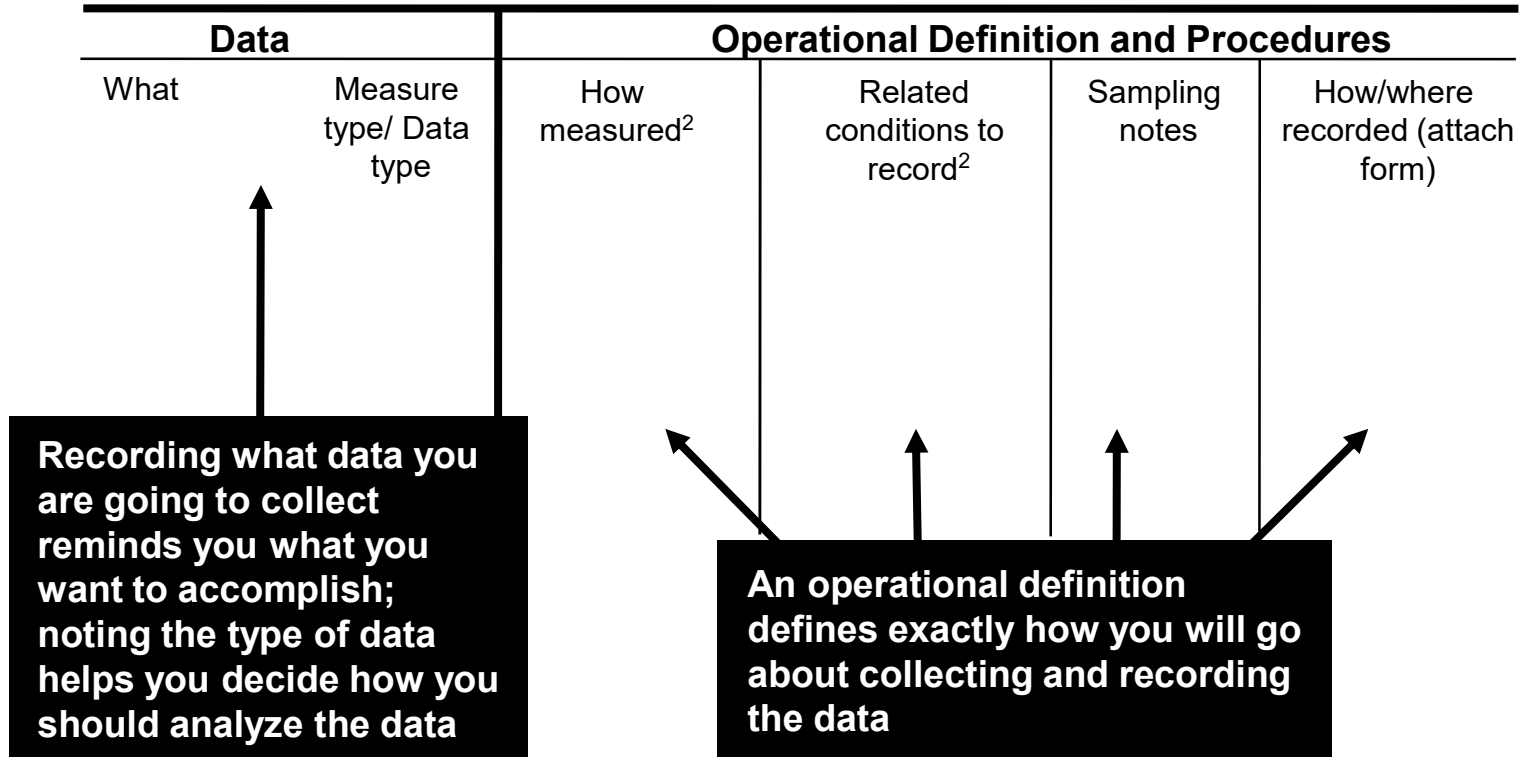
# Data Collection Plan Features

## Data Collection Plan

Project \_\_\_\_\_

What questions do you want to answer?

**Being clear about your question will help you make sure you collect the right data**



# Data Collection Plan Features, cont.

How will you ensure consistency and stability?

**What will you do to make sure the data collected at one point in time are comparable to data collected at other times? That is, that no biases have been introduced in the way the data are collected?**

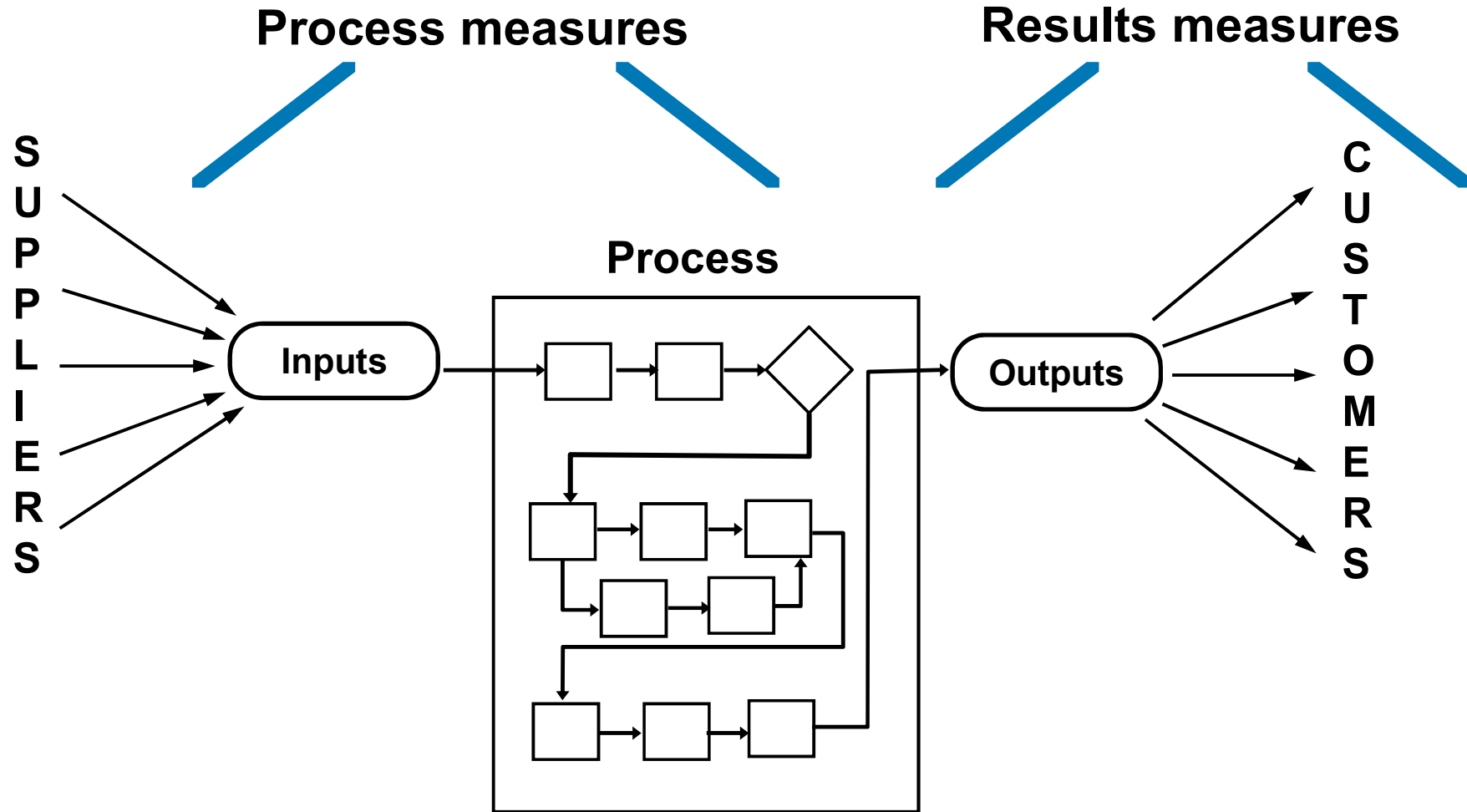
What is your plan for starting data collection? (Attach details if necessary)

**Just how will you go about collecting the data?**

How will the data be displayed? (Sketch below)

**Thinking about how you will display the data will help you make sure you're getting the right kind of data to answer the question you have in mind**

# Where to Collect Data: Process or Result?



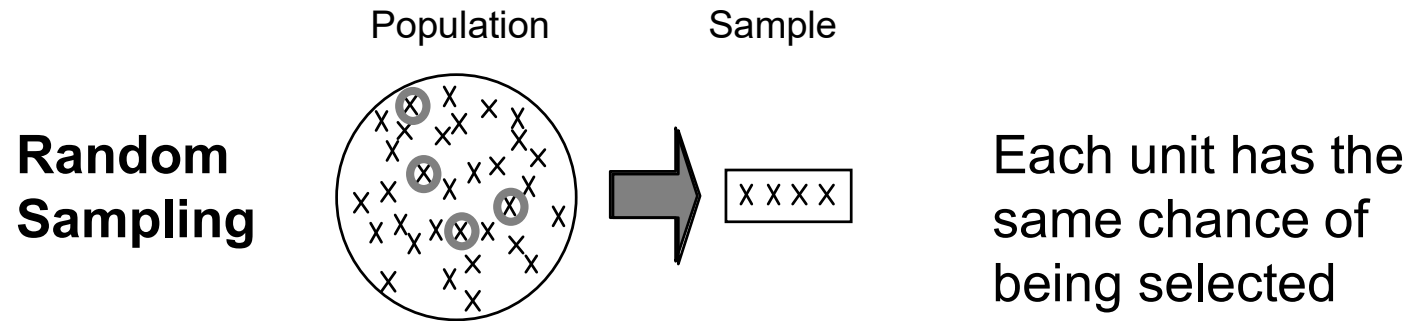
# Examples of Data

Type/How Obtained	Examples
<p><b>Continuous:</b> (or “variables”)</p> <p>Measuring instrument or a calculation</p>	<p><b>Service:</b> Elapsed time to complete transaction, average length of phone calls</p> <p><b>Manufacturing:</b> Elapsed cycle time, metal purity, gauge production rates, weight, length, speed</p> <p><b>Both:</b> Budget vs. actual (dollars), average customer satisfaction score, amount purchased</p>
<p><b>Discrete:</b> Percentage or proportion</p> <p>Count occurrences and non occurrences</p>	<p><b>Service:</b> Proportion of late applications, incorrect invoices</p> <p><b>Manufacturing:</b> Proportion of defective items, reworked items, damaged items, late shipments</p> <p><b>Both:</b> Proportion of employees absent, incomplete orders</p>
<p><b>Discrete:</b> Count</p> <p>Count occurrences in an area of opportunity</p>	<p><b>Service:</b> Number of applications, errors, complaints, etc.</p> <p><b>Manufacturing:</b> Number of computer malfunctions, machine breakdowns, accidents</p>
<p><b>Discrete:</b> Attribute</p> <p>Observation</p>	<p><b>Service:</b> Type of application, type of request</p> <p><b>Manufacturing:</b> Type of product</p> <p><b>Both:</b> Type of customer, type of method used (new vs. old), location of activity (city/state)</p>
<p><b>Discrete:</b> Ordinal</p> <p>Observation or ranking</p>	<p><b>Both:</b> Customer rating (1=very satisfied/ 5= very dissatisfied); day of week (MTWRF), date, time order</p>

# Anexas Consultancy Services

## Sampling Approaches

# Selecting a Random Sample



- Many people think they can select a random sample from a list without the aid of statistical techniques
- Statistician Brian Joiner likes to point out that “haphazard sampling is not the same as random sampling”
- When trying to choose a sample, do not merely choose whatever items you like from a list or population

# Minitab Follow-Along: Select a Random Sample

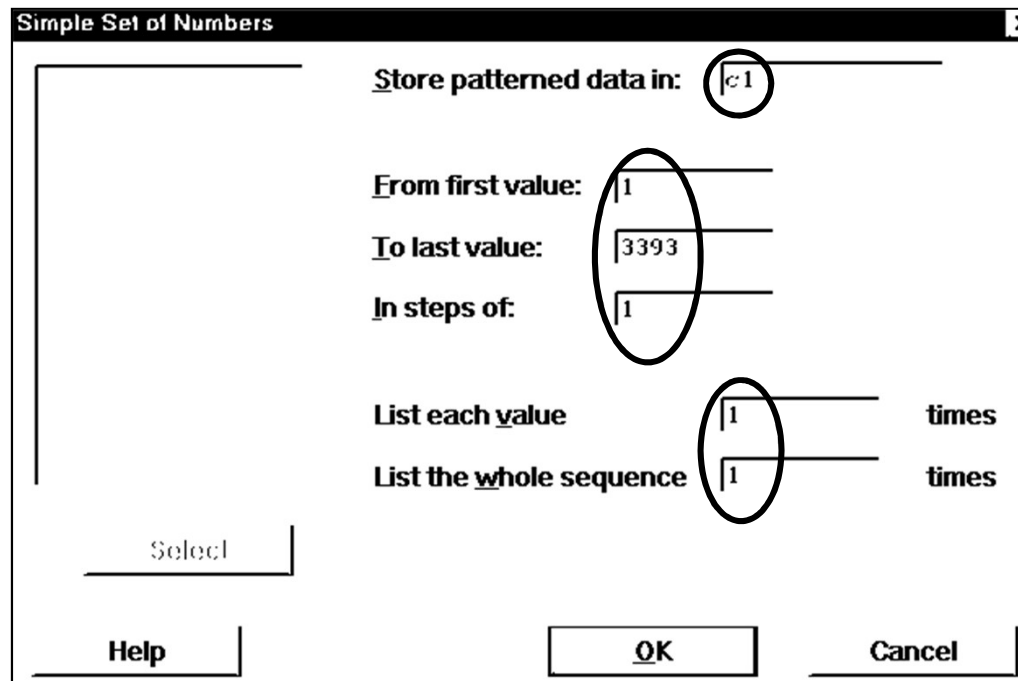
**Background:** Suppose you have 3,393 customers listed in alphabetical order and you wish to randomly sample 100 of them to determine their expectations regarding cycle time for resolving complaints



# Minitab Follow-Along: Select a Random Sample, cont.

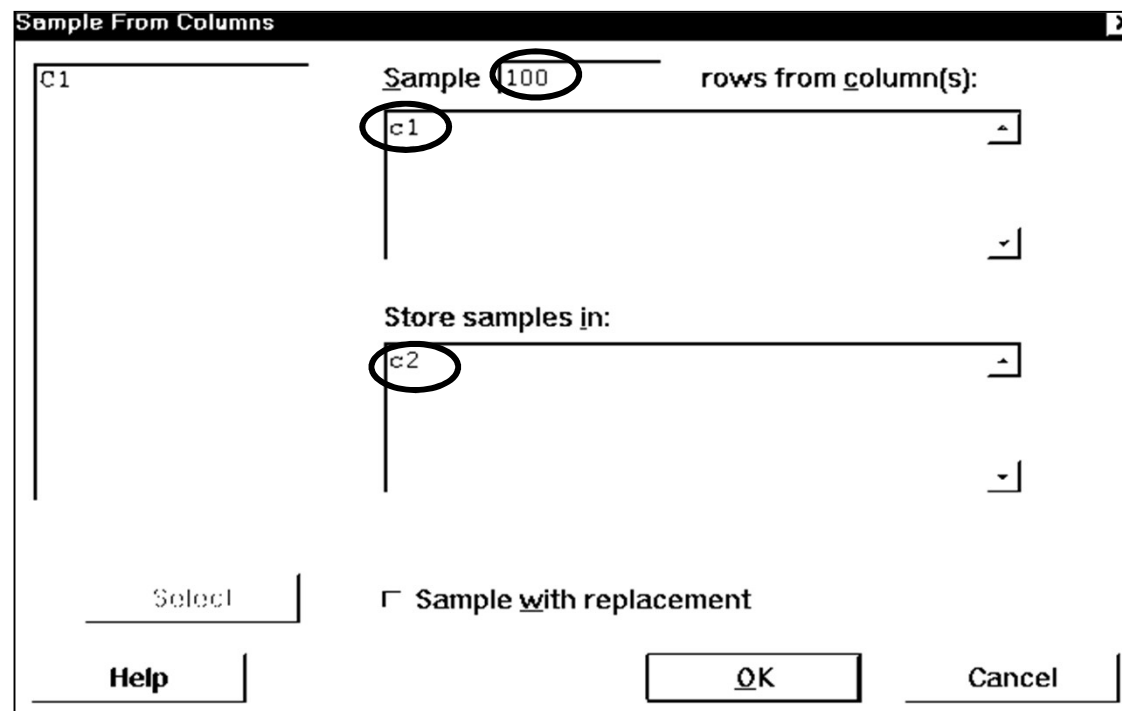
## Instructions:

1. Open a blank worksheet in Minitab
2. Generate ID numbers between 1 and 3,393 and store them in C1  
Calc > Make Patterned Data > Simple Set of Numbers



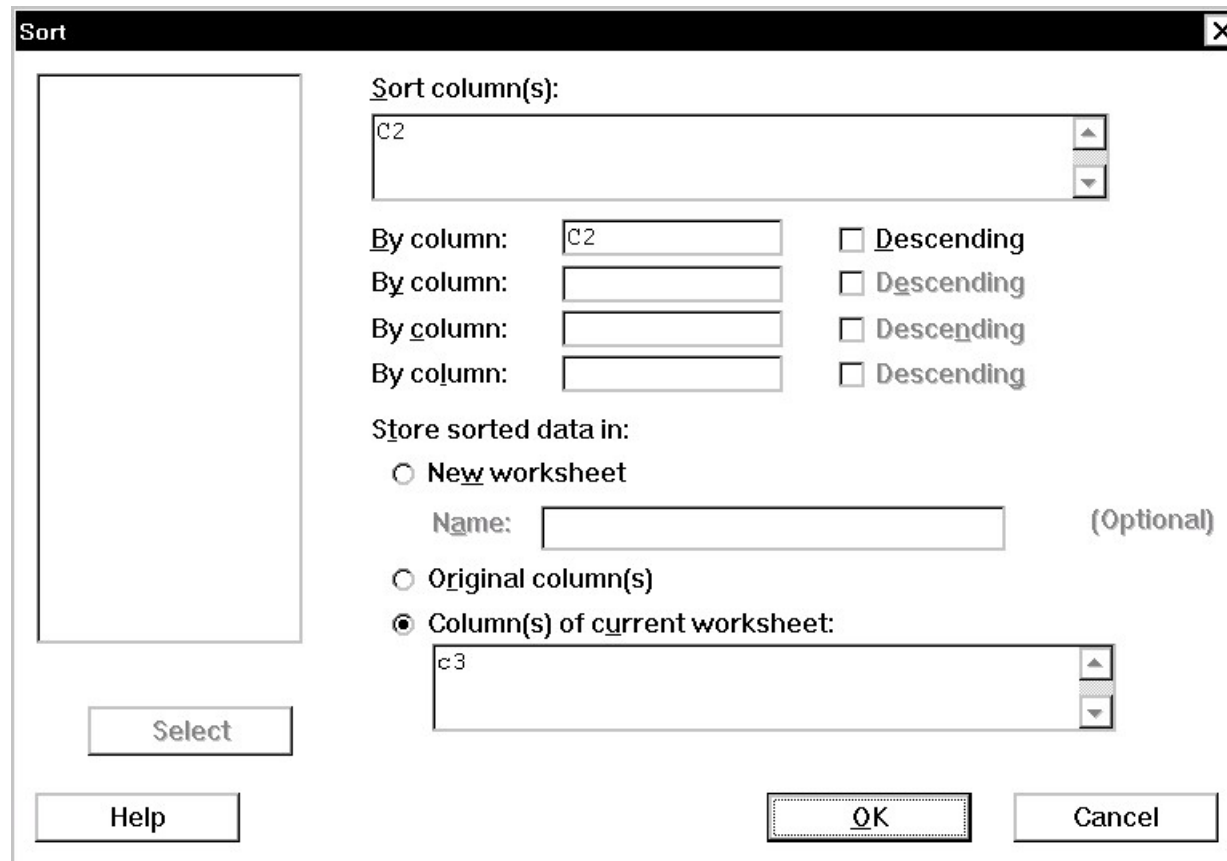
# Minitab Follow-Along: Select a Random Sample, cont.

3. Randomly select 100 ID numbers from C1 and store them in C2  
Calc > Random data > Sample from Columns > ...



# Minitab Follow-Along: Select a Random Sample, cont.

- Sort the sample ID numbers in order and store them in C3  
Data > Sort...



# Minitab Follow-Along: Select a Random Sample, cont.

5. Compare your sample with your neighbor's sample. Did you get the same ID numbers?
6. How would you use the ID numbers in C3 to select your sample?

# How Random Sampling Ensures Representativeness of a Population

- Even with small populations, there are many ways that a random sample could turn out
  - Example: There are 1.73E13 (17.3 trillion) ways of selecting 10 units randomly from 100 units

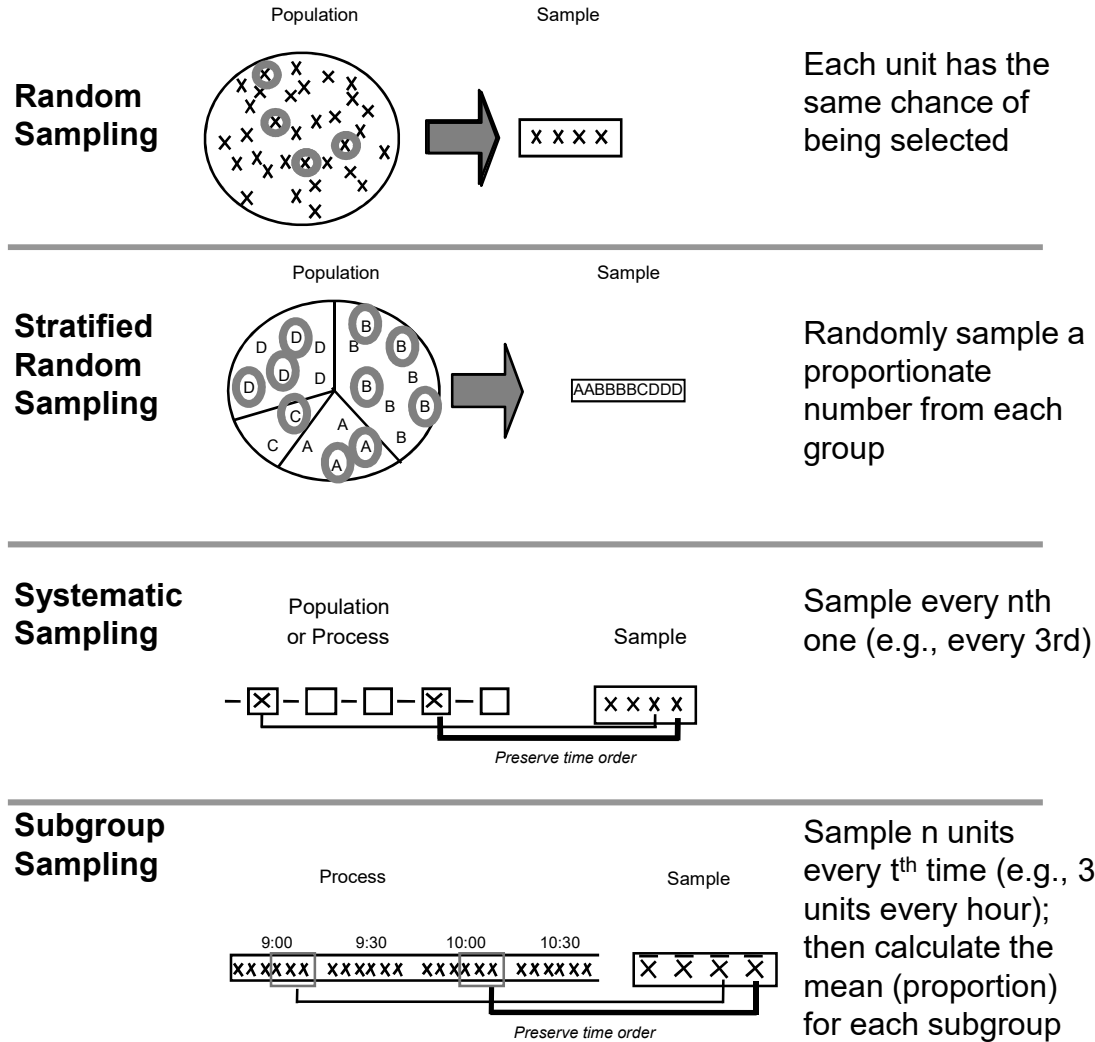
**Number of Ways to Select n Units From N**

Sample size (n)	Population size (N)		
	100	1000	10000
10	1.73 E <sup>13</sup>	2.63E <sup>23</sup>	2.74E <sup>33</sup>
100	1	6.39E <sup>139</sup>	6.52E <sup>241</sup>
1000		1	8.73E <sup>1409</sup>

# How Random Sampling Ensures Representativeness of a Population, cont.

- There are relatively few ways to choose a sample that is systematically different from the rest of the units
  - Example: It is possible to randomly draw the 10 smallest (or largest) units from a list of 100, but the chance is astronomically small (2 in 17.3 trillion)
- The vast majority of the trillions of combinations will adequately represent the diversity of the population as a whole
- Random sampling ensures the sample is not chosen to make a predetermined (biased) case—even unconsciously—because each unit has the same chance of being chosen

# Sampling Approaches



# Anexas Consultancy Services

## Measurement Systems Analysis (MSA)



# Why Measure?

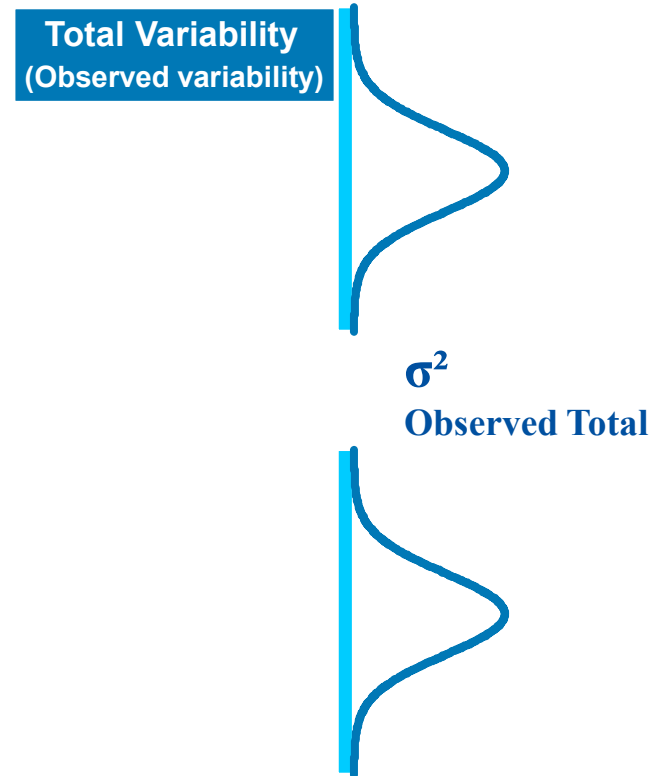
- To understand a decision:
  - Meet standards & specifications
  - Detection / reaction oriented
  - Short-term results
- Stimulate continuous improvement:
  - Where to improve?
  - How much to improve?
  - Is improvement cost effective?
  - Prevention oriented
  - Long-term strategy

**“If you cannot measure, you cannot improve!”  
--Taguchi**

# Observed Variation

**Process A**

**Process B**

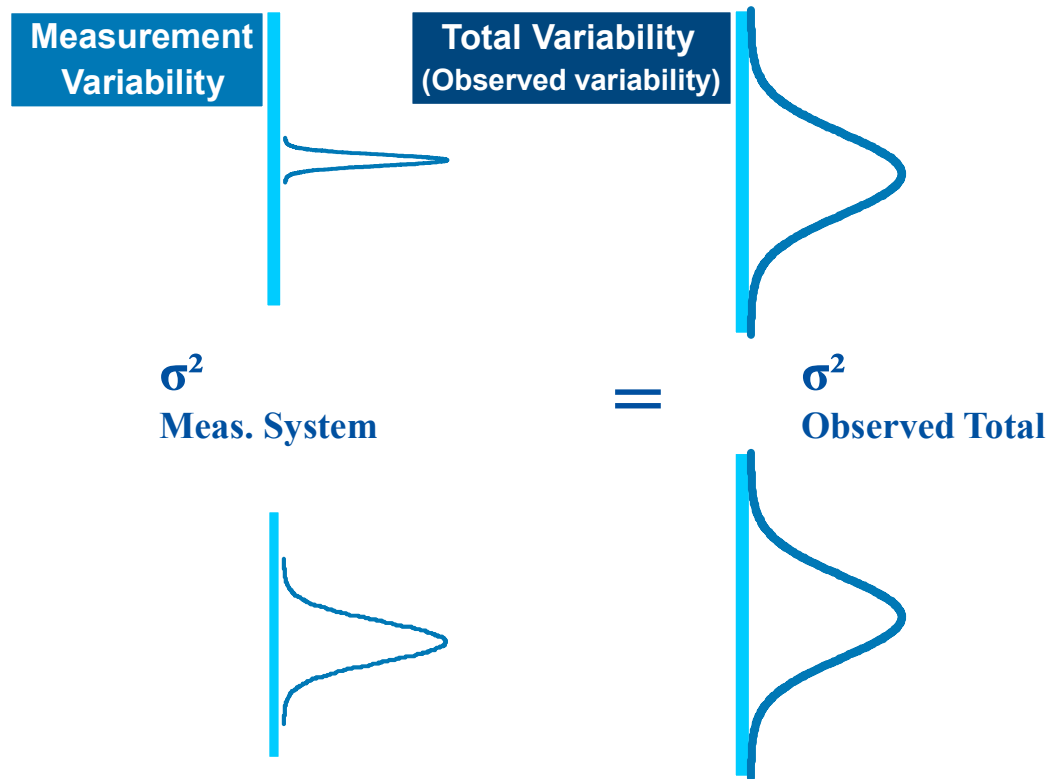


**Which process is best?**

# Measurement Variation

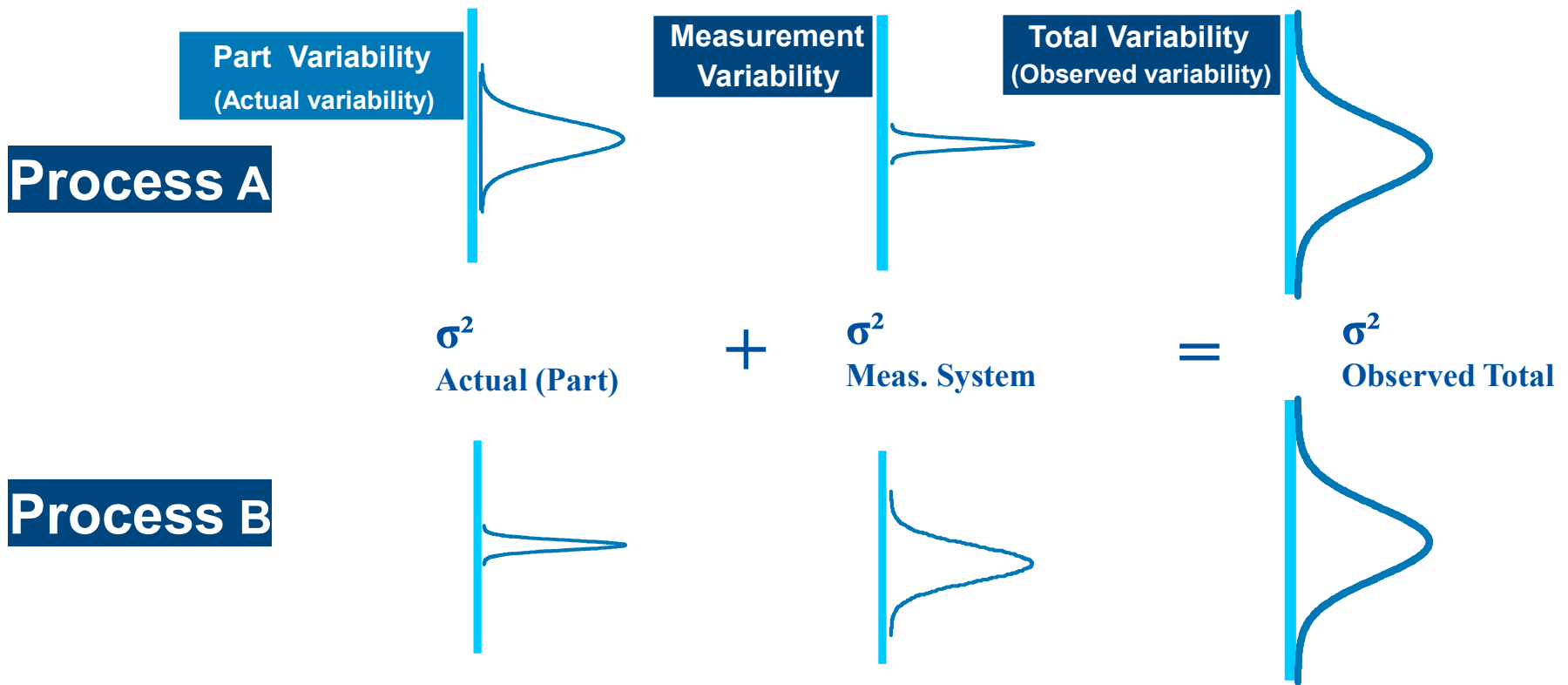
**Process A**

**Process B**



**Which process is best?**

# Part Variation



Which process is best?

# What is an MSA

- Scientific and objective method of analyzing the validity of a measurement system
- A “tool” which quantifies:
  - 1) Equipment Variation
  - 2) Appraiser (Operator) Variation
  - 3) The Total Variation of a Measurement System
- MSA is NOT just Calibration
- MSA is NOT just Gage Repeatability & Reproducibility (R&R)

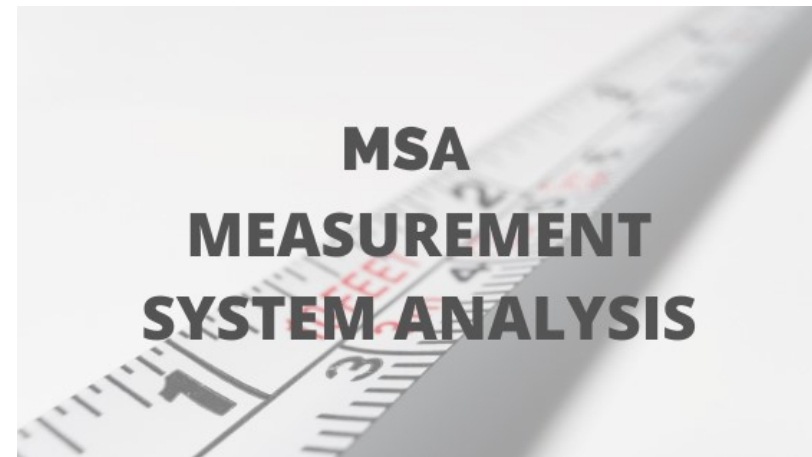
**Measurement System Analysis is often a “project within a project”**

# Anexas Consultancy Services

## Components of Measurement Error

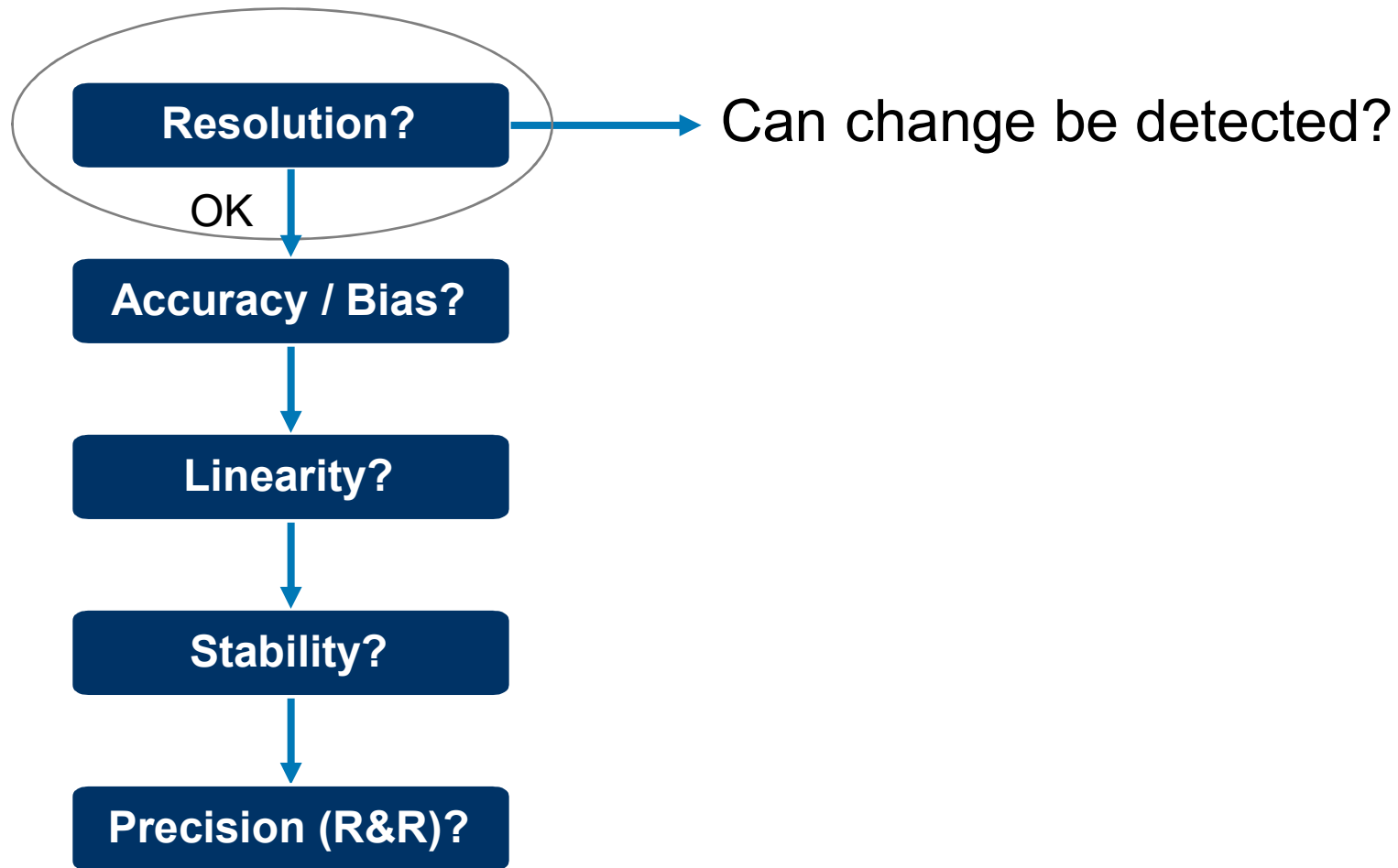
# Components of Measurement Error

- Resolution / Discrimination
- Accuracy (bias effects)
- Linearity
- Stability (consistency)
- Repeatability (test-retest)
- Reproducibility



**Each component of measurement error can contribute to variation causing wrong decisions to be made**

# Resolution / Discrimination





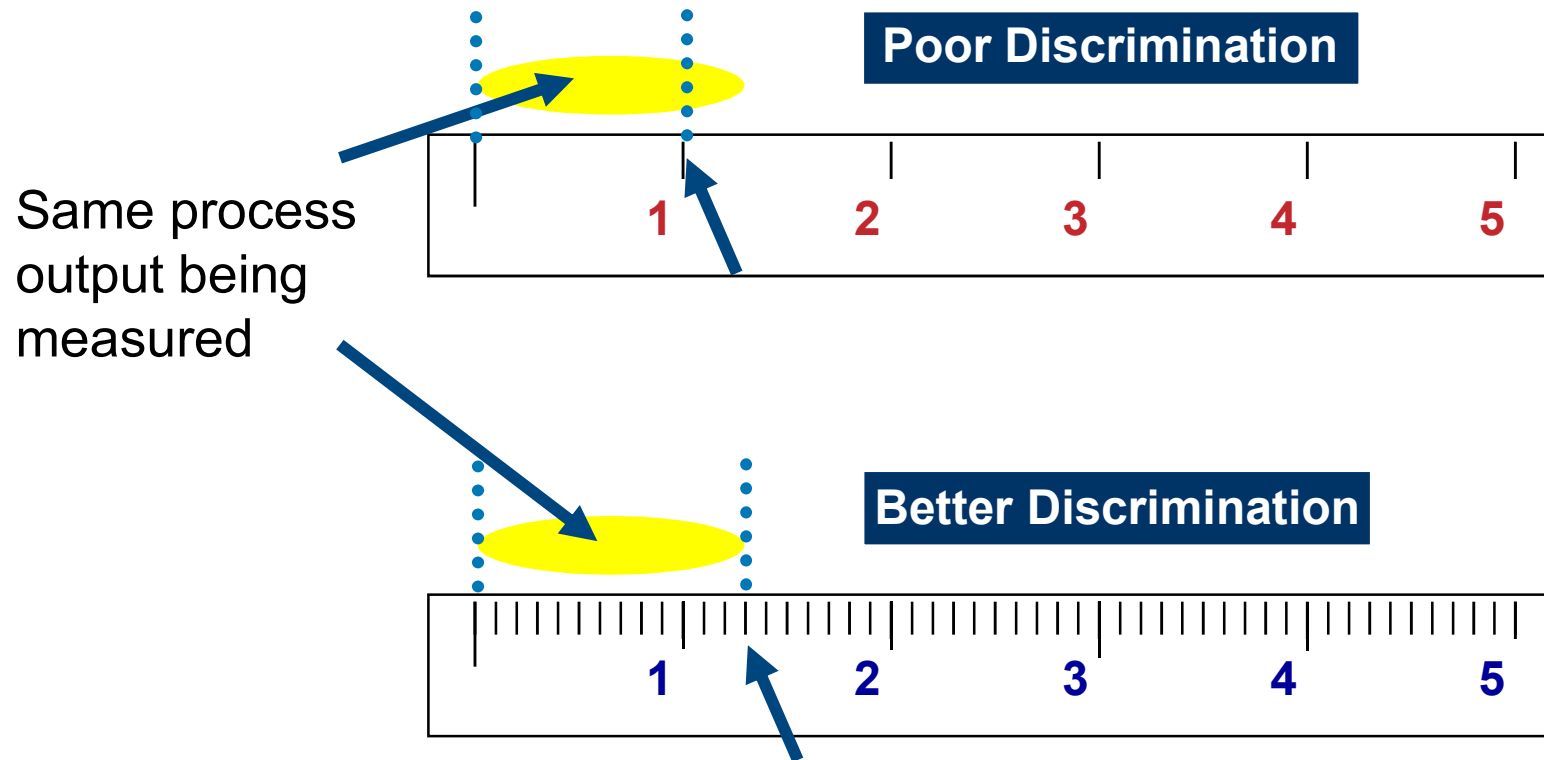
# Resolution

- Simplest measurement system problem
- Poor resolution is a common issue
- Impact is rarely recognized and/or addressed
- Easily detected
- No special studies are necessary
- No “known standards” are needed

# Definitions:

- Resolution / Discrimination
  - Capability to detect the smallest tolerable changes
- Inadequate Measurements Units
  - Measurement units too large to detect variation present
- Guideline: “10 Bucket Rule”
  - Increments in the measurement system should be one-tenth the product specification or process variation

# Resolution / Discrimination



# Resolution

Given this form :

<b>Customer Name</b>	_____
<b>Date Received</b>	_____
<b>Date Issued</b>	_____

What percent of requests are processed within 4 hours of receipt?

# Resolution

Given this NEW form :

<b>Customer Name</b>	_____
<b>Date Received</b>	_____
<b>Time Received</b>	_____
<b>Date Issued</b>	_____
<b>Time Issued</b>	_____

What percent of requests are processed (issued) within 4 hours of receipt?

# Resolution

On Hold complaints per hour

Complaint	Number
Transfers	50
Disputes	210
Information	143
Other	<u>12</u>
Total	415

What is the customer's biggest complaint?

# Resolution

## On Hold complaints per hour

Complaint	Number
Transfers	50
Set up or Maintenance Disputes	70
Service Received Disputes	60
Billing Amount Disputes	80
Update Account Information	115
Request Information	28
Other	<u>12</u>
Total	415

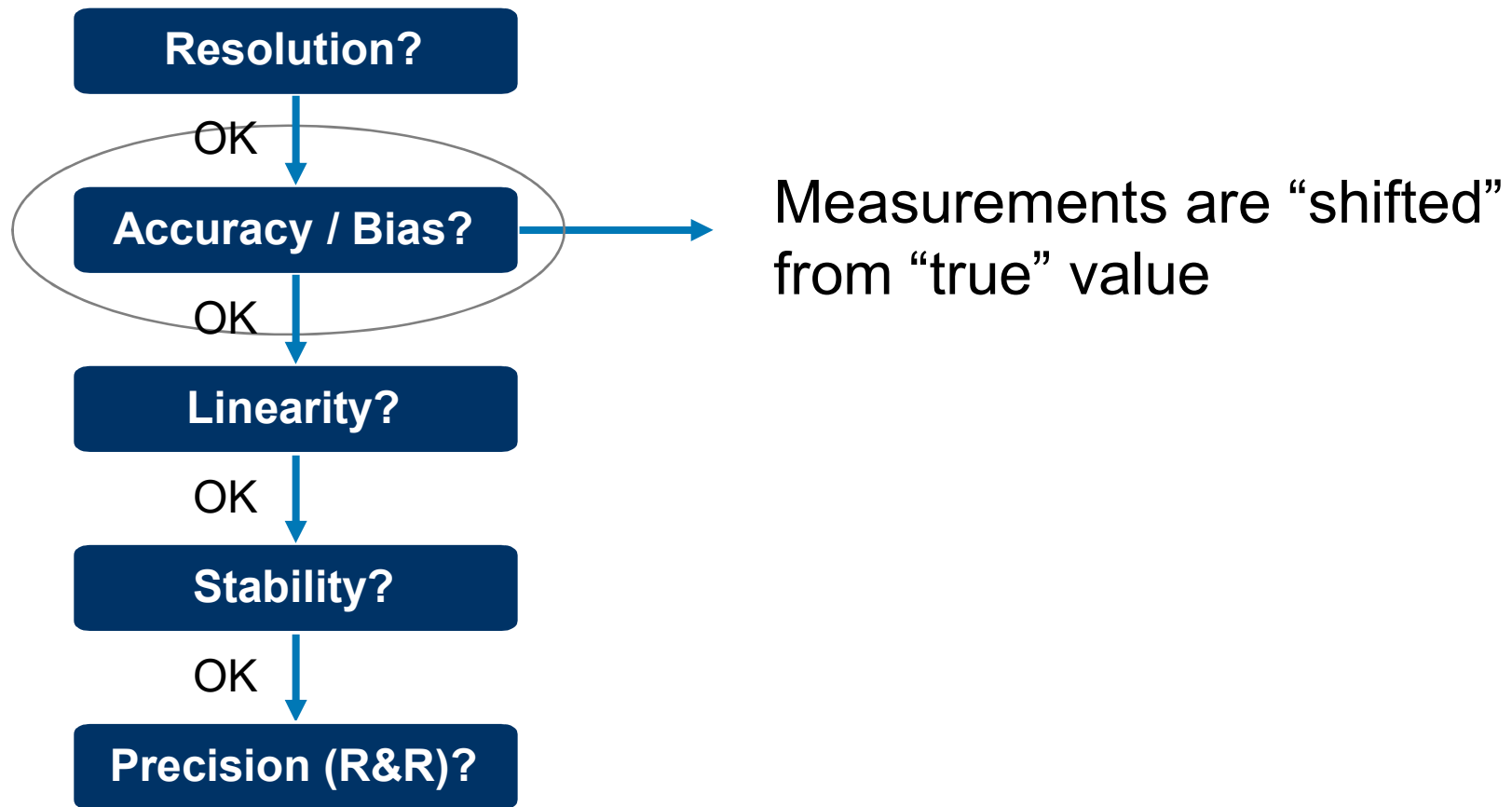
What is the customer's biggest complaint?

# Resolution Actions

- Measure to as many decimal places as possible
- Use a device that can measure smaller units
- Live with it, but document that the problem exists
- Larger sample size may overcome problem
- Priorities may need to involve other considerations:
  - Engineering tolerance
  - Process Capability
  - Cost and difficulty in replacing device



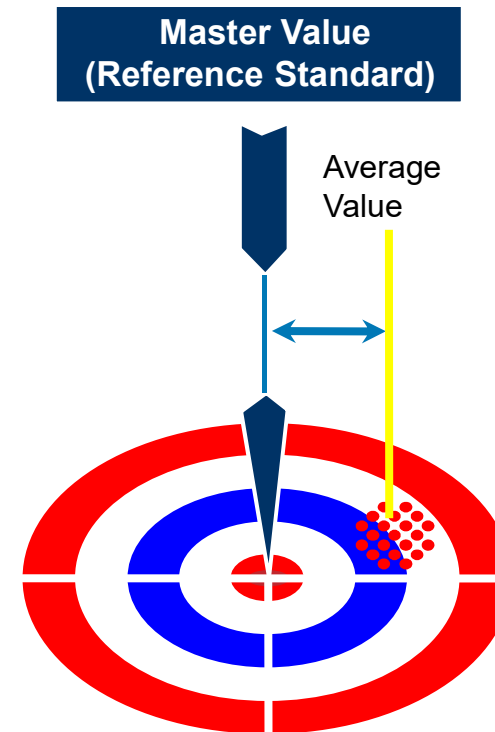
# Accuracy / Bias



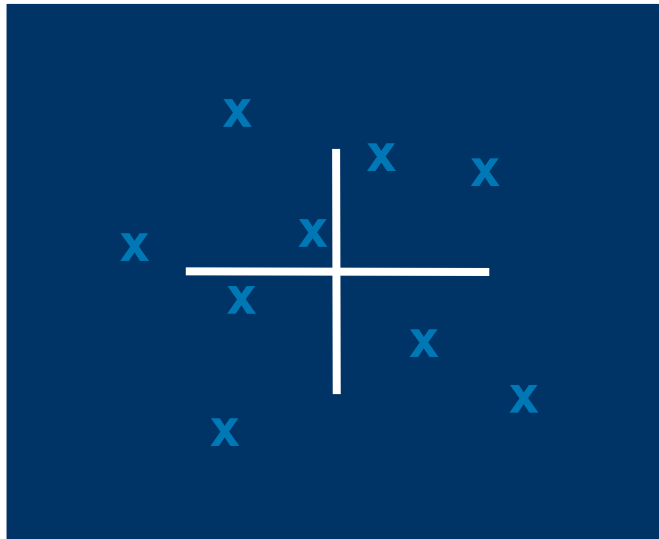
# Accuracy / Bias

Difference between the observed average value of measurements and the master value

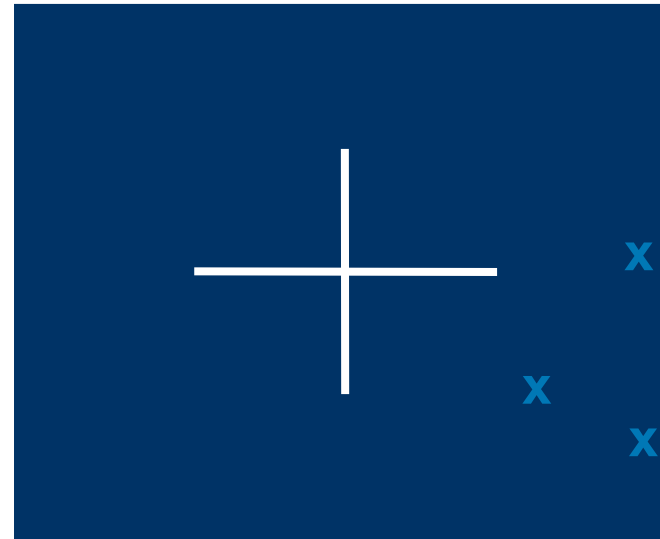
Master value is an accepted, traceable reference standard



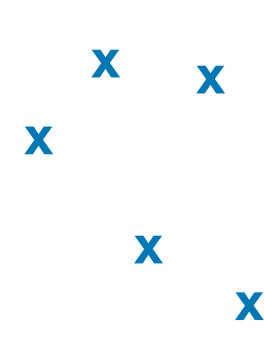
# Accuracy / Bias



More accurate



Less accurate



# Accuracy / Bias

Customers Surveyed . . .

*“Please Comment on Our Service . . .”*

Do You Have Any Complaints?

No

Yes

(If “Yes,” Please complete the following):

1. List the Problems

---

---

---

2. Date 

---

3. Time 

---

4. Name of Rep. 

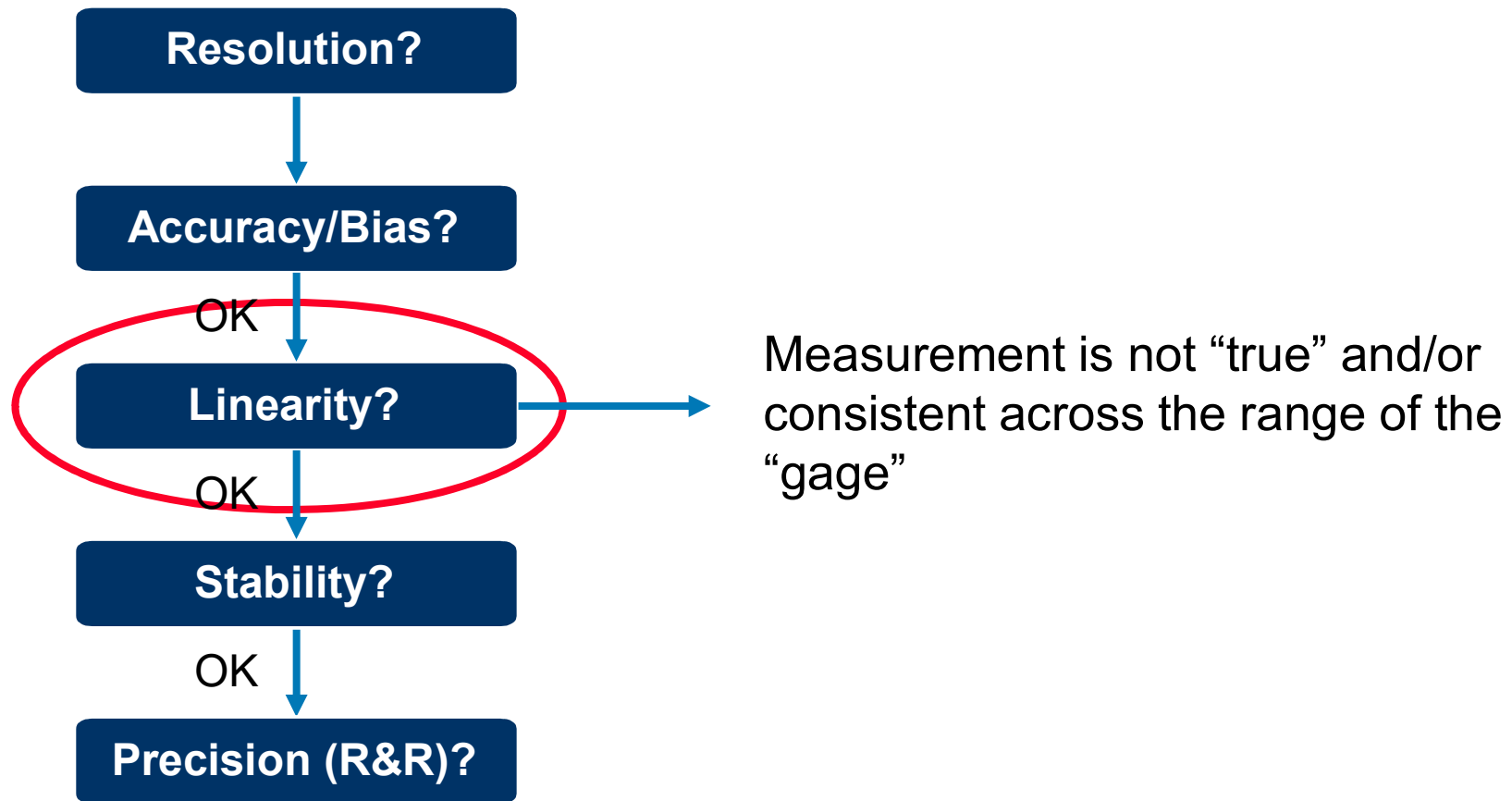
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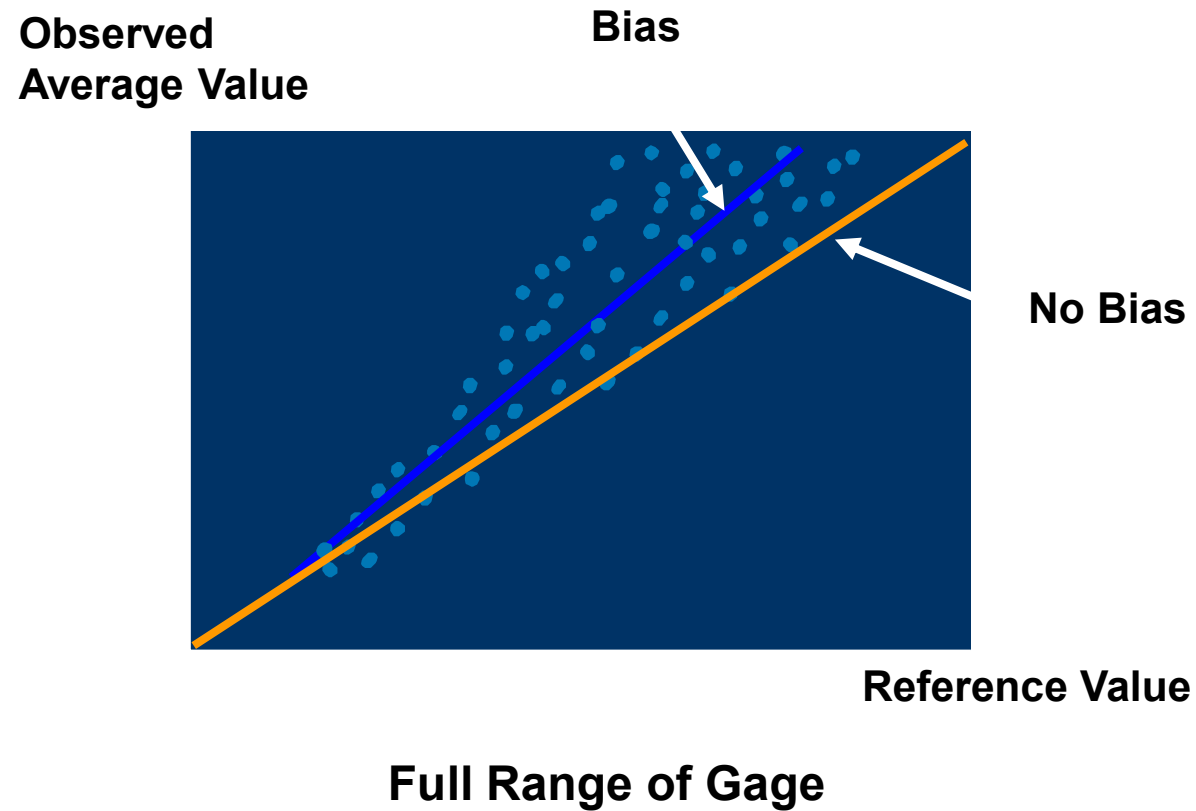
# Accuracy / Bias Actions

- Calibrate when needed / scheduled
- Use operations instructions
- Review specifications
- Review software logic
- Create Operational Definitions

# Linearity



# Linearity



# Linearity – Attribute Example

Survey scoring:

—	Super Outstanding!	10
—	Outstanding!	9
—	Incredible	8
—	Excellent	7
—	Great	6
—	Very Good	5
—	Good	4
—	OK	3
—	Fair	2
—	Poor	1

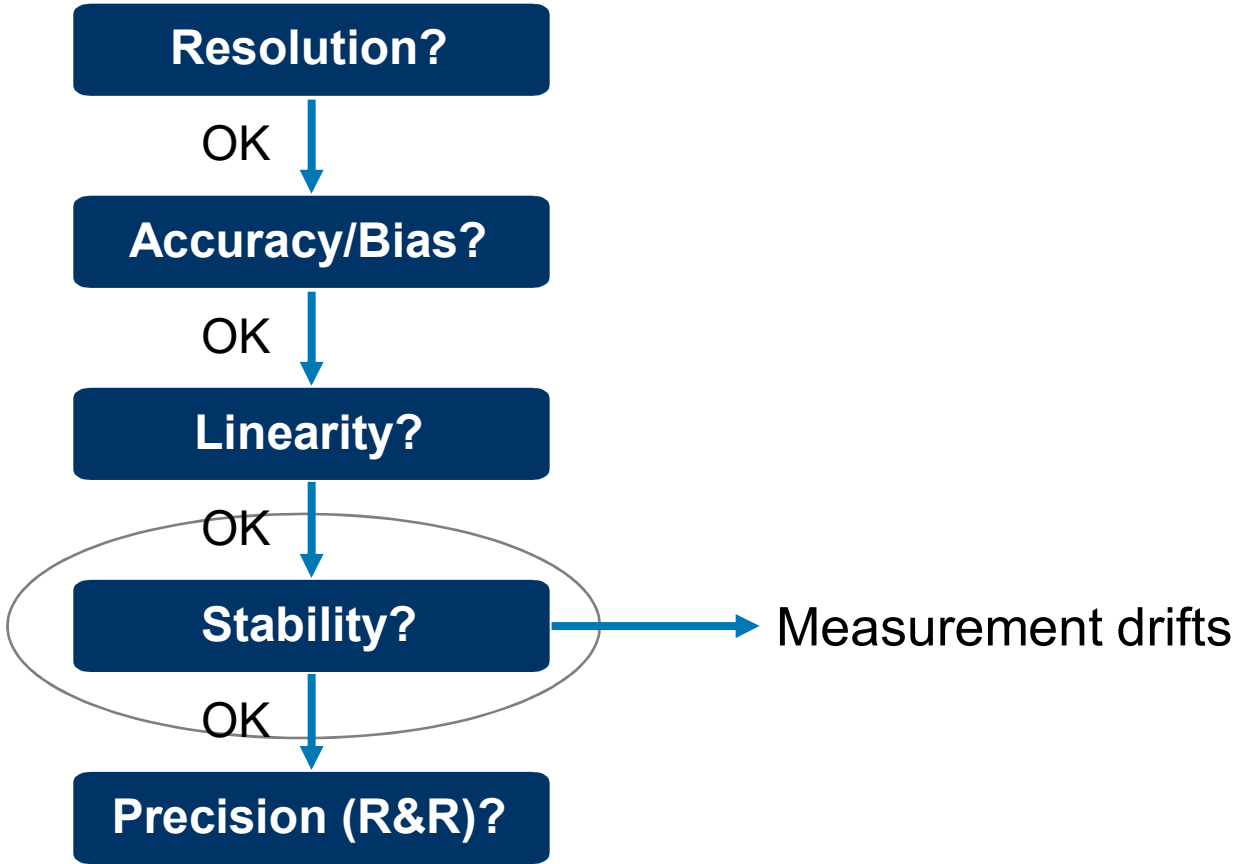
Is this a fair scale?



# Linearity Actions

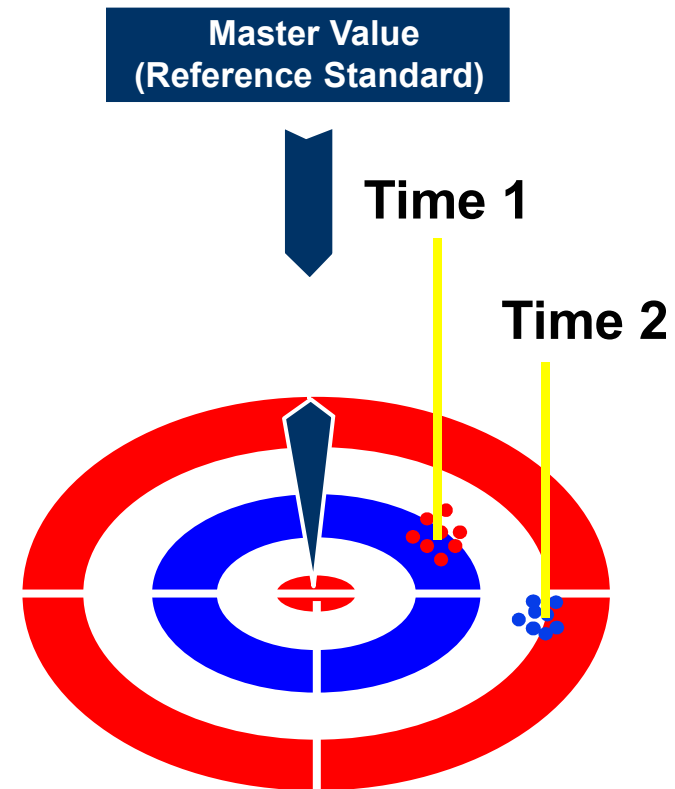
- Use only in restricted range
- Rebuild
- Use with correction factor / table / curve
- Sophisticated study required and will not be discussed in this course

# Stability



# Stability

- Measurements remain constant and predictable over time
  - for both mean and standard deviation
- No drifts, sudden shifts, cycles, etc.
- Evaluated using control charts



# Stability – Attribute Example

**Request Form #101**

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**Rev. 1/25/95**

**Request Form #101**

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**Rev. 8/4/00**

**Request Form #101**

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**Rev. 4/12/99**

**Request Form #101**

_____	_____	_____
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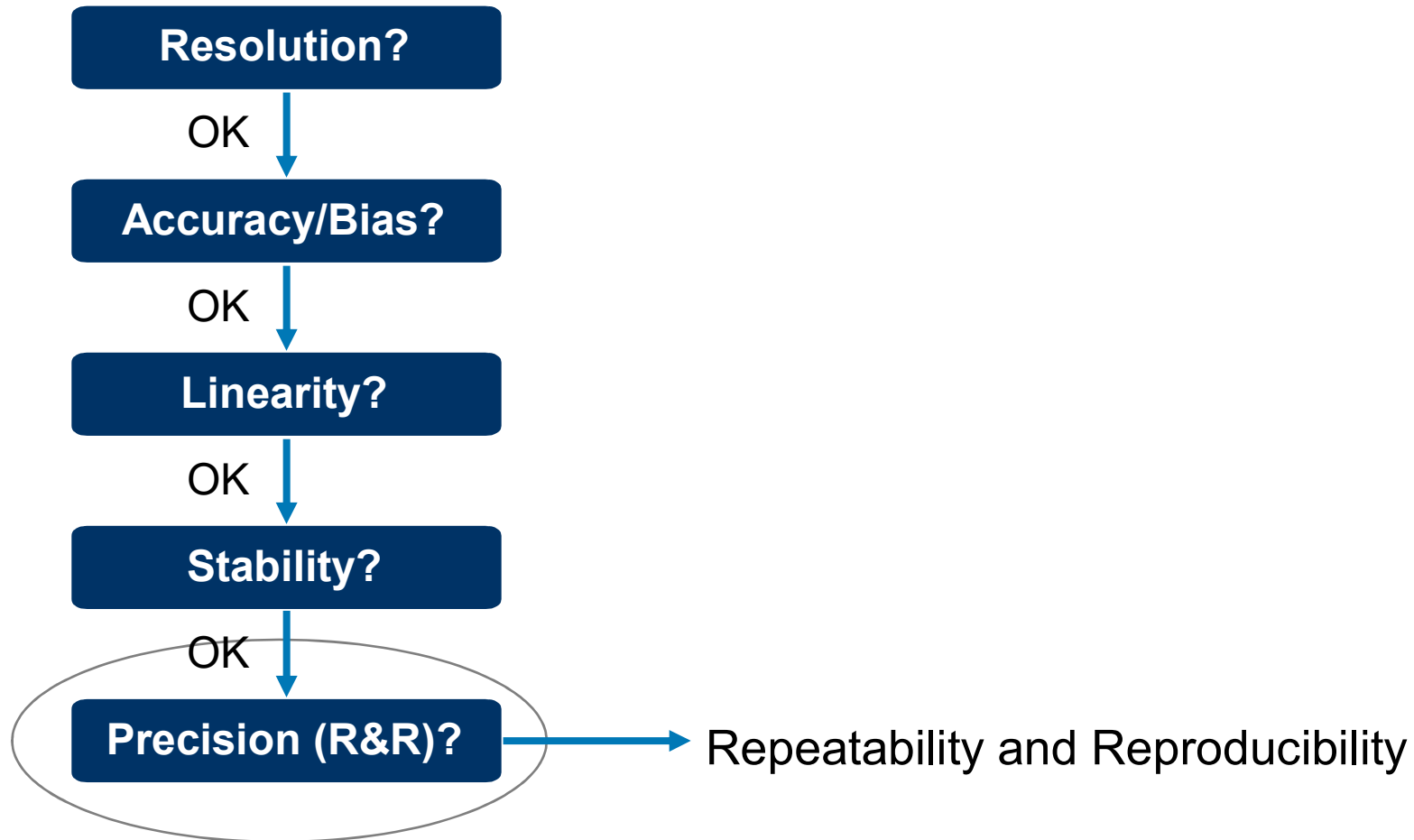
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**Rev. 1/13/96**

# Stability Actions

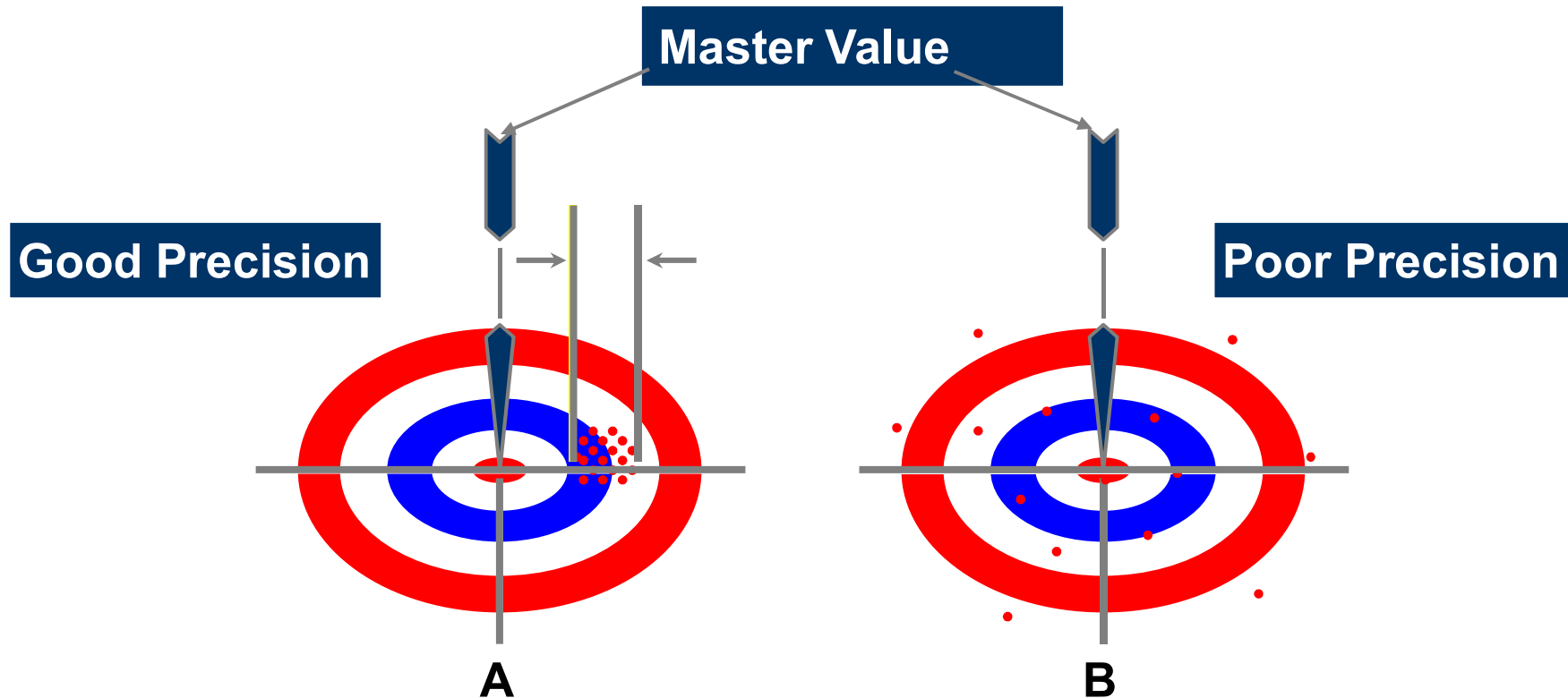
- Change / adjust components
- Establish “life” timeframe
- Use control charts
- Use / update current SOP

# Precision



# Precision

$$\sigma^2_{\text{total}} = \sigma^2_{\text{product / process}} + \sigma^2_{\text{repeatability}} + \sigma^2_{\text{reproducibility}}$$



Also known as Gage R & R

# Repeatability (A Component of Precision)

- Variation that occurs when repeated measurements are made of the same item under absolutely identical conditions
  - Same:
    - operator
    - set-up
    - units
    - environmental conditions
  - Short-term

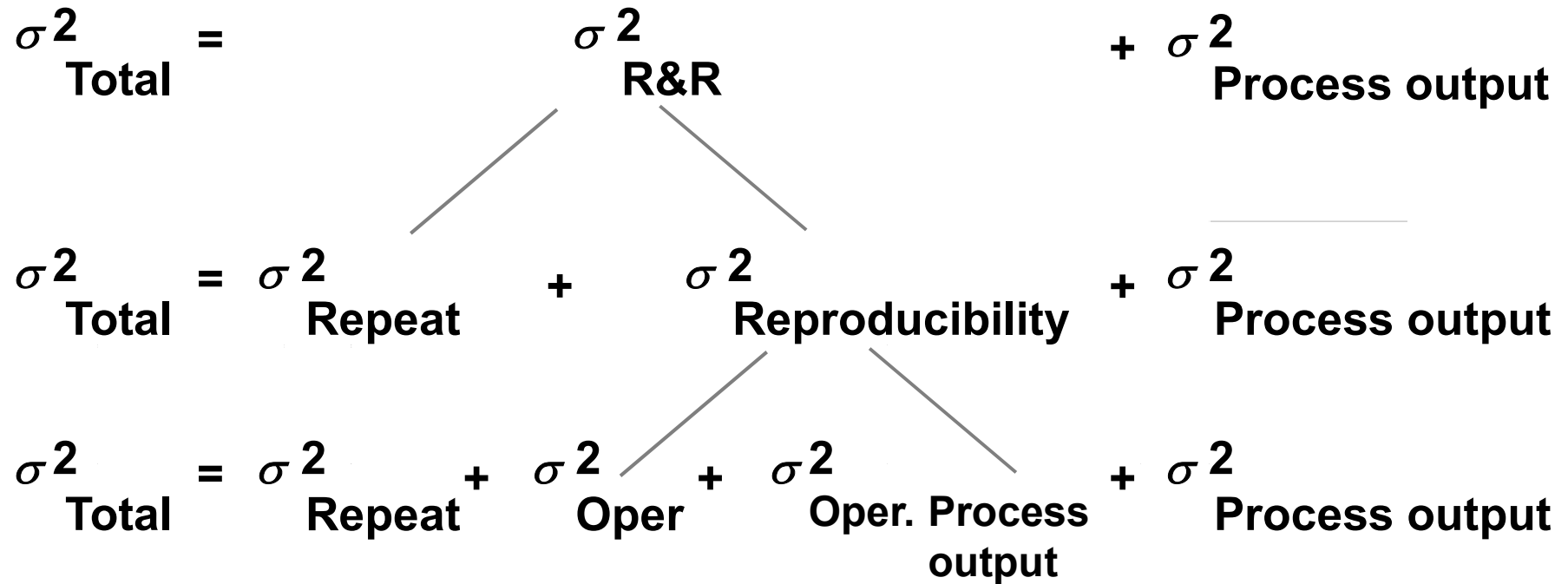


# Reproducibility (A Component of Precision)

- The variation that results when different conditions are used to make the measurements
  - Different:
    - operators
    - set-ups
    - test units
    - environmental conditions
    - locations
    - companies
  - Long-term

# R & R

## The Big Picture: Linking Them All Together



# R & R Actions

- Repeatability
  - Repair, replace, adjust equipment
  - SOP
- Reproducibility
  - Training
  - SOP

# Anexas Consultancy Services

## Attribute Measurement Studies

# Purpose of Attribute MSA

- Assess standards against customers' requirements
- Determine if all appraisers use the same criteria
- Quantify repeatability and reproducibility of operators
- Identify how well measurement system conforms to a “known master”
- Discover areas where:
  - Training is needed
  - Procedures are lacking
  - Standards are not defined

# Attribute MSA – Excel Method

- Allows for R & R analysis within and between appraisers
- Test for effectiveness against standard
- Limited to nominal data at two levels

# Attribute MSA Example:

↓	C1-T	C2-T		C3-T		C4-T		C5-T		C6-T		C7-T	
	Attribute	Appraiser 1	Trial 1	Appraiser 1	Trial 2	Appraiser 2	Trial 1	Appraiser 2	Trial 2	Appraiser 3	Trial 1	Appraiser 3	Trial 2
1	Pass	Pass		Pass		Pass		Pass		Pass		Pass	
2	Pass	Pass		Pass		Pass		Pass		Pass		Pass	
3	Pass	Pass		Pass		Pass		Pass		Pass		Pass	
4	Pass	Pass		Pass		Pass		Pass		Fail		Pass	
5	Fail	Fail		Fail		Fail		Fail		Pass		Fail	
6	Fail	Pass		Pass		Pass		Pass		Pass		Pass	
7	Pass	Pass		Pass		Pass		Pass		Pass		Pass	
8	Pass	Pass		Pass		Pass		Pass		Pass		Pass	
9	Fail	Fail		Fail		Fail		Fail		Fail		Fail	
10	Pass	Pass		Pass		Pass		Pass		Pass		Pass	
11	Pass	Pass		Pass		Pass		Pass		Pass		Pass	
12	Pass	Pass		Pass		Pass		Pass		Pass		Pass	
13	Pass	Pass		Pass		Pass		Pass		Pass		Pass	

Open file *Attribute MSA.mpj*

# Scoring Example:

% APPRAISER SCORE - >	100.00%	78.57%	100.00%
% SCORE VS. ATTRIBUTE - >	78.57%	64.29%	71.43%
		SCREEN % EFFECTIVE SCORE - >	57.14%
		SCREEN % EFFECTIVE SCORE vs. ATTRIBUTE - >	42.86%

- 100% is target for all scores
  - <100% indicates training required
- % Appraiser score = repeatability
- Screen % Effectiveness Score = reproducibility
- % Score vs. Attribute
  - individual error against a known population
- Screen % Effective vs. Attribute
  - Total error against a known population



# Attribute MSA – Minitab Method

- Allows for R & R analysis within and between appraisers
- Test for effectiveness against standard
- Allow nominal data with two levels
- Allows for ordinal data with more than two levels

# Minitab Method -- Data Entry

↓	C1-T	C2-T		C3-T		C4-T		C5-T		C6-T		C7-T	
	Attribute	Appraiser 1	Trial 1	Appraiser 1	Trial 2	Appraiser 2	Trial 1	Appraiser 2	Trial 2	Appraiser 3	Trial 1	Appraiser 3	Trial 2
1	Pass	Pass		Pass		Pass		Pass		Pass		Pass	
2	Pass	Pass		Pass		Pass		Pass		Pass		Pass	
3	Pass	Pass		Pass		Pass		Pass		Pass		Pass	
4	Pass	Pass		Pass		Pass		Pass		Fail		Pass	
5	Fail	Fail		Fail		Fail		Fail		Pass		Fail	
6	Fail	Pass		Pass		Pass		Pass		Pass		Pass	
7	Pass	Pass		Pass		Pass		Pass		Pass		Pass	
8	Pass	Pass		Pass		Pass		Pass		Pass		Pass	
9	Fail	Fail		Fail		Fail		Fail		Fail		Fail	
10	Pass	Pass		Pass		Pass		Pass		Pass		Pass	
11	Pass	Pass		Pass		Pass		Pass		Pass		Pass	
12	Pass	Pass		Pass		Pass		Pass		Pass		Pass	
13	Pass	Pass		Pass		Pass		Pass		Pass		Pass	

- Same data as Excel example
- Arranged in multiple columns
- Data can also be stacked in single column

# Attribute Study – Minitab Analysis

Tool Bar Menu > Stat > Quality Tools > Attribute Gage R&R Study

Minitab - Attribute MSA.MPJ - [Worksheet 1 \*\*\*]

File Edit Data Calc Stat Graph Editor Tools Window Help Assistant

Basic Statistics  
Regression  
ANOVA  
DOE  
Control Charts  
Quality Tools  
Reliability/Survival  
Multivariate  
Time Series  
Tables  
Nonparametrics  
Equivalence Tests  
Power and Sample Size

	C2-T	C4-T	C5-T	C6-T	C7-T	C8-T
	Appraiser 1 Trial 1				Appraiser 3 Trial 2	
1	Pass				Pass	
2	Pass				Pass	
3	Pass				Pass	
4	Pass				Pass	
5	Fail				Fail	
6	Pass				Pass	
7	Pass				Pass	
8	Pass				Pass	
9	Fail	Fail	Fail		Fail	
10	Pass	Pass	Pass		Pass	
11	Pass	Pass	Pass		Pass	
12	Pass	Pass	Pass		Pass	
13	Pass	Pass	Pass		Pass	
14	Pass	Pass	Pass		Pass	
15	Fail	Fail	Fail		Fail	
16	Pass	Pass	Pass		Pass	
17	Pass	Pass	Pass		Pass	
18	Pass	Pass	Pass	Pass	Pass	Pass
19	Fail	Fail	Fail	Fail	Fail	Fail
20	Pass	Pass	Pass	Pass	Pass	Pass
21	Pass	Pass	Pass	Pass	Pass	Pass
22	Fail	Fail	Pass	Pass	Pass	Pass
23	Pass	Pass	Pass	Pass	Pass	Pass
24	Pass	Pass	Pass	Pass	Fail	Pass
25	Fail	Fail	Fail	Fail	Fail	Fail

Run Chart...  
Pareto Chart...  
Cause-and-Effect...  
Individual Distribution Identification...  
Johnson Transformation...  
Capability Analysis  
Capability Sixpack  
Tolerance Intervals...  
Gage Study  
Create Attribute Agreement Analysis Worksheet...  
Attribute Agreement Analysis...  
Acceptance Sampling by Attributes...  
Acceptance Sampling by Variables  
Multi-Vari Chart...  
Symmetry Plot...

**Attribute Agreement Analysis**  
Evaluate the consistency and accuracy of subjective ratings that are made by multiple appraisers.

# Attribute Study – Minitab Analysis (Continued)

1. Select “Single Column” if data is stacked

1. Select “Multiple Columns” if data is unstacked

2. Enter number of appraisers and trials

3. Enter name of column with “Known”

4. Select OK

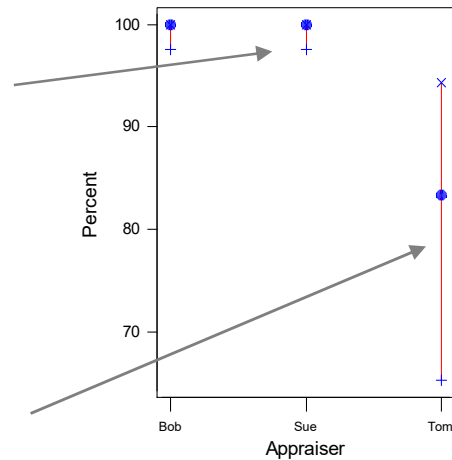
The screenshot shows the Minitab Attribute Study dialog box. On the left, a list of variables includes C1 Attribute, C2 Appraiser 1, C3 Appraiser 1, C4 Appraiser 2, C5 Appraiser 2, C6 Appraiser 3, and C7 Appraiser 3. The 'Data are arranged as' section has two radio buttons: 'Single column:' (unselected) and 'Multiple columns:' (selected). Below this, a text box contains the text: 'Appraiser 1 Trial 1'-  
Appraiser 3 Trial 2'. Below the text box is the instruction '(Enter trials for each appraiser together)'. There are two input fields: 'Number of appraisers:' with the value '3' and 'Number of trials:' with the value '2'. Below these is the 'Appraiser names (optional):' label and an empty text box. The 'Known standard/attribute:' field contains the text 'Attribute' and is labeled '(Optional)'. At the bottom, there is a checkbox labeled 'Categories of the attribute data are ordered' which is unchecked. On the right side of the dialog, there are four buttons: 'Information...', 'Options...', 'Graphs...', and 'Results...'. At the bottom right, there are 'OK' and 'Cancel' buttons. Arrows from the numbered callouts point to the 'Multiple columns:' radio button, the 'Number of appraisers' and 'Number of trials' fields, the 'Attribute' text in the 'Known standard/attribute' field, and the 'OK' button.

# Attribute MSA – Minitab Graphical Output

Assessment Agreement

Date of study: 1/03/2001  
 Reported by: Jose  
 Name of product: XYZ Report  
 Misc:

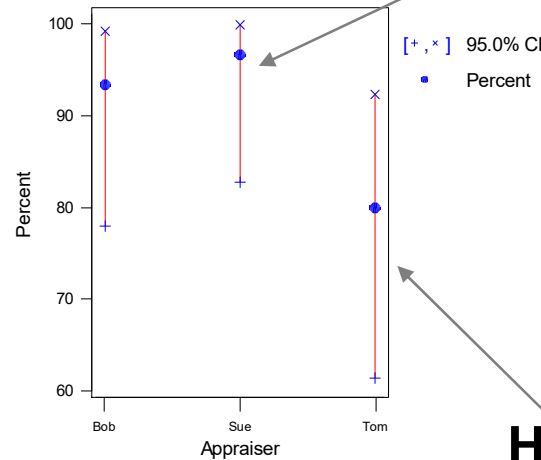
Within Appraiser



**Lower variation within appraiser**

**Higher variation within appraiser**

Appraiser vs Standard



**Lower variation appraiser vs. standard**

**Higher variation appraiser vs. standard**

**Not included if no "Known"**

# Attribute MSA – Session Window Results

- Each Appraiser vs Standard

- Assessment Agreement

- Appraiser # Inspected # Matched Percent (%) 95.0% CI

Bob	30	28	93.3 ( 77.9, 99.2)
Sue	30	29	96.7 ( 82.8, 99.9)
Tom	30	24	80.0 ( 61.4, 92.3)

**Individual vs. Standard**

- # Matched: Appraiser's assessment across trials agrees with standard.

- Assessment Disagreement

- Appraiser # Pass/Fail Percent (%) # Fail/Pass Percent (%) # Mixed Percent (%)

Bob	1	3.3	1	3.3	0	0.0
Sue	1	3.3	0	0.0	0	0.0
Tom	1	3.3	0	0.0	5	16.7

**Disagreement assessment (repeatability)**

- # Pass/Fail: Assessments across trials = Pass / standard = Fail.

- # Fail/Pass: Assessments across trials = Fail / standard = Pass.

- # Mixed: Assessments across trials are not identical.

- Between Appraisers

- Assessment Agreement

- # Inspected # Matched Percent (%) 95.0% CI

30	24	80.0 ( 61.4, 92.3)
----	----	--------------------

**Between appraisers (reproducibility)**

- # Matched: All appraisers' assessments agree with each other.

- All Appraisers vs Standard

- Assessment Agreement

- # Inspected # Matched Percent (%) 95.0% CI

30	23	76.7 ( 57.7, 90.1)
----	----	--------------------

**Total agreement (against known)**

- # Matched: All appraisers' assessments agree with standard.

# Anexas Consultancy Services

## Variables Measurement Studies

# Six Step Variables MSA

1. Conduct initial gage calibration (or verification)
2. Perform trials and data collection
3. Obtain statistics via Minitab
4. Analyze, interpret results
5. Check for inadequate measurement units
6. On-going evaluation
  - What would be your long-term gage plan ?



# Trials and Data Collection

- Generally two to three operators
- Generally 5-10 process outputs to measure
- Each process output is measured 2-3 times (replicated) by each operator

**Randomization is Critical**

# Randomization, Repeats, Replicates

## **Randomization**

- Runs are made in an arbitrary vs. patterned order
- Average out effects of noise or unknown factors
- Tradeoff - Invalid results versus slight inconvenience (if any)

## **Repeats**

- Running more than one sample of a single run
- Results are averaged

## **Replication**

- Running entire experiment in a time sequence
- MSA allows for repeatability study

# Variables MSA – Minitab Example

USL=1.5  
LSL=0.5

**Replicate 1**                      **Replicate 2**  
**(Randomized order)**

Part Trial 1	Operator Trial 1	Response Trial 1	Parts Trial 2	Operator Trial 2	Response Trial 2
1	1	0.60000	8	1	0.85000
2	1	1.00000	2	1	1.00000
3	1	0.80000	10	1	0.60000
4	1	0.95000	6	1	1.00000
5	1	0.45000	5	1	0.55000
6	1	1.00000	4	1	0.85000
7	1	0.95000	7	1	0.95000
8	1	0.80000	1	1	0.65000
9	1	1.00000	9	1	1.00000
10	1	0.70000	3	1	0.85000
1	2	0.55000	1	2	0.55000
2	2	0.95000	2	2	1.05000
3	2	0.75000	3	2	0.80000
4	2	0.75000	4	2	0.80000
5	2	0.40000	5	2	0.40000
6	2	1.05000	6	2	1.00000
7	2	0.90000	7	2	0.95000
8	2	0.70000	8	2	0.75000
9	2	0.95000	9	2	1.00000
10	2	0.50000	10	2	0.55000
1	3	0.55000	1	3	0.50000
2	3	1.00000	2	3	1.05000
3	3	0.80000	3	3	0.80000
4	3	0.80000	4	3	0.80000
5	3	0.50000	5	3	0.45000
6	3	1.05000	6	3	1.00000

**Open file Variable *MSA.mtw***

# MSA Using Minitab

**10 Process Outputs  
3 Operators  
2 Replicates**

- Have Operator 1 measure all samples once (as shown in the outlined block)
- Then, have Operator 2 measure all samples once
- Continue until all operators have measured samples once (this is Replicate 1)
- Repeat these steps for the required number of Replicates
- Enter data into Minitab in 3 columns as shown

**USL=1.5  
LSL=0.5**

**Replicate 1**

**Replicate 2  
(Randomized order)**

Part Trial 1	Operator Trial 1	Response Trial 1	Parts Trial 2	Operator Trial 2	Response Trial 2
1	1	0.60000	8	1	0.85000
2	1	1.00000	2	1	1.00000
3	1	0.80000	10	1	0.60000
4	1	0.95000	6	1	1.00000
5	1	0.45000	5	1	0.55000
6	1	1.00000	4	1	0.85000
7	1	0.95000	7	1	0.95000
8	1	0.80000	1	1	0.65000
9	1	1.00000	9	1	1.00000
10	1	0.70000	3	1	0.85000
1	2	0.55000	1	2	0.55000
2	2	0.95000	2	2	1.05000
3	2	0.75000	3	2	0.80000
4	2	0.75000	4	2	0.80000
5	2	0.40000	5	2	0.40000
6	2	1.05000	6	2	1.00000
7	2	0.90000	7	2	0.95000
8	2	0.70000	8	2	0.75000
9	2	0.95000	9	2	1.00000
10	2	0.50000	10	2	0.55000
1	3	0.55000	1	3	0.50000
2	3	1.00000	2	3	1.05000
3	3	0.80000	3	3	0.80000
4	3	0.80000	4	3	0.80000
5	3	0.50000	5	3	0.45000
6	3	1.05000	6	3	1.00000
7	3	0.95000	7	3	0.95000
8	3	0.80000	8	3	0.80000

# Stack the Data

The screenshot shows the Minitab interface with a data table and the 'Stack' command menu open. The 'Stack Blocks of Columns' dialog box is in the foreground, showing options to stack data from multiple blocks of columns into one block.

	C1	C2	C3	C4	C5	C6
	Part Trial 1	Operator Trial 1	Response Trial 1	Parts Trial 2	Operator Trial 2	Response Trial 2
1	1	1	0.60000	8	1	0.85000
2	2	1	1.00000	2	1	1.00000
3	3	1	0.80000	10	1	0.60000
4	4	1	0.95000	6	1	1.00000
5	5	1	0.45000	5	1	0.55000
6	6	1	1.00000	4	1	0.85000
7	7	1	0.95000	7	1	0.95000
8	8	1	0.80000	1	1	0.65000
9	9	1	1.00000	9	1	1.00000
10	10	1	0.70000	3	1	0.85000
11	1	2	0.55000	1	2	0.55000
12	2	2	0.95000	2	2	1.05000
13	3	2	0.75000	3	2	0.80000
14	4	2	0.75000	4	2	0.80000
15	5	2	0.40000			
16	6	2	1.05000			
17	7	2	0.90000			
18	8	2	0.70000			
19	9	2	0.95000			
20	10	2	0.50000			
21	1	3	0.55000			
22	2	3	1.00000			
23	3	3	0.80000			
24	4	3	0.80000			
25	5	3	0.50000			
26	6	3	1.05000			
27	7	3	0.95000			
28	8	3	0.80000			
29	9	3	1.05000			
30	10	3	0.80000			

Your data in Minitab should initially look like this. You will need to **STACK** your data so that all like data is in one column only.

Use the commands

- > **Data**
- > **Stack**
- > **Stack Blocks of Columns**

(Stack all Process Outputs, Operators, and Responses so that they are in one column only).

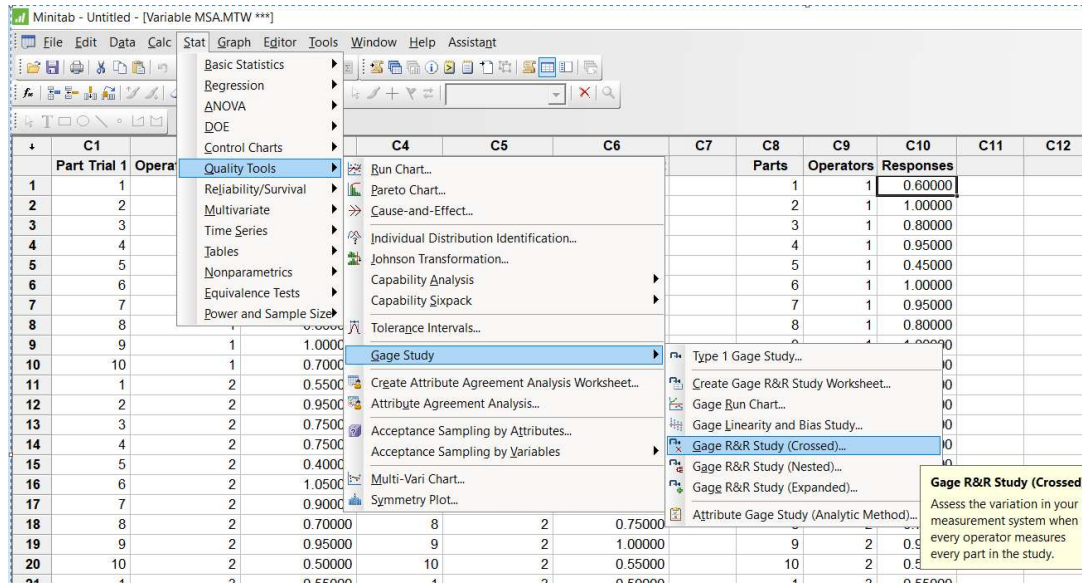
Now you are ready to run the macro for data analysis

# Stacked and Ready for Analysis

C8	C9	C10	C11
Parts	Operators	Responses	
1	1	0.60000	
2	1	1.00000	
3	1	0.80000	
4	1	0.95000	
5	1	0.45000	
6	1	1.00000	
7	1	0.95000	
8	1	0.80000	
9	1	1.00000	
10	1	0.70000	
1	2	0.55000	
2	2	0.95000	
3	2	0.75000	
4	2	0.75000	
5	2	0.40000	
6	2	1.05000	
7	2	0.90000	
8	2	0.70000	
9	2	0.95000	

**NOTE:**  
**C08 C09, C10 are the columns in which the respective data are found in our example. You must have ALL data STACKED in these columns and named those columns as mentioned**

# Prepare the Analysis

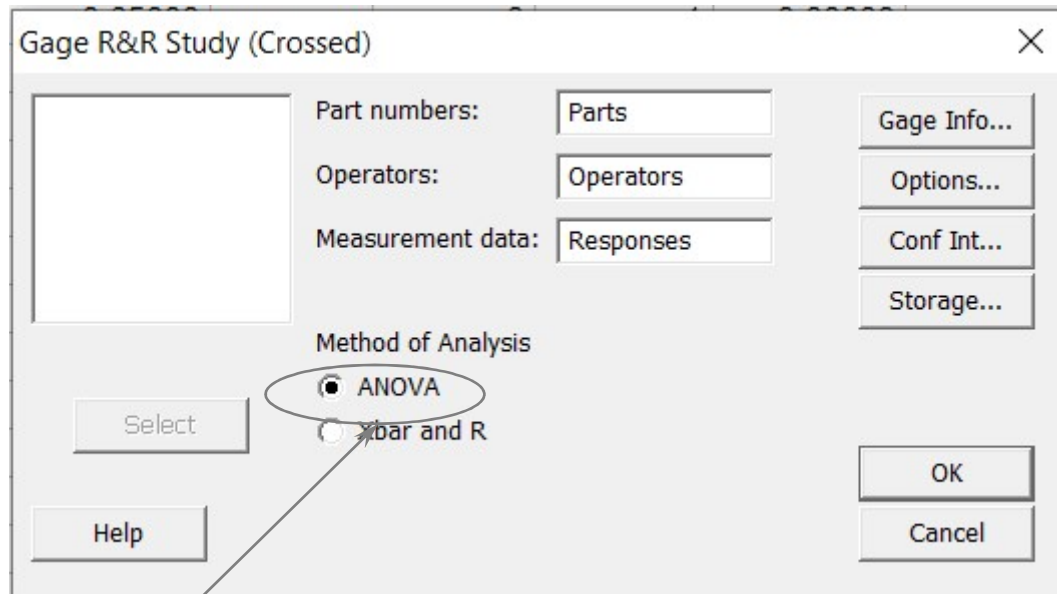


Use the commands  
 > Stat > Quality Tools  
 > Gage R&R Study (Crossed)  
 Each process output  
 measured by each  
 operator

OR

> Gage R&R Study (Nested)  
 For “destructive tests”  
 where each process  
 output is measured  
 uniquely by each  
 operator

# Choose Method of Analysis



**ANOVA method is preferred**  
**- Gives more information**



# MSA Output:

## Session Window

Two-Way ANOVA Table With Interaction

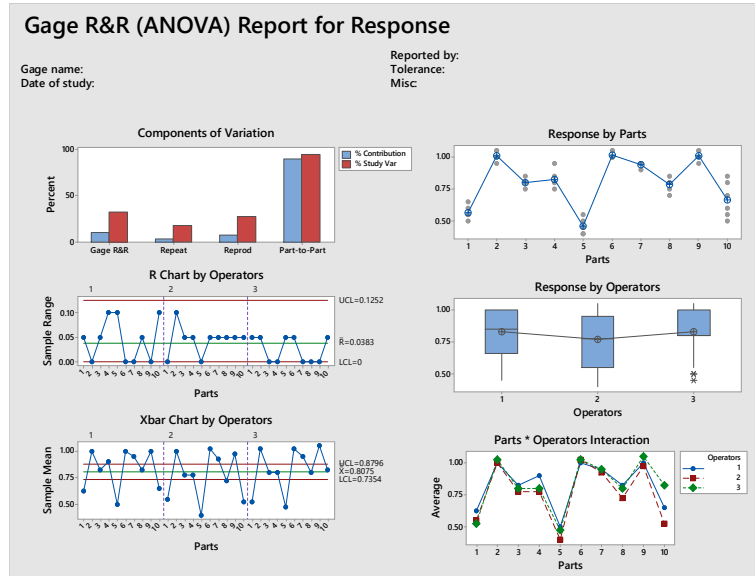
Source	DF	SS	MS	F	P
Part	9	2.05871	0.228745	39.7178	0.00000
Operator	2	0.04800	0.024000	4.1672	0.03256
Operator*Part	18	0.10367	0.005759	4.4588	0.00016
Repeatability	30	0.03875	0.001292		
Total	59	2.24912			

### Gage R&R

Source	VarComp	%Contribution	
		(of VarComp)	(%SV)
Total Gage R&R	0.004437	10.67	
Repeatability	0.001292	3.10	
Reproducibility	0.003146	7.56	
Operator	0.000912	2.19	
Operator*Part	0.002234	5.37	
Part-To-Part	0.037164	89.33	
Total Variation	0.041602	100.00	
	StdDev	Study Var	%Study Var
Source	(SD)	(5.15*SD)	(%SV)
Total Gage R&R	0.066615	0.34306	32.66
Repeatability	0.035940	0.18509	17.62
Reproducibility	0.056088	0.28885	27.50
Operator	0.030200	0.15553	14.81
Operator*Part	0.047263	0.24340	23.17
Part-To-Part	0.192781	0.99282	94.52
Total Variation	0.203965	1.05042	100.00

Number of Distinct Categories = 4

## Graphs



**What does all this mean?**

# Graphical Output - 6 Graphs in All

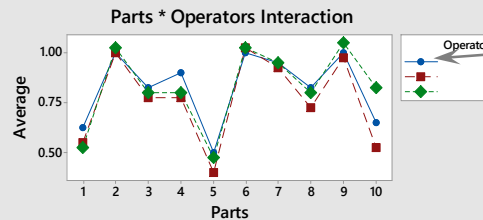
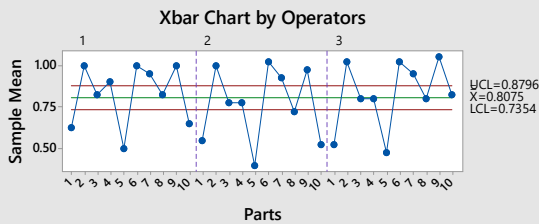
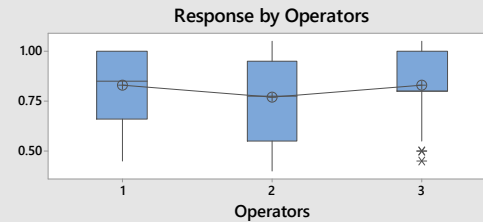
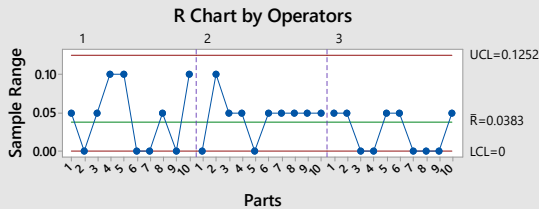
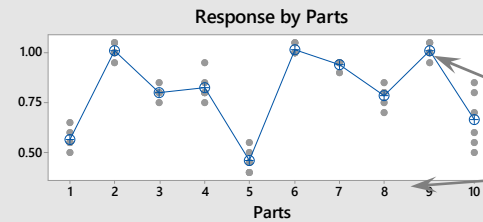
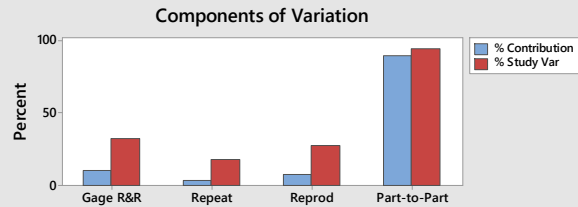
MSA  
HEALTH  
SIDE

MSA  
TROUBLESHOOT  
SIDE

## gauge R&R (ANOVA) Report for Response

Gage name:  
Date of study:

Reported by:  
Tolerance:  
Misc:



If only 1 operator, you won't get these graphs

If nested study, you won't get this graph

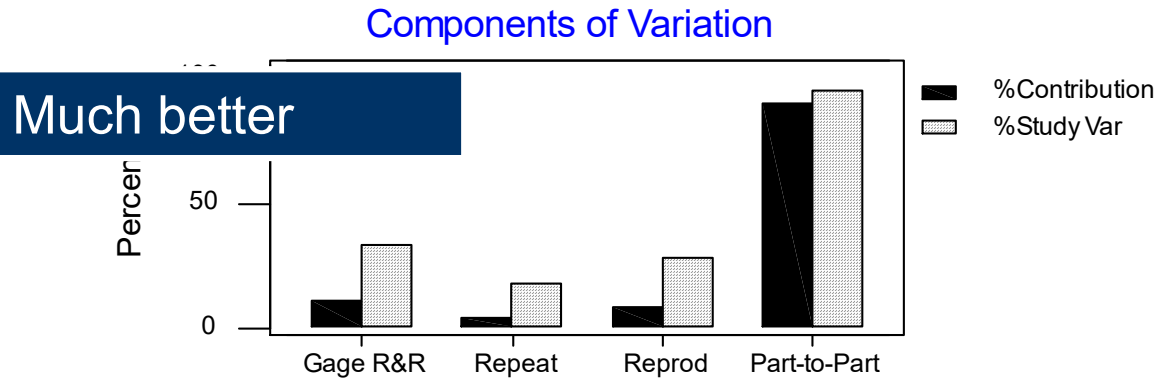
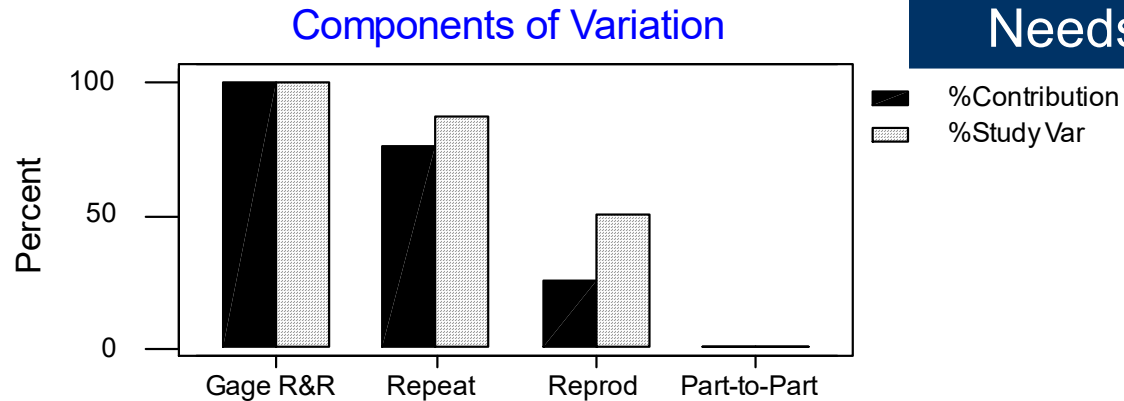
# Graphical Output Metrics

## CHART OUTPUT

- **Xbar Chart: Shows sampled process output variety**
  - **Reproducibility / bias**
- **R Chart: Helps identify unusual measurements**
  - **Resolution / repeatability**
- **Bar Chart: Distinguishes R & R from Process Output-  
Process Output**
  - **Location of variation**

**These are your leading graphical indicators**

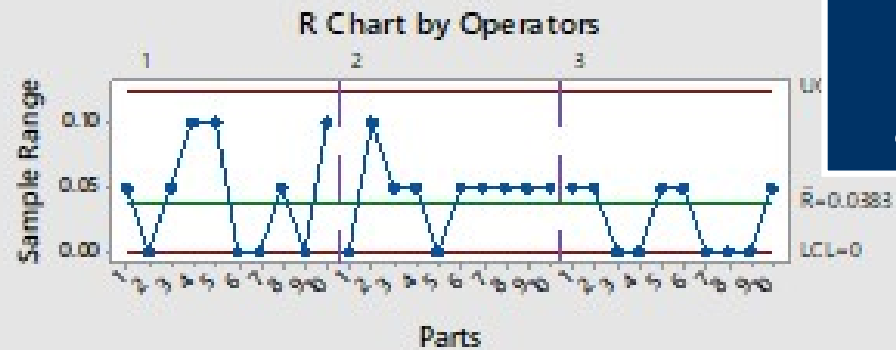
# Bar Charts for Components



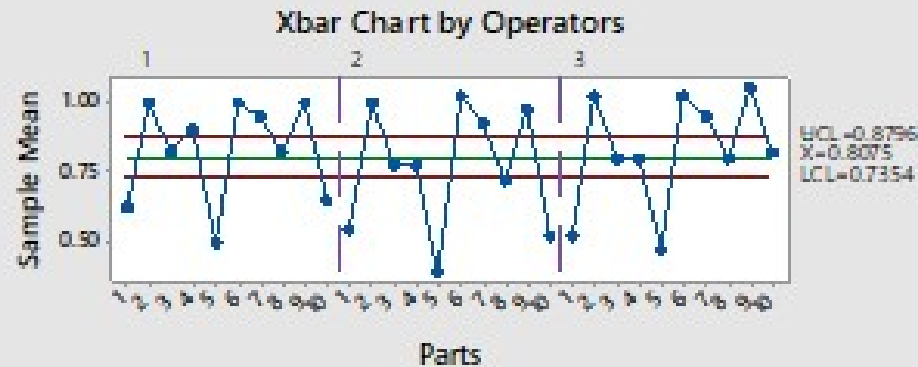
**Answers: “Where is the variation?”**

# Closer Look at the Xbar & R Charts

R Chart:  
Exposes gage  
resolution  
& stability



Xbar Chart:  
Test of sensitivity,  
bias, & population  
variety



**Xbar: at least 50% outside limits; R chart: in control**

# Tabular Output Metrics

## %Contribution

Source	VarComp	%Contribution (of VarComp)
Total Gage R&R	0.004437	10.6
Repeatability	0.001292	3.10
Reproducibility	0.003146	7.56
Operator	0.000912	2.19
Operator*Part	0.002234	5.37
Part-To-Part	0.037164	89.33
Total Variation	0.041602	100.00

## %Study

Source	StdDev (SD)	Study Var (5.15*SD)	%Study Var (%SV)	%Tolerance (SV/Toler)
Total Gage R&R	0.066615	0.34306	32.6	68.6
Repeatability	0.035940	0.18509	17.62	37.02
Reproducibility	0.056088	0.28885	27.50	57.77
Operator	0.030200	0.15553	14.81	31.11
Operator*Part	0.047263	0.24340	23.17	48.68
Part-To-Part	0.192781	0.99282	94.52	198.56
Total Variation	0.203965	1.05042	100.00	210.08
Number of Distinct Categories			4	

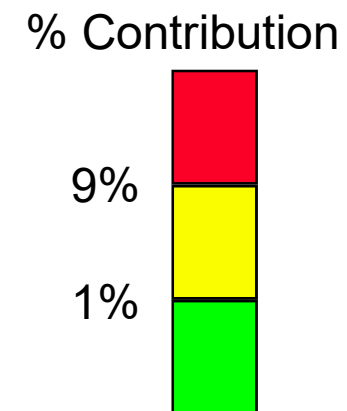
## %Tolerance

## Number of Distinct Categories

# % Contribution

$$\% \text{ Contribution} = \frac{\sigma^2_{\text{R\&R}}}{\sigma^2_{\text{Total}}} * 100$$

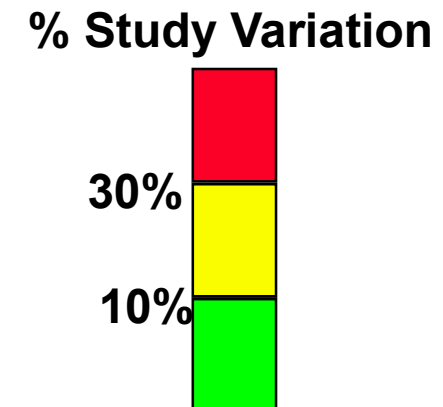
- Measurement System Variation (R & R) as a percentage of Total Observed Process Variation
- Includes both repeatability and reproducibility



# % Study Variation

$$\% \text{ Study Variation} = \frac{\sigma_{\text{R\&R}}}{\sigma_{\text{Total}}} * 100$$

- Looks at standard deviations instead of variance
- Measurement System Standard Deviation (R & R) as a percentage of Total Observed Process Standard Deviation
- Includes both repeatability and reproducibility





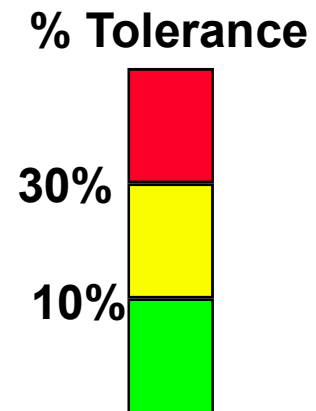
# % Tolerance

## Precision to Tolerance P/T

$$\% \text{ Tolerance} = \frac{5.15 * \sigma_{R\&R}}{\text{Tolerance}} * 100$$

- Measurement error as a percent of tolerance
- Includes both repeatability and reproducibility
- 5.15 Study Variation = 99%

Acceptance  
Criteria

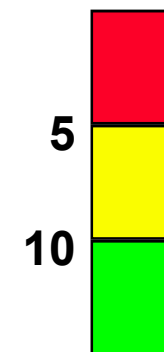


# Distinct Categories

$$\text{Number of Distinct Categories} = \sqrt{2 * \left[ \frac{\sigma^2_{\text{Process Output}}}{\sigma^2_{\text{R\&R}}} \right]}$$

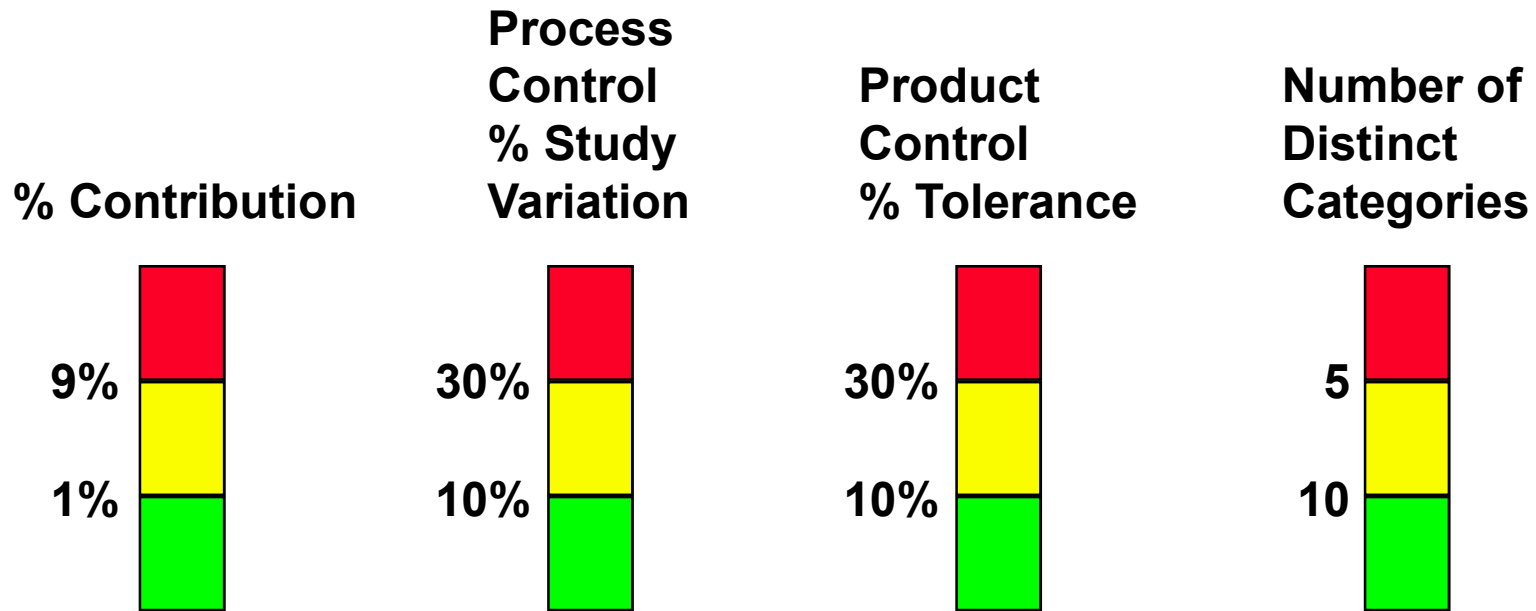
- Number of divisions that the Measurement System can accurately measure across the process variation
- How well a measurement process can detect process output variation--process shifts and improvement
- Less than 5 indicates Attribute conditions

Number of Distinct Categories



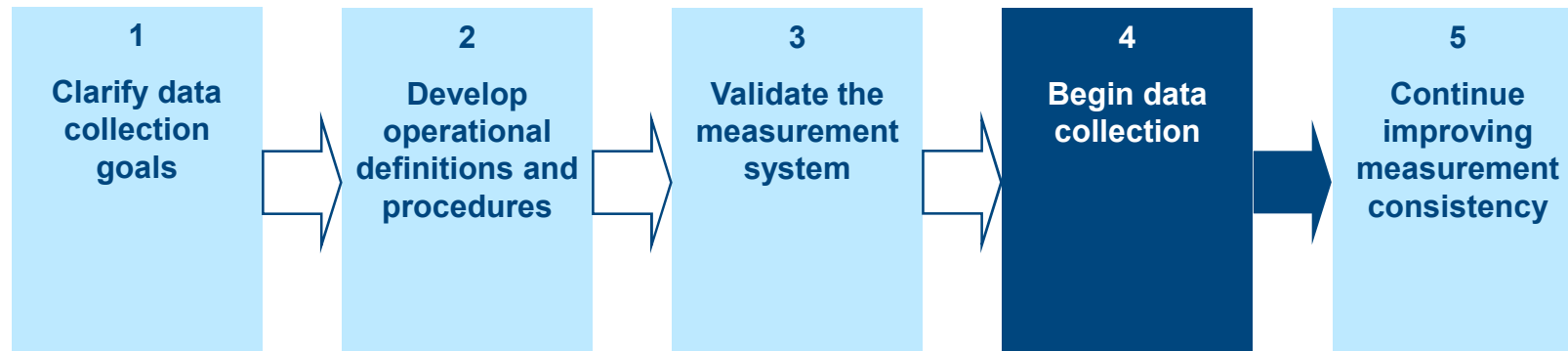
# Acceptability Summary

## Tabular Method



**Desirable to Have All 4 Indicators Say “Go”**

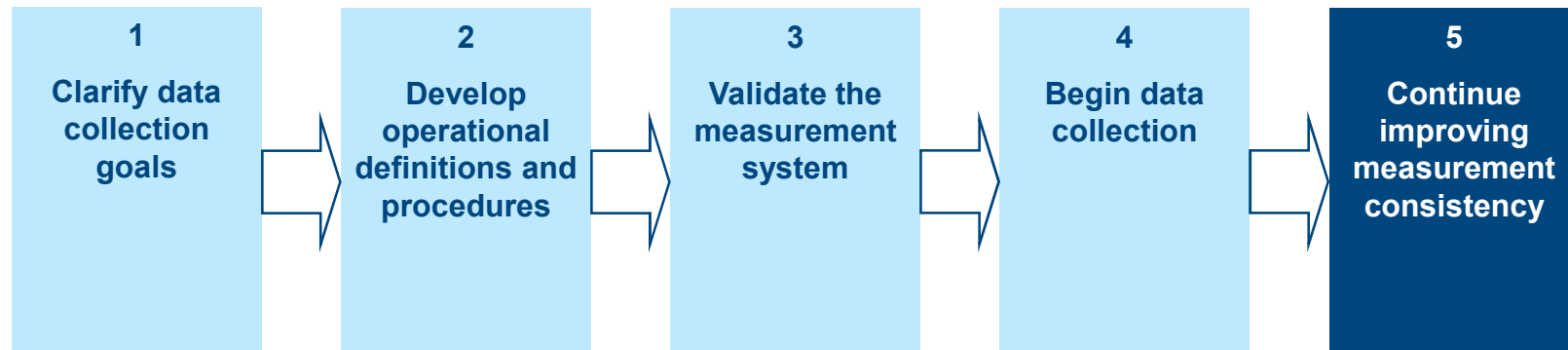
# Data Collection, Step 4: Begin the Data Collection



## Goal

- To ensure a smooth start-up of data collection

# Data Collection, Step 5: Continue Improving Measurement Consistency and Stability



## Goal

- To check that data collection procedures are being followed and that changes are made as necessary to adapt to changing conditions

## A Final Note: Do You Really Need New Data?

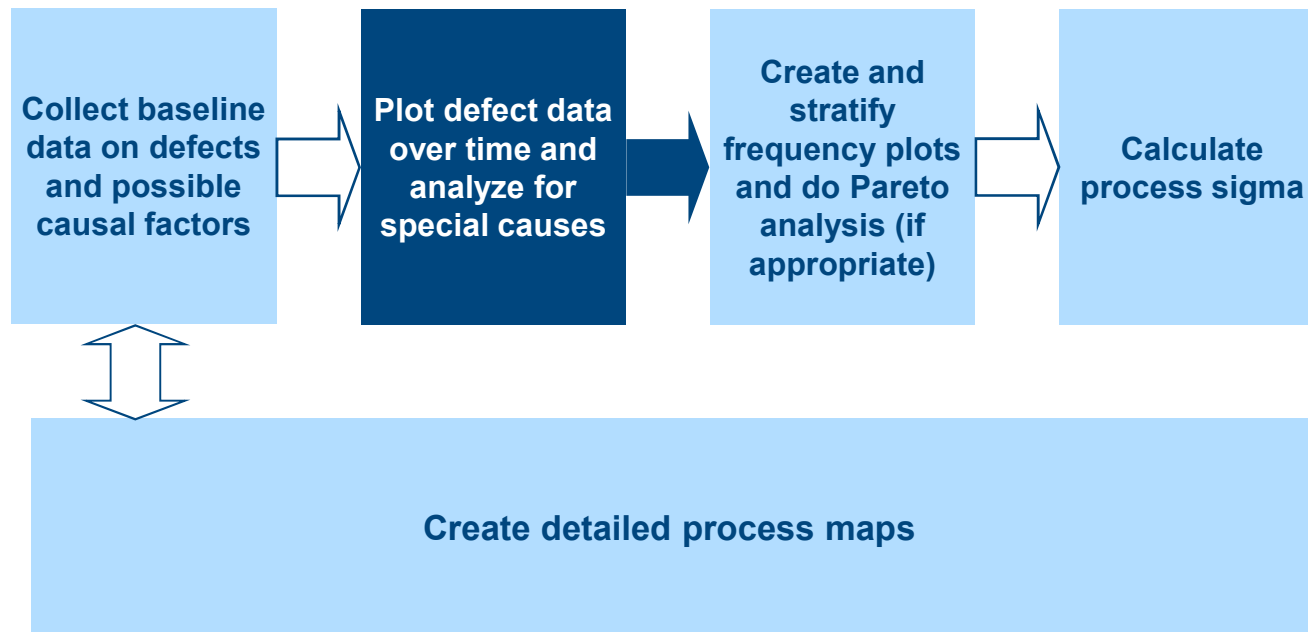
- Data collection can be time-consuming and expensive
- Before you collect new data, you may want to:
  - See if you have existing data that fits your needs—but use with caution
  - Use **observation** of the workplace as a way to gather information

# Anexas Consultancy Services

## Data Analysis I

Identifying Patterns over Time

# You Are Here



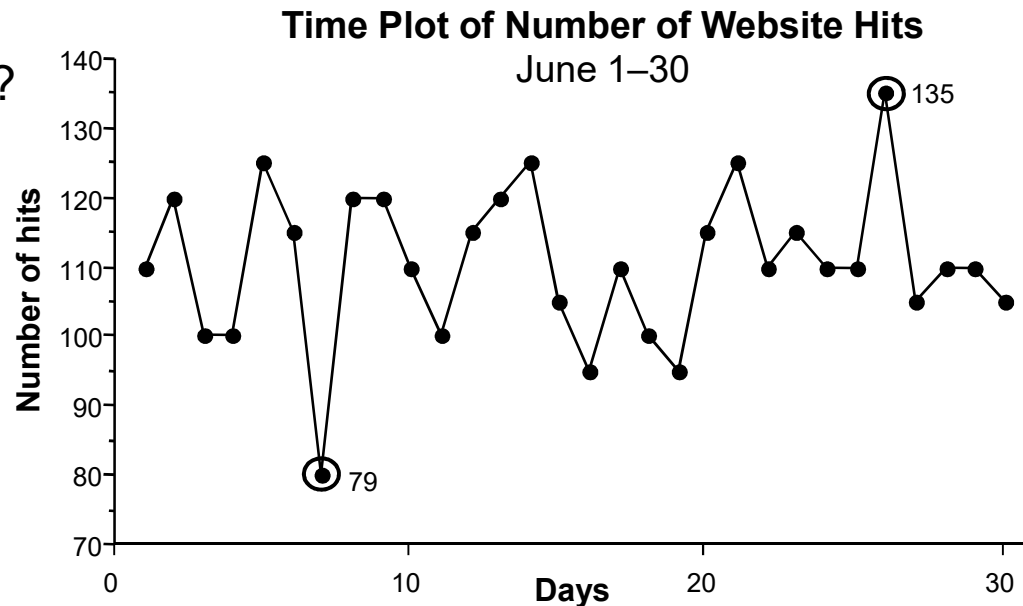


# Goals

- Given a situation involving a specific product or service, explain the customer's view of quality in terms of the Taguchi Loss Function
- Differentiate between common- and special-cause variation and select an appropriate response to each
- Given a set of data and an improvement problem, determine whether or not to use a time plot or control chart
- Given a set of data and an improvement problem, create and interpret a time plot using the run chart rules
- Given a set of data and an improvement problem, create and interpret an individuals control chart

# Case Study: Website Hits

- The Information Services manager for a small company was told that the preceding month the new website had gotten 79 hits/day early in the month and 135 hits/day near the end of the month
  - His questions:
    - Was the 79 more typical? Or the 135? Was there a clear trend upwards?
    - How does looking at the data this way help?
  - His staff charted the hits/day for June



# Anexas Consultancy Services

## Data Analysis II

Looking for Patterns Not Related to Time

# Goals

Given a set of data and an improvement problem:

- Determine whether or not to use a Box plot
- Determine whether or not to use a Pareto chart
- Create and interpret a Box plot
- Create and interpret a Pareto chart

# Anexas Consultancy Services

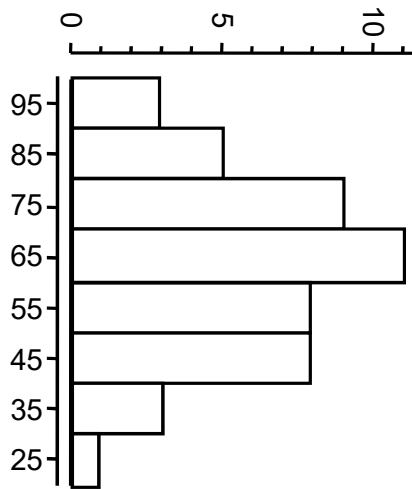
Studying the Distribution  
of Numeric Data

# Anexas Consultancy Services

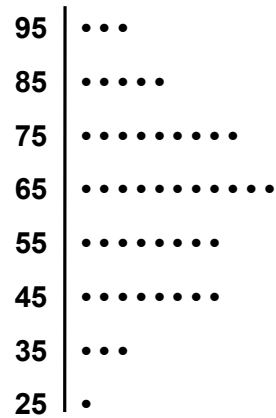
## Minitab Exercises

### Part 1: Histograms, Box Plots

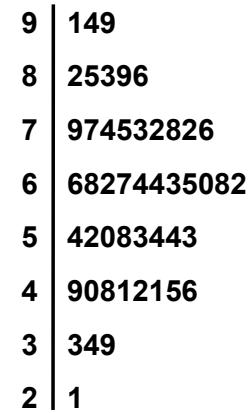
# Types of Frequency Plots



Histogram



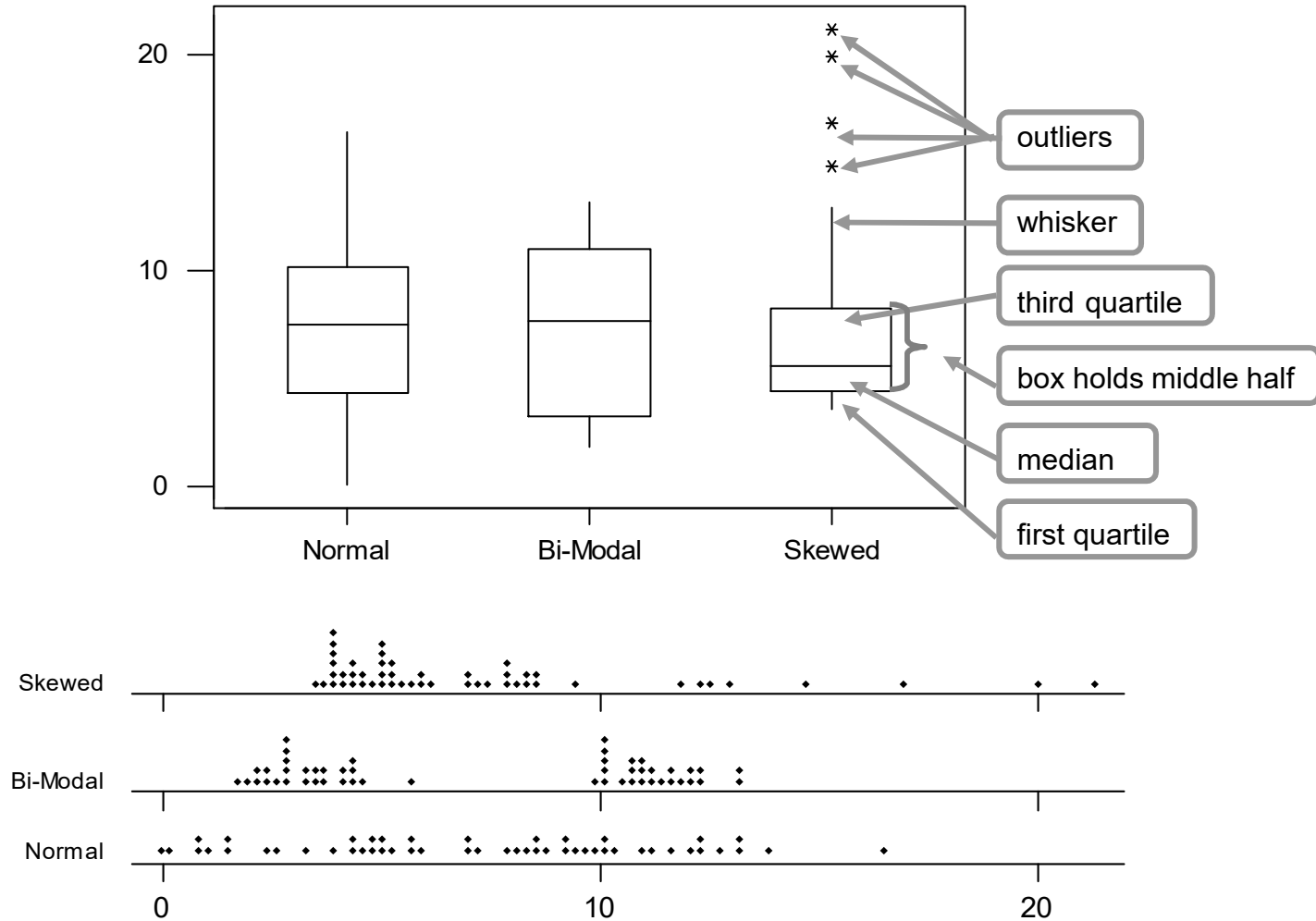
Dot Plot



Stem and Leaf Plot

# Box Plots—Shorthand Histograms

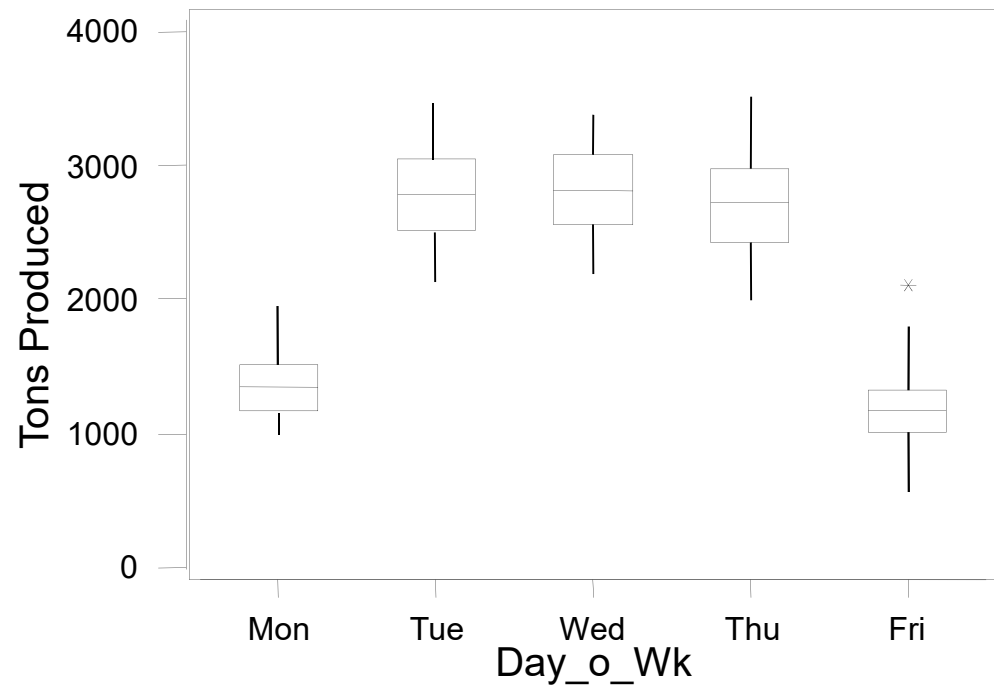
Box plot and dot plot of the same sets of data





# Advantages of Box Plots

- Box plots let you look at the shape, center, and spread of many sets of data at the same time
- This allows you to look for similarities and differences across processes or strata



- These data show the daily volume of paper produced at a paper mill, stratified by day of the week
- What conclusions do you draw?

# Anexas Consultancy Services

## Minitab Exercises

Data Analysis II,  
Part 2: Pareto Charts

# Pareto Charts: Uses

- Understand the pattern of occurrence for a problem
- Judge the **relative impact** of various parts of a problem
- Track down the biggest contributors to a problem
- Decide where to focus efforts

WORK LESS ACHIEVE MORE

80-20 PRINCIPLE

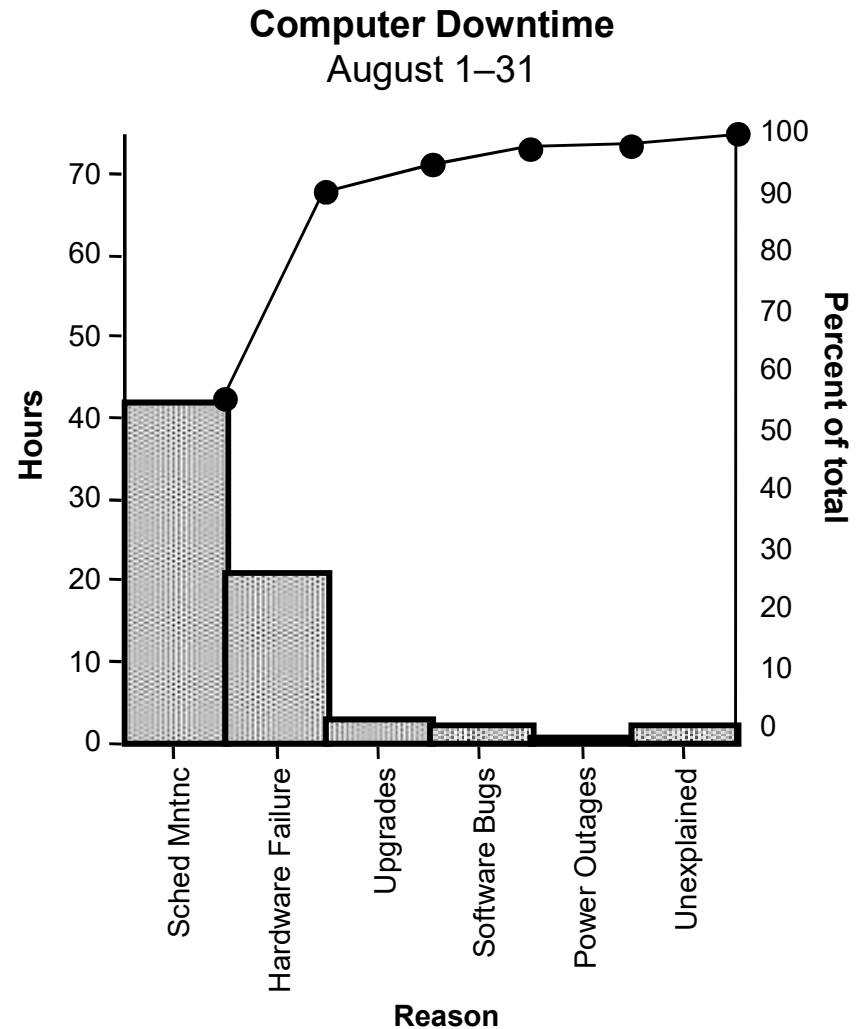
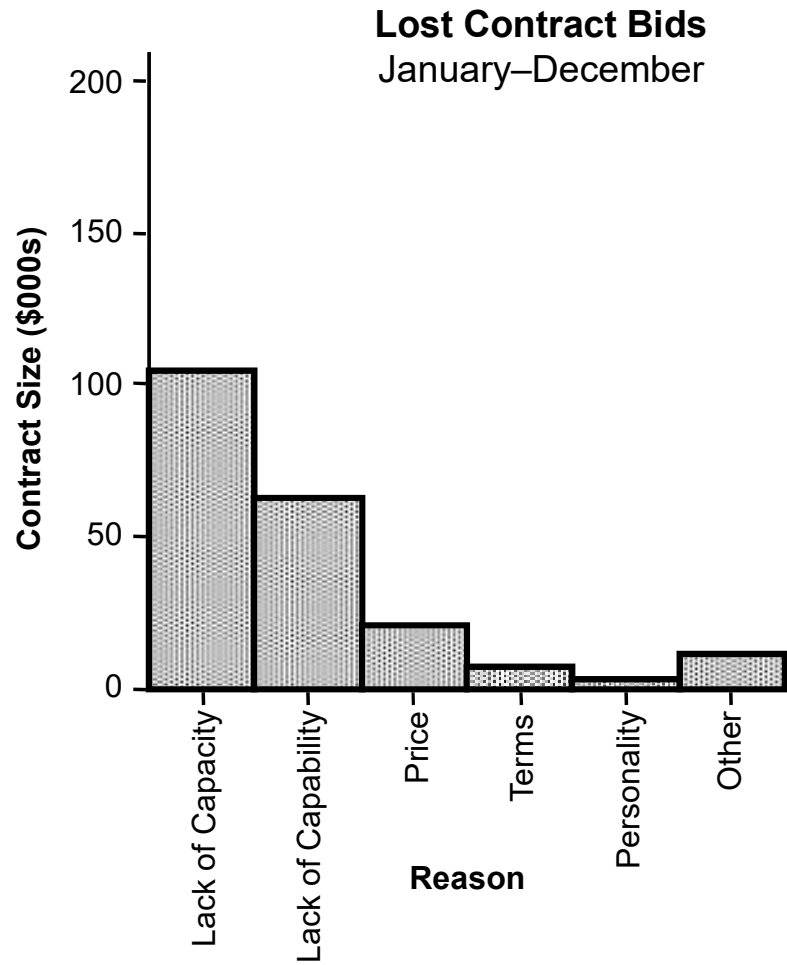


# The Pareto Principle

- The Pareto principle, often called the “80/20 rule,” says that, in many situations, roughly 80% of the problems are caused by only 20% of the contributing factors
- The Pareto principle implies that we can frequently solve a problem by identifying and attacking its “vital few” sources



# Examples of Pareto Charts



# Anexas Consultancy Services

Determining Process Sigma Level

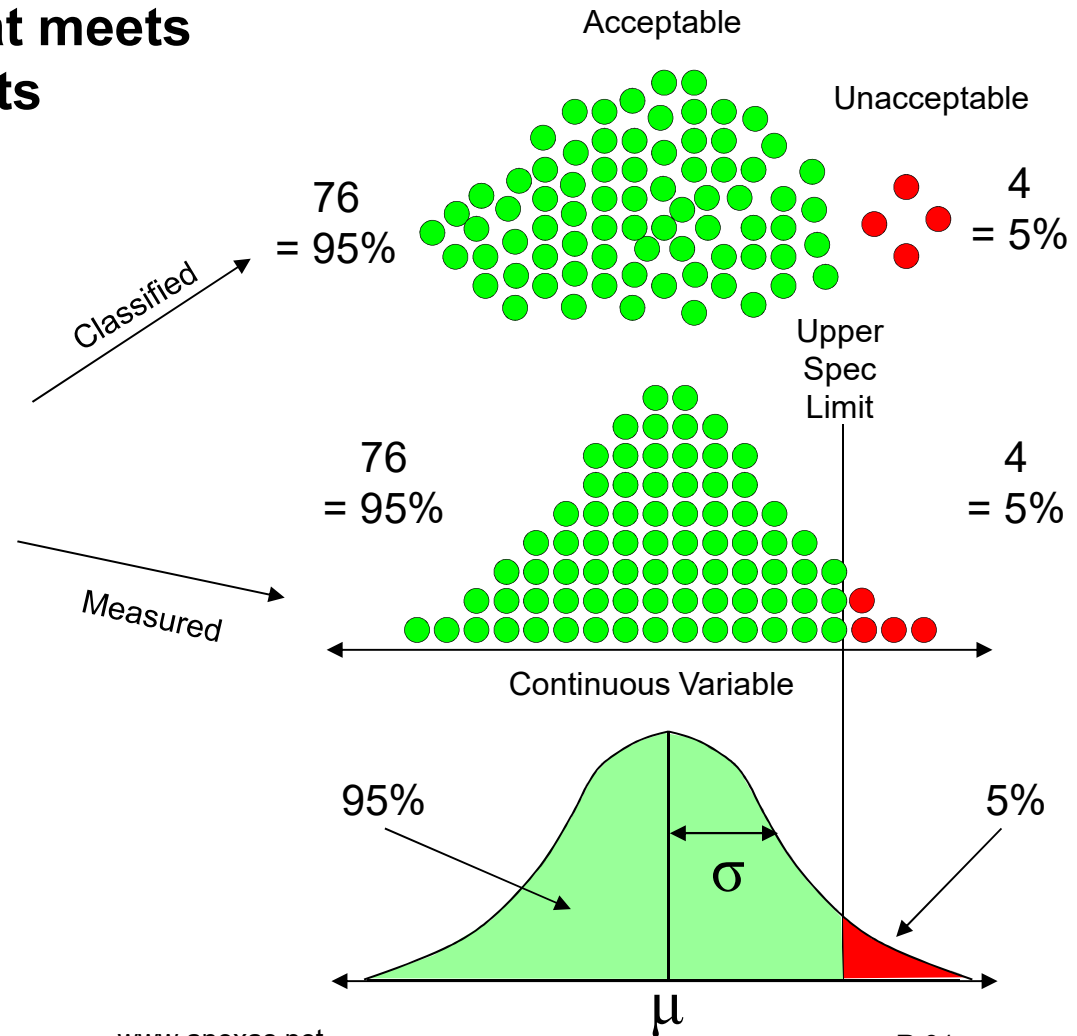
# Goals

- Explain the value of measuring process performance
- Understand the various ways to measure process performance
- Calculation of Sigma level using DPMO method
- Automated Sigma level calculator

# Process Yield

## Percentage of output that meets customer requirements

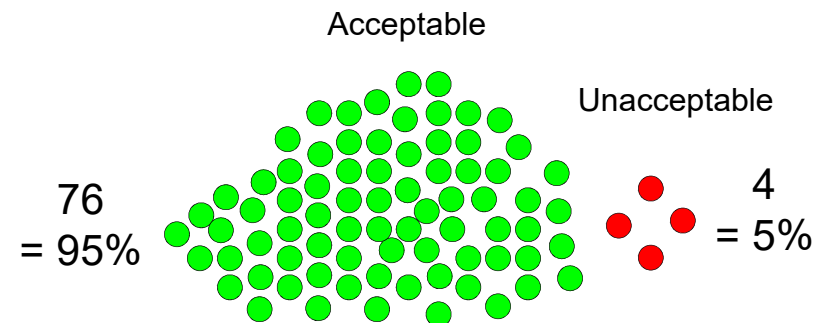
- Method 1: Actual count
- Method 2: Estimated yield (continuous only)





# Actual Count Method

- It involves simple and straightforward counting
- However, there must be at least five units in each category:
  - Usually there is no problem finding five good units
  - When quality is already good, large sample sizes may be required in order to find five bad units

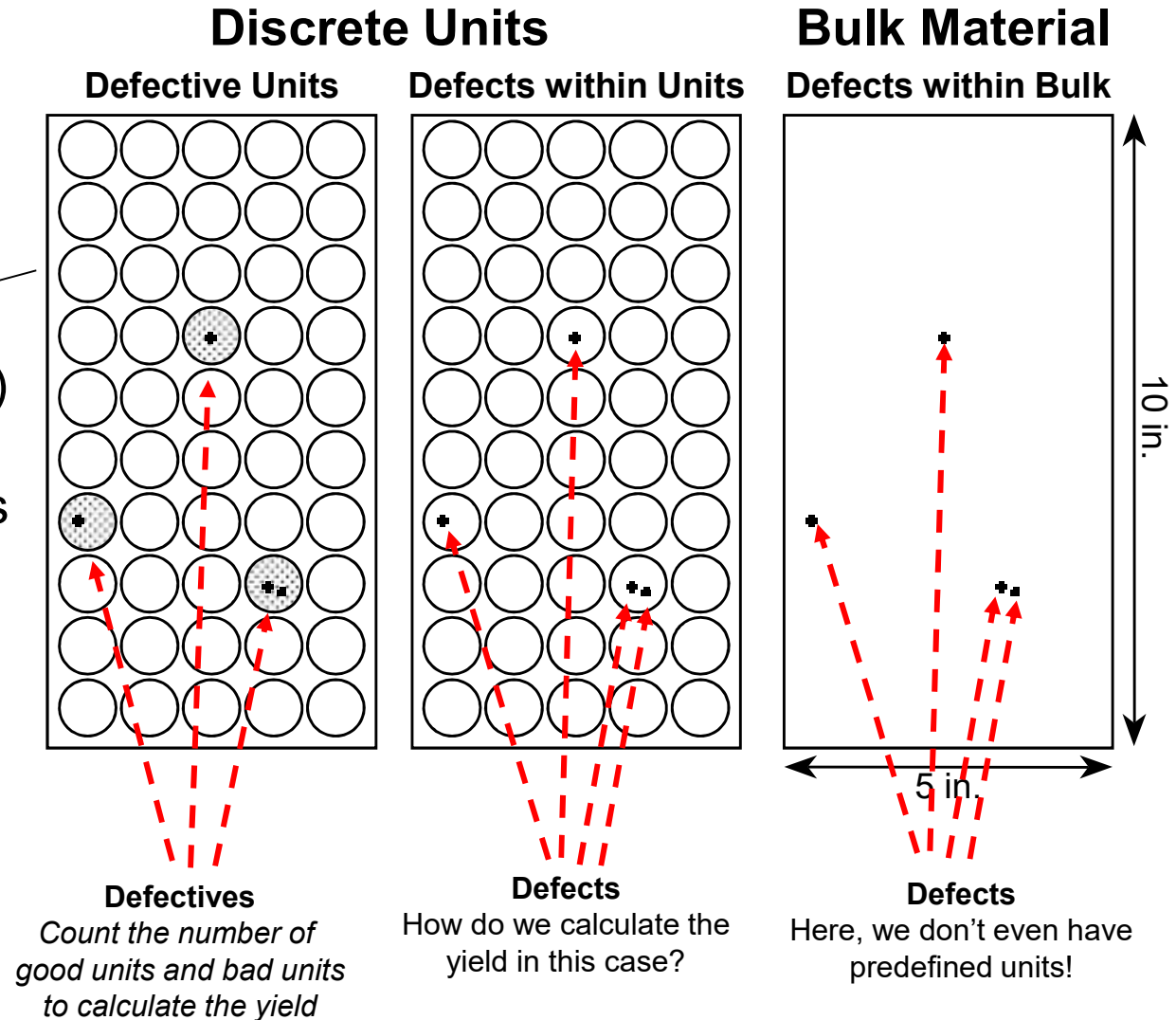


# Defects and Defectives

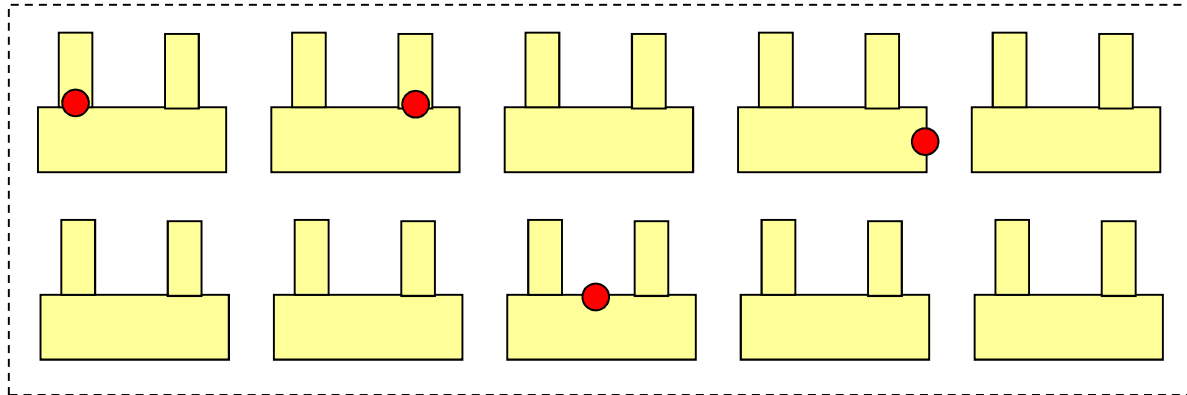
**Unit:** The item produced or processed

**Defective:** ←  
A unit of (unsalable) output that contains one or more defects

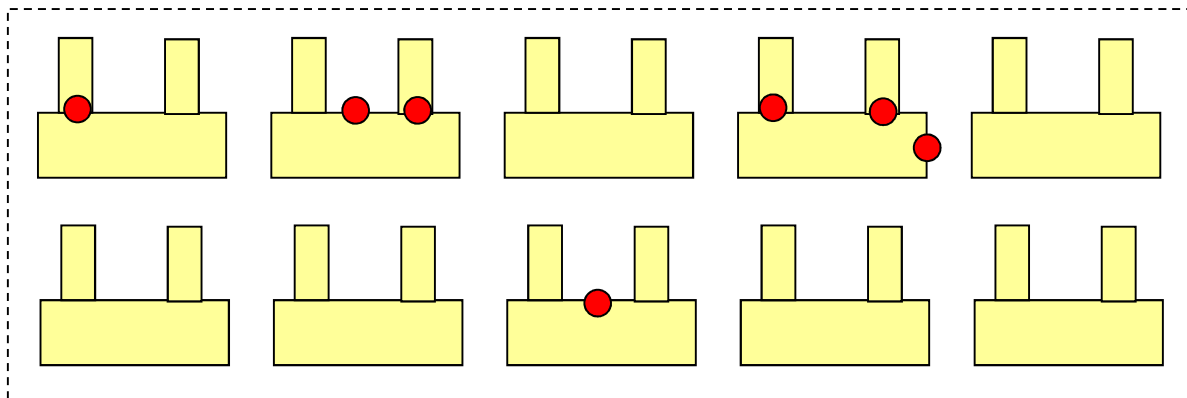
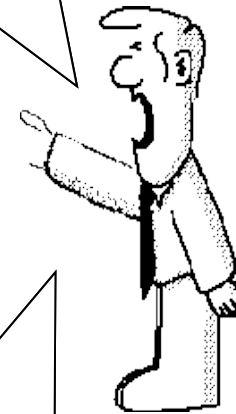
**Defect:** A feature that does not meet customers' requirements



# Difficulty: Multiple Defects per Unit?



Hey! Both these samples have 80% yield!



But this one has nearly twice as many defects!

# Sigma Level Calculation

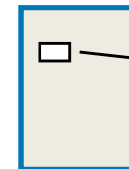
**Unit:** the item produced or processed

**Defect:** any event that does not meet the specification of a CTQ as defined by the customer

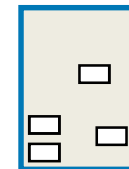
**Defect opportunity:** any event which can be measured that provides a chance of not meeting a customer requirement (specification)



*Form*



*Critical field with missing Information*



*# Critical fields on the form*

# Calculate process sigma : formula

Calculate the number of Defects Per Million Opportunities

(No. of Defects)

$$\text{DPMO} = \frac{\text{No. of Defects}}{\text{No. Of Units} \times \text{No. of opportunities}} \times 1\,000\,000$$

In the Sigma table, look at the Sigma value relating to the DPMO determined

# Sigma Level Table

Long term Yield Rendement Long terme	Process Sigma Sigma du processus	Defects per 1,000,000 Défauts par 1.000.000	Long term Yield Rendement Long terme	Process Sigma Sigma du processus	Defects per 1,000,000 Défauts par 1.000.000
99.99966%	6.0	3.4	93.320%	3.0	66,800
99.9995%	5.9	5	91.920%	2.9	80,800
99.9992%	5.8	8	90.320%	2.8	96,800
99.9990%	5.7	10	88.50%	2.7	115,000
99.9980%	5.6	20	86.50%	2.6	135,000
99.9970%	5.5	30	84.20%	2.5	158,000
99.9960%	5.4	40	81.60%	2.4	184,000
99.9930%	5.3	70	78.80%	2.3	212,000
99.9900%	5.2	100	75.80%	2.2	242,000
99.9850%	5.1	150	72.60%	2.1	274,000
99.9770%	5.0	230	69.20%	2.0	308,000
99.9670%	4.9	330	65.60%	1.9	344,000
99.9520%	4.8	480	61.80%	1.8	382,000
99.9320%	4.7	680	58.00%	1.7	420,000
99.9040%	4.6	960	54.00%	1.6	460,000
99.8650%	4.5	1,350	50%	1.5	500,000
99.8140%	4.4	1,860	46%	1.4	540,000
99.7450%	4.3	2,550	43%	1.3	570,000
99.6540%	4.2	3,460	39%	1.2	610,000
99.5340%	4.1	4,660	35%	1.1	650,000
99.3790%	4.0	6,210	31%	1.0	690,000
99.1810%	3.9	8,190	28%	0.9	720,000
98.930%	3.8	10,700	25%	0.8	750,000
98.610%	3.7	13,900	22%	0.7	780,000
98.220%	3.6	17,800	19%	0.6	810,000
97.730%	3.5	22,700	16%	0.5	840,000
97.130%	3.4	28,700	14%	0.4	860,000
96.410%	3.3	35,900	12%	0.3	880,000
95.540%	3.2	44,600	10%	0.2	900,000
94.520%	3.1	54,800	8%	0.1	920,000

# Automated Sigma Level Calculator given in Template

*Double click on the spreadsheet and enter data in the shaded cells.*

**Define the following**

- CTQ:** your customer CTQ
- Target:** your customer required target
- Defect:** Describe here how you would identify a defect
- Unit:** How do you define a unit
- Opportunity:** What is the rationale behind the # of opportunities

## ***DPMO***

- 1 Number Of Units Processed
- 2 Total Number Of Defects Made (Include Defects Made And Later Fixed)
- 3 Number Of Defect Opportunities Per Unit
- 4 Solve For Defects Per Million Opportunities
- 5 Sigma will calculate

	enter
N=	100
D=	20
O=	4

50000

Sigma= 3.14

# Some Rules for Counting Opportunities

- To facilitate before/after comparisons:
  - Use the same opportunity definitions as have been used in the past, provided they are operational
  - Keep the same number of opportunities per unit





# DPMO: Making DPO “Visible”

- Defects per million opportunities (DPMO)
- This is “scaling up” to make the number “visible”

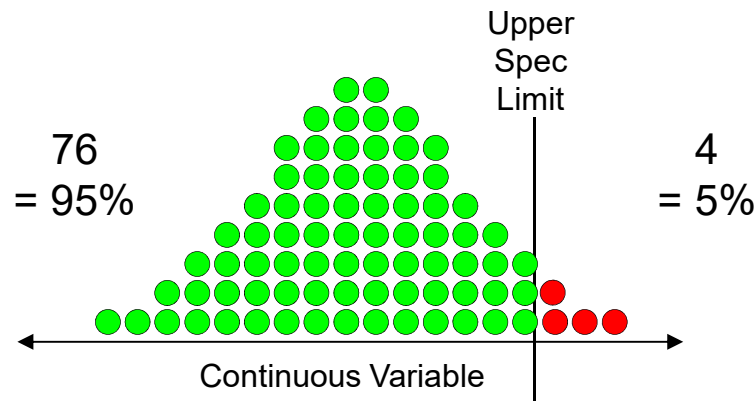
But what if I don't produce a million units or opportunities?



You don't need a sample of 100 in order to calculate percentage, which means “per hundred.” It's just a way of clearing out decimal places.

# Process Yield: Continuous Variables (Actual Count)

- For continuous variables, we have two ways to calculate yield:
  - The first is simple and straightforward counting, as before
  - But there must be at least five units in each category (in spec and out of spec):
    - Usually there is no problem finding five units in spec
    - When quality is already good, larger sample sizes may be required to find five units that are out of spec

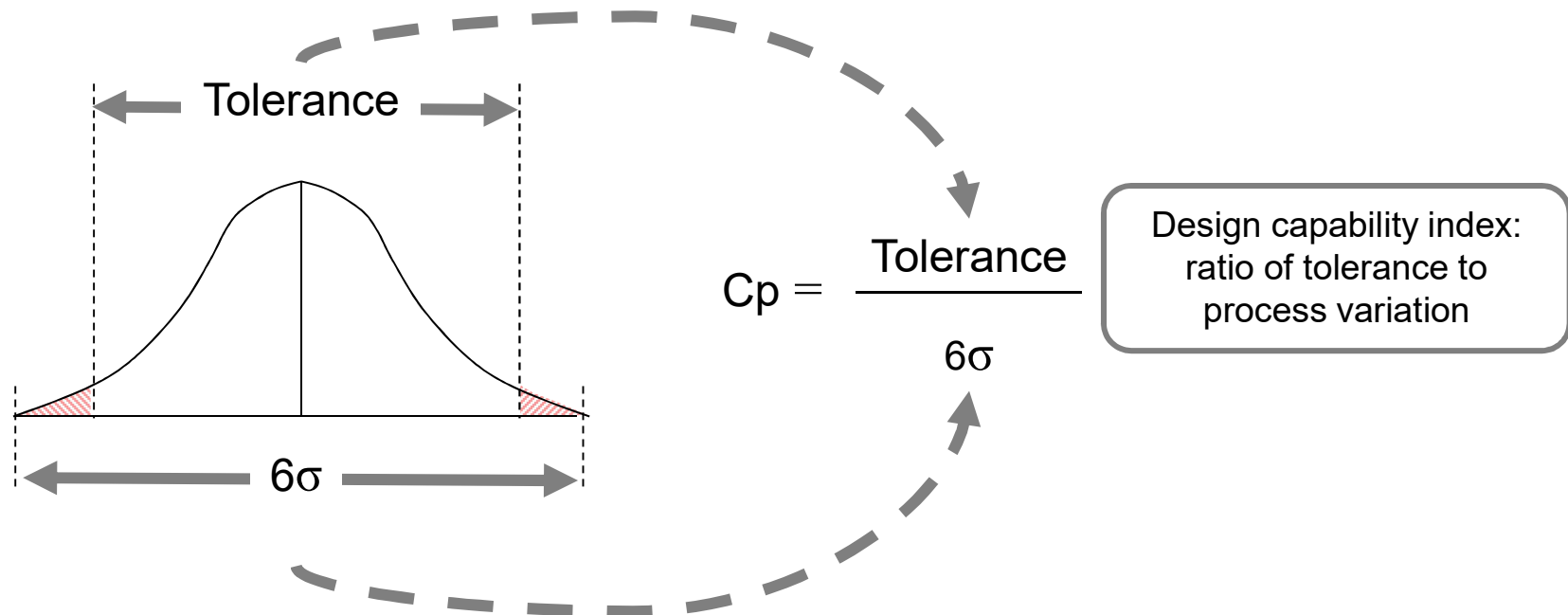


# Anexas Consultancy Services

## Process Capability Indices

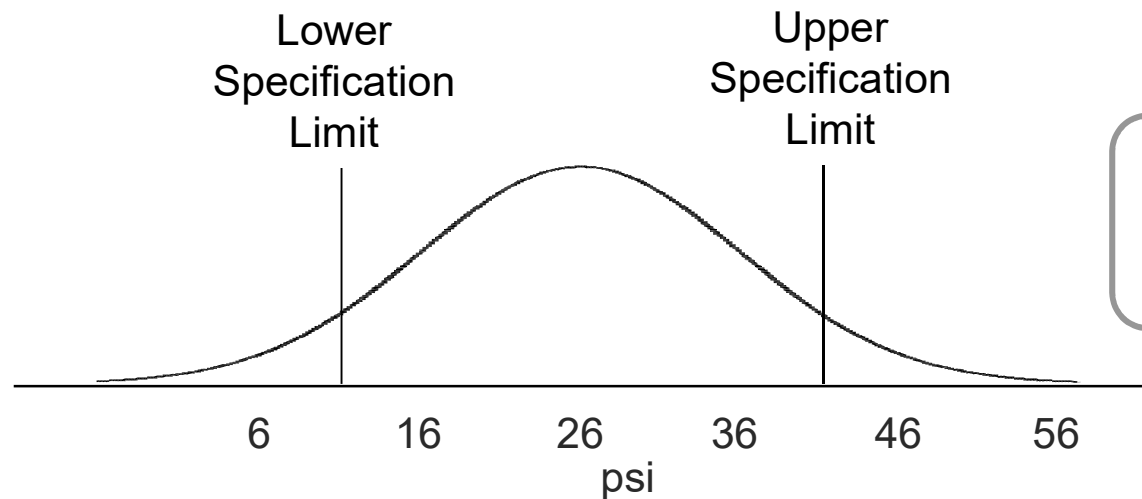
# Capability Indices

- **Specification limits:** what the process must do
- **Statistical distribution:** what the process can do
- **The collision:** when what we **can** do is not as good as what we **must** do:



# Design Capability Index Cp: Example

- Specification limits: 10–42 psi seal strength
- Process standard deviation:  $s = 10$



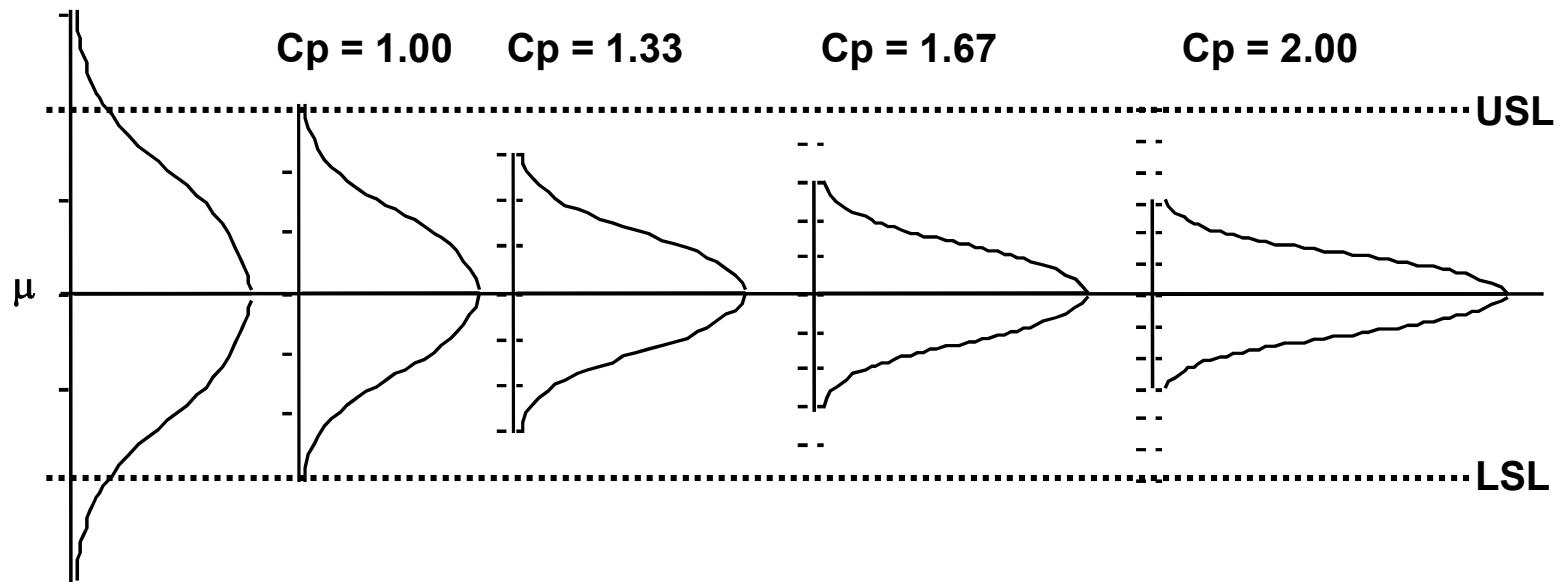
Example:

$$C_p = (42 - 10) / (6 \times 10)$$

$$C_p = 32 / 60 = 0.533$$

# Design Capability Index Cp: Example, cont.

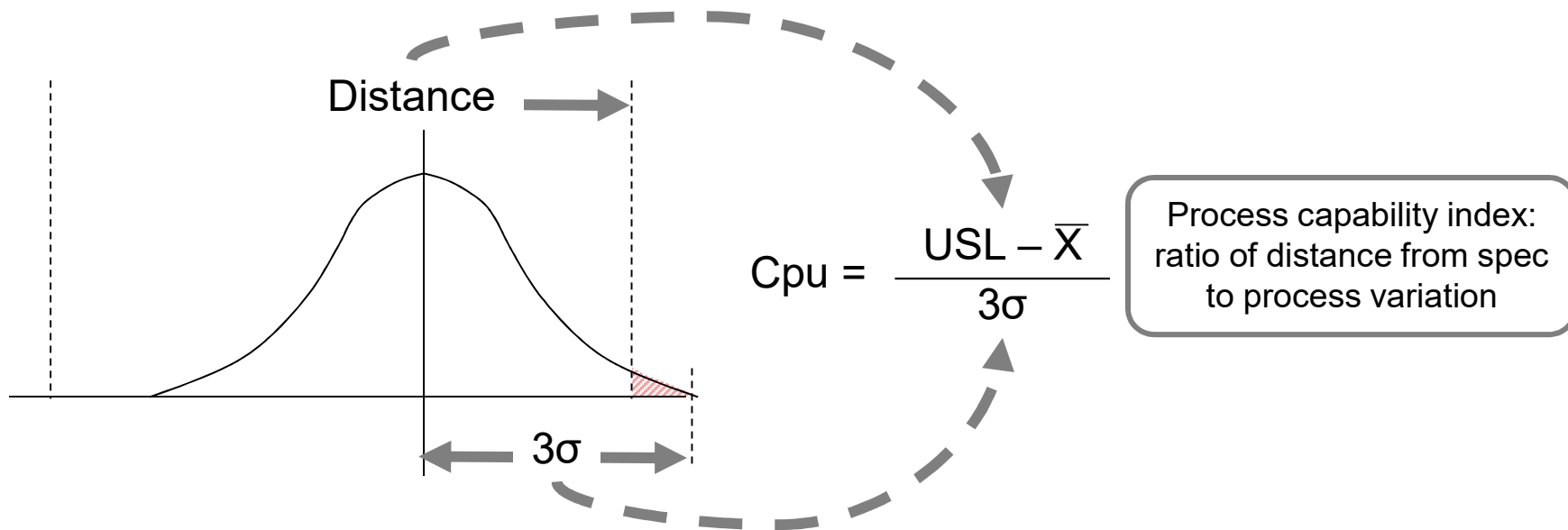
**Cp = 0.67**



- Cp < 1 means the distribution does not fit between the specification limits
- Cp = 1 means it fits (except for the tails)
- Cp = 1.33 means the distribution occupies three-quarters of the tolerance range
- Cp = 1.67 means the distribution occupies 60% of the tolerance range
- Cp = 2.00 means the distribution occupies half the tolerance range

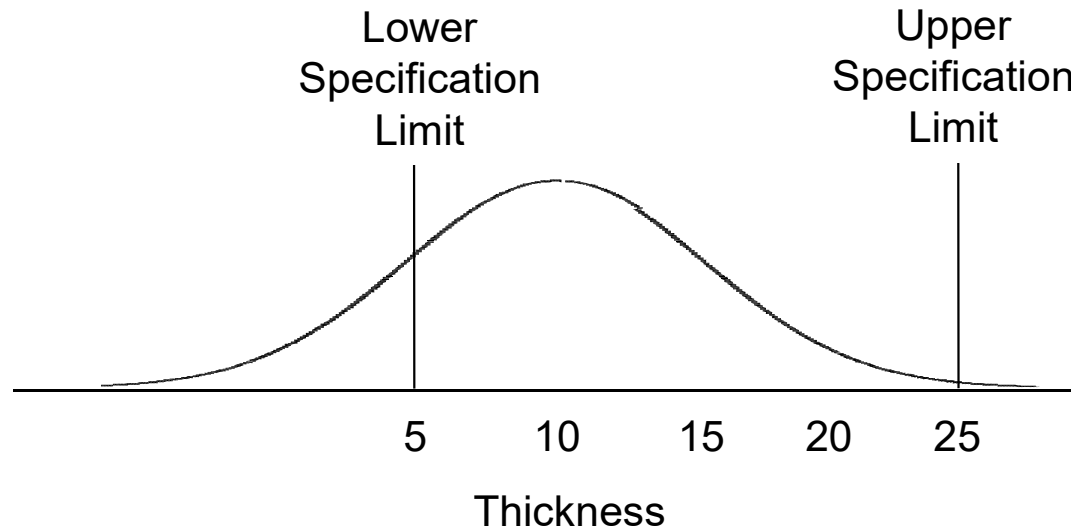
## Difficulty: What If the Process Is Off-Centered?

- Cpk takes the targeting (mean) of the process into account:
  - Distance from the mean to the nearest spec limit
- $Cpk = \min\{Cpu, Cpl\}$  where Cpu and Cpl are calculated for the upper limit and lower limit, respectively



# Process Capability Index Cpk: Example

- Specification limits: 5–25 cm thickness
- Process mean:  $\bar{X} = 10$
- Process standard deviation:  $s = 5$



$$C_{pu} = (25 - 10) / (3 \cdot 5) = 1$$

$$C_{pl} = (10 - 5) / (3 \cdot 5) = 0.333$$

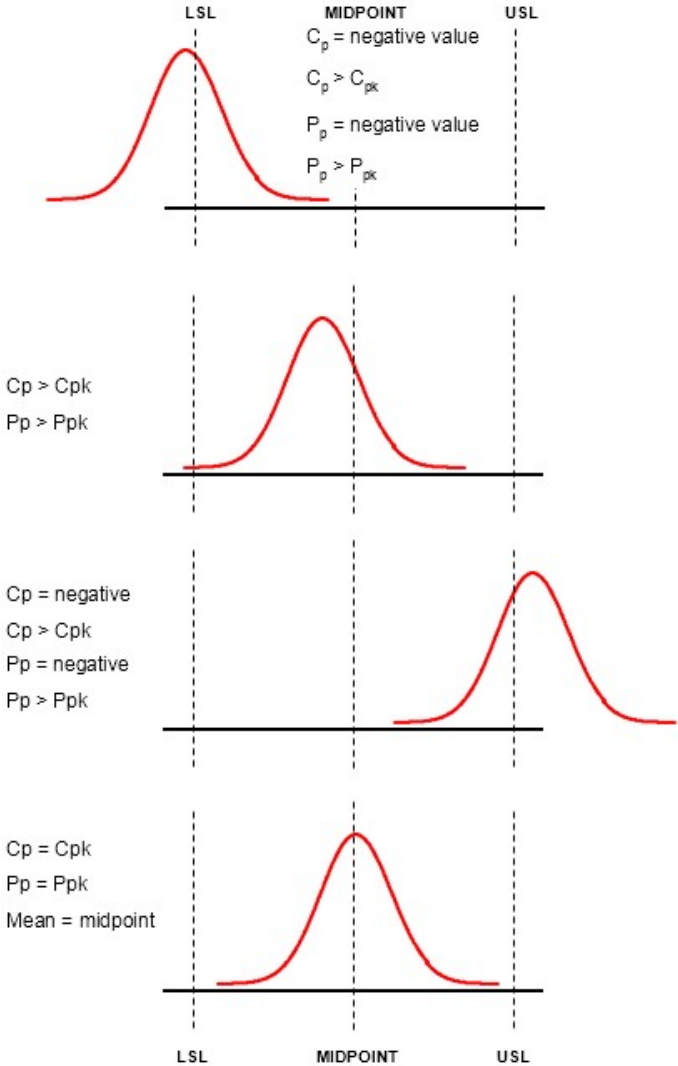
$$C_{pk} = \text{minimum}\{C_{pu}, C_{pl}\}$$

$$C_{pk} = 0.33$$

- Calculate for upper limit ( $C_{pu}$ ) and lower limit ( $C_{pl}$ )
  - $C_{pk}$  is the minimum of the two



# Process Capability Indices



# Drawbacks of Capability Indices

- They apply only to continuous variables:
  - There is no index for discrete variables
- One index number per variable:;
  - There is no “overall index”
- Assumes Normal distribution

# TDU: Total Defects per Unit

- Calculate proportion out of spec for each continuous quality
- Calculate DPU for all discrete qualities combined
- Add them together:

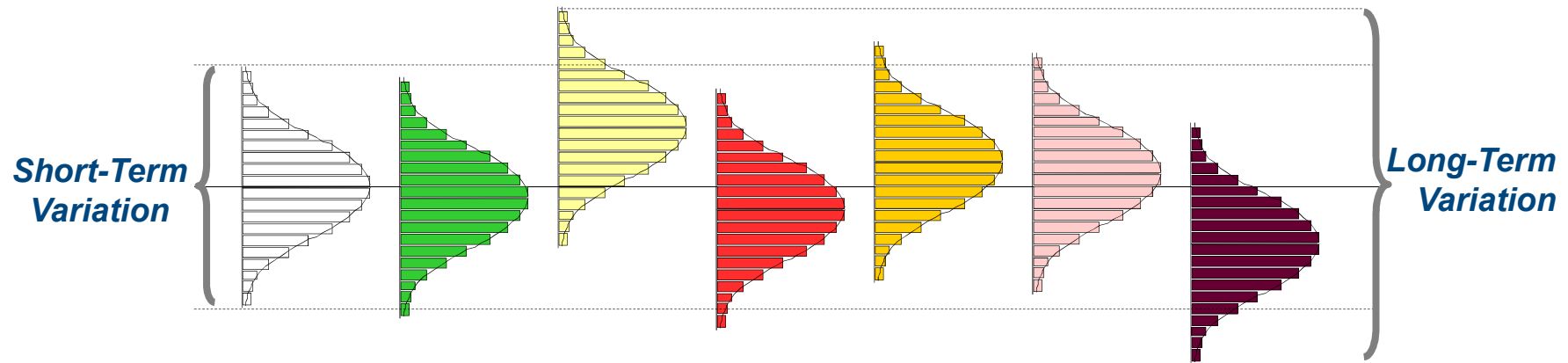
Bag thickness:	0.005
Bag strength:	0.002
<u>Discrete DPU:</u>	<u>0.010</u>
TDU:	0.017

- Rolled throughput yield:  $RTY = e^{-TDU}$ :

$$RTY = \exp(-0.017) = 0.983 = 98.3\%$$

# One Last Difficulty: Process Shift

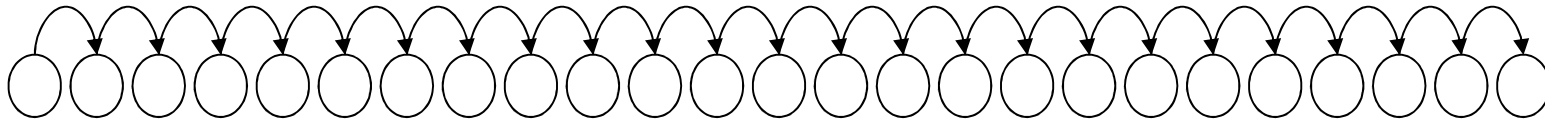
- Normal curve percentiles assume no shift in the mean
  - But even well-controlled processes experience some shift:



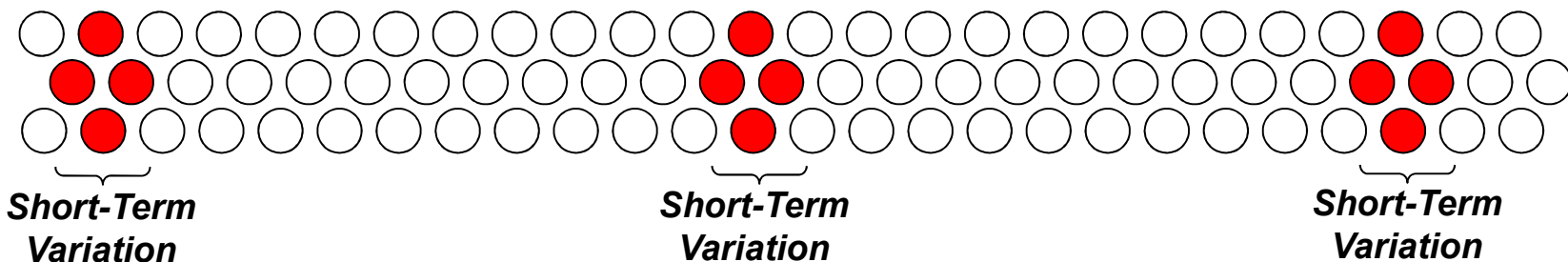
- **Short-term variation:** variation from cycle to cycle of process
- **Long-term variation:** includes time-to-time shifting of process:
  - Typically greater than short-term variation
- Magnitude of difference is the “shift”

# Long-Term Variation vs. Short-Term Variation

- **Long-term variation** is the standard deviation of an entire data series over “a reasonable period of time”
- **Short-term variation** can be calculated in three ways:
  - Serial production: moving ranges



- Mass production: subgroup ranges

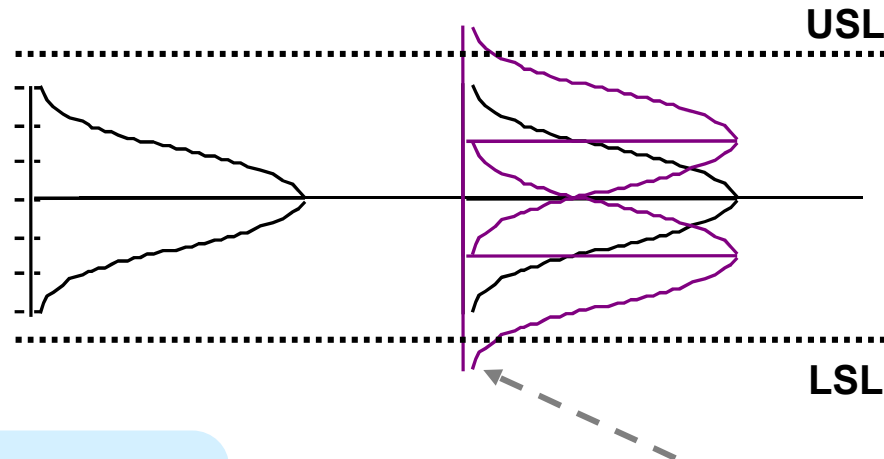


- Selecting time periods in a data series when the process seems especially stable

# Effect of Long-Term Variation on Yield

Random Process Variation

Imperfect Process Control



Process seems very capable of holding the specs

But when shift is taken into account, there are problems

- We need to evaluate the process performance, taking long-term shift into account

# Process Performance vs. Process Capability

- Process performance index uses the same formulas:
  - But with long-term standard deviation instead of short-term

**Capability Index  
(short-term st. dev.)**

**Performance Index  
(long-term st. dev.)**

$$C_p = \frac{USL - LSL}{6s_{ST}}$$

$$P_p = \frac{USL - LSL}{6s_{LT}}$$

$$C_{pu} = \frac{USL - \bar{X}}{3s_{ST}}$$

$$P_{pu} = \frac{USL - \bar{X}}{3s_{LT}}$$

$$C_{pl} = \frac{\bar{X} - LSL}{3s_{ST}}$$

$$P_{pl} = \frac{\bar{X} - LSL}{3s_{LT}}$$

$$C_{pk} = \min\{C_{pu}, C_{pl}\}$$

$$P_{pk} = \min\{P_{pu}, P_{pl}\}$$

**Takes  
targeting  
into account**

# Anexas Consultancy Services

**Sigma Performance Level**



# Sigma Performance Level

- Sigma performance level is an all-inclusive process/product performance indicator figure
  - Includes capability and shift
  - Covers discrete and continuous variables
- Sigma performance level =  $3 \cdot Cpk$
- Sigma short-term performance level = Z-score + 1.5
- Even if dealing with discrete variables, we use the Z-score that corresponds with the yield
  - This gives us the capability if there were no shift
- Add 1.5 to the Z-score as a “standard amount of shift”

# Calculating Process Performance

- All of these calculations have been automated for convenience
- Open the Excel file: **PrCapCal**
- Go to the sheet: Method 1: Yield and Sigma

# Interpreting the Excel Spreadsheet

## Method 1: Yield and Sigma

1. Determine number of defect opportunities per unit	<b>O =</b>
2. Determine number of units processed	<b>N =</b>
3. Determine total number of defects made (include defects made and later fixed)	<b>D =</b>
4. Calculate Defects Per Opportunity	$DPO = \frac{D}{N \times O} =$
5. Calculate Yield	$Yield = (1 - DPO) \times 100 =$
6. Look up Sigma in the Process Sigma Table	<b>Process Sigma =</b>

10
453
4
0.00088
99.912%
4.63

1. Enter the number of opportunities per unit (O)

2. Enter the number of units fully inspected (N)

3. Enter the number of defects found in the inspections (D)

4. Spreadsheet returns the opportunity-based yield...

5. ... and the sigma performance level

# Interpreting the Excel Spreadsheet, cont.

## Method 2: Process Capability

	Enter		
Xbar		10	
SST		4	
SLT		5	
Target			Delete if no Target
USL		14	Delete if no USL
LSL		5	Delete if no LSL
		1.83 sigma	
		370511 ppm	264305 ppm
		62.949% LT yld	73.569% Pot yld
		0.27 Ppk	0.33 Cpk
		0.33 Z LT	0.63 Z ST
		0.3 shift	

1. Enter the process mean, X-bar

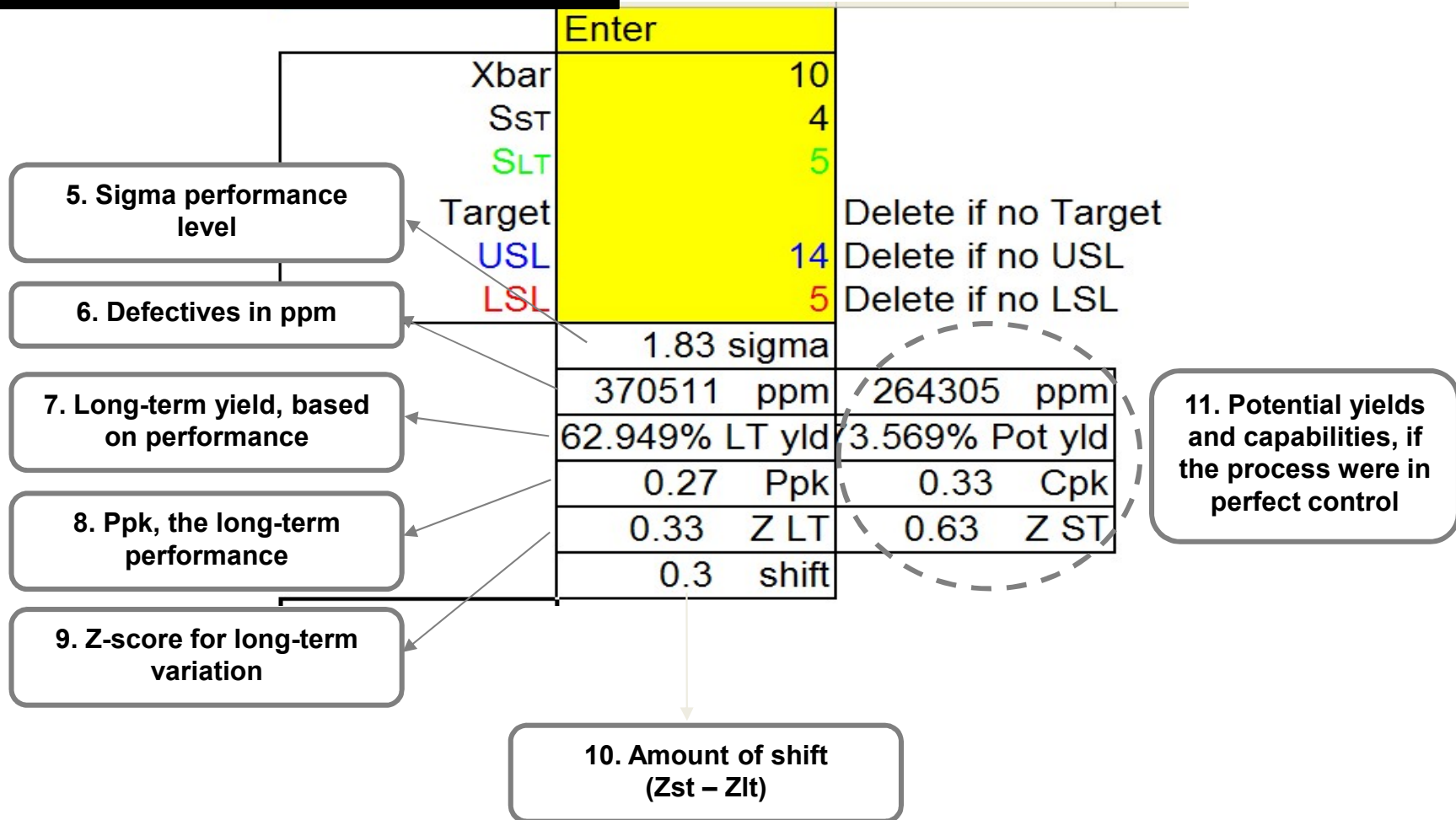
2. Enter the *short-term* st. dev.

3. Enter the *long-term* st. dev.

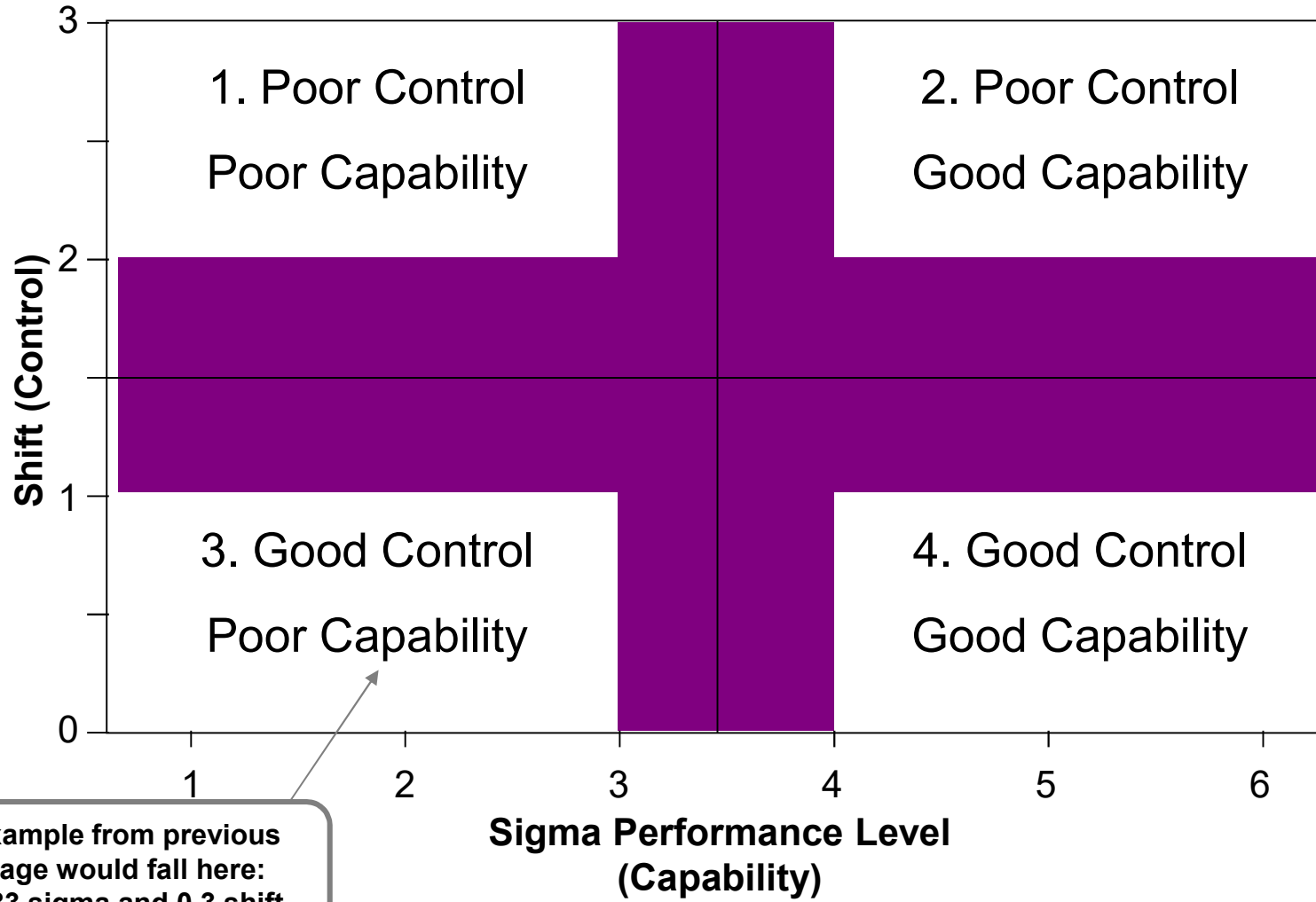
4. Enter the specification limits, as appropriate.  
**NOTE: Be sure to delete entries if there is no limit or target**

# Interpreting the Excel Spreadsheet, cont.

## Method 2: Process Capability



# Looking at Sigma and Shift



Example from previous page would fall here:  
1.83 sigma and 0.3 shift

# Exercise: Calculating Yield

**Objective:** Practice using Method 1 to calculate yield:

- Open *PrCapCal* Excel Spreadsheet
- Go to Method 1: Yield & Sigma

**Instructions:**

- Compute the yield for each of the following situations:
  - 5000 units, 1 opportunity per unit, 250 defects
  - 187 units, 3 opportunities per unit, 209 defects
  - 1000 units, 21 opportunities per unit, 15 defects
- A table has been provided below to aid in your calculations

**Time:** 5 minutes

## Exercise: Calculating Yield, cont.

**Objective:** Practice using Method 2 to calculate yield:

- Open *PrCapCal* Excel Spreadsheet
- Go to Method 2: Process Capability

**Instructions:**

- Calculate yield for the situations described below
- Data are roughly Normally distributed:

$\bar{X} = 28$	$s = 2.9$	$USL = 30$	$LSL = 25$
$\bar{X} = 50$	$s = 10$	$USL = 35$	$LSL = \text{none}$
$\bar{X} = -4.7$	$s = 13$	$USL = 15$	$LSL = -15$

**Time:** 5 minutes



# Summary

- Yield: Actual count method:  $1 - p$ :
  - Difficulty: Multiple defects in a unit
- DPU:  $D/N$ :
  - Difficulty: Comparing processes with different complexities
- Opportunity-based yield:  $1 - D/(N*O)$ :
  - Difficulty: Small sample sizes typical of continuous variables
- Yield: Normal approximation method:  $C_p$ :
  - Difficulty: Non-Normal data (transformations not covered)
  - Difficulty: Process is off-centered

## Summary, cont.

- Capability index: Cpk:
  - Difficulty: Process shift
- Performance index: Ppk:
  - Difficulty: Combining discrete and continuous
- Sigma performance level

# Anexas Consultancy Services

## Process/Product Performance Indicators for Efficiency

Cycle Time

Overall Equipment Effectiveness

## Cycle Time: Little's Law

$$\text{Cycle Time} = \frac{\text{Work in Process}}{\text{Exit Rate}}$$

- **Exit rate** is the count of how many units leave the system in a given time
- **Work in process** is the count of how many units are in the process at a given time
- **Cycle time** should match the “heartbeat” of the process:
  - Called **takt time**
  - This enables one-piece flow, just-in-time, and other benefits

## Example of Little's Law

- The Procurement Department processes 14 orders per hour:
  - There is a backlog of 74 unprocessed orders
  - A 75th order is put into the queue
  - How long must the 75th order wait to be processed?

$$5.4 \text{ Hours} = \frac{75 \text{ Orders}}{14 \text{ Orders per Hour}}$$

# Overall Equipment Effectiveness (OEE)

- OEE measures three main equipment production process components:
  - Availability of equipment:
    - Breakdown loss
    - Setup and adjustment loss
  - Performance efficiency:
    - Minor stoppage loss
    - Idling loss
    - Reduced speed loss
  - Rate of quality products:
    - Quality defect and rework loss

# Calculating Availability

$$\text{Availability} = \frac{\text{Actual operating time}}{\text{Planned operating time}} \times 100$$

- **Planned operating time:**
  - The daily or monthly time available for operation minus all forms of scheduled stops (e.g., breaks in the production schedule, lunch, rest breaks, stoppage for routine maintenance, team meetings, training, etc.)
- **Downtime:**
  - The total time taken for unscheduled stoppages (e.g., breakdowns, retooling, changeover, adjustments, changes, etc.)
- **Actual operating time:**
  - The amount of time the machine is not idle
  - Equals: Planned operating time – Downtime

# Calculating Performance Efficiency

$$\text{Performance efficiency} = \frac{\text{Actual amount processed}}{\text{Ideal amount processed}} \times 100$$

- Ideal amount processed:
  - The ideal amount that can be processed per time unit (minute, hour, etc.) as determined by equipment design specifications
  - Times the operating time (in minutes, hours, etc.)
- Processed amount:
  - The actual amount processed



# Calculating Rate of Quality Products

## Rate of quality products

$$= \frac{(\text{Actual amount processed} - \text{Amount defective})}{\text{Actual amount processed}} \times 100$$

- Amount processed:
  - The total amount of product, good **or** bad, produced in a given amount of time
- Amount defective:
  - The total processed amount that contains defects
- Notice that this is the **first-time yield** for that equipment

# OEE: Establishing the Baseline

**OEE = Availability**

× **Performance efficiency**

× **Rate of quality products**

# Why Do We Need to Measure OEE?

- Calculates actual effectiveness of equipment
- Accounts for loss factors
- Provides data for improvement
- Makes work easier
- Helps to understand actual plant capacity
- Helps to develop accurate capacity plans
- Helps to attain a world-class benchmark and beyond

# Calculating OEE

1. Enter data as required

1. AVAILABILITY	
Total Shift Time (in minutes)	480
<b>a) Scheduled Stops (in minutes)</b>	
Scheduled lunch break	20
Scheduled rest breaks	10
Routine/preventive maintenance	10
Other:	15
Other:	5
<b>TOTAL (Scheduled Stops)</b>	<b>60</b>
Planned Operating Time (in minutes)	420
<b>b) Unscheduled Stops (Downtime)</b>	
Equipment/die changeover	35
Breakdown	10
Readjustment	45
Stop and inspect	10
Other:	
Other:	
<b>Total (Unscheduled Stops/Downtime)</b>	<b>100</b>
<b>c) Actual Operating Time</b>	<b>320</b>
<b>% AVAILABILITY</b>	<b>76.2%</b>

## 2. PERFORMANCE EFFICIENCY

Actual Amount Processed	5
Equipment Capacity (Ideal Units per Minute)	0.025
Ideal Amount Processed	8
<b>PERFORMANCE EFFICIENCY</b>	<b>62.5%</b>

## 3. RATE OF QUALITY PRODUCTS (FTY)

Amount Defective	1
<b>RATE OF QUALITY PRODUCTS</b>	<b>80.0%</b>

## 4. OVERALL EQUIPMENT EFFECTIVENESS

<b>OVERALL EQUIPMENT EFFECTIVENESS</b>	<b>38.1%</b>
--	--------------

2. Spreadsheet returns
1. Percentage availability
  2. Performance efficiency
  3. Rate of quality products
  4. OEE

1a. Use the calculator to convert to minutes or units/min

The Calculator	
Enter Hours	To get Minutes
1	60
Enter seconds	To get Minutes
60	1
Enter Sec./Unit	To get Units/Min
60	1
Enter Minutes/Unit	To get Units/Min.
40	0.025
Enter Hours/Unit	To get Units/Min.
1	0.016666667

# Exercise: Calculating OEE

**Objective:** Practice calculating overall equipment effectiveness

**Instructions:**

- Work in pairs or trios
- Using the data provided below, calculate the OEE

**Time:** 10 minutes

# Answer: Calculating OEE

1. AVAILABILITY	
Total Shift Time (in minutes)	480
<b>a) Scheduled Stops (in minutes)</b>	
Scheduled lunch break	30
Scheduled rest breaks	20
Routine/preventive maintenance	30
Other:	
Other:	
<b>TOTAL (Scheduled Stops)</b>	80
Planned Operating Time (in minutes)	400
<b>b) Unscheduled Stops (Downtime)</b>	
Equipment/die changeover	65
Breakdown	50
Readjustment	40
Stop and inspect	
Other:	
Other:	
<b>Total (Unscheduled Stops/Downtime)</b>	155
<b>c) Actual Operating Time</b>	245
<b>% AVAILABILITY</b>	<b>61.3%</b>

2. PERFORMANCE EFFICIENCY	
Actual Amount Processed	25
Equipment Capacity (Ideal Units per Minute)	0.2
Ideal Amount Processed	49
<b>PERFORMANCE EFFICIENCY</b>	<b>51.0%</b>

3. RATE OF QUALITY PRODUCTS (FTY)	
Amount Defective	5
<b>RATE OF QUALITY PRODUCTS</b>	<b>80.0%</b>

4. OVERALL EQUIPMENT EFFECTIVENESS	
	<b>25.0%</b>

**Note: Good units plus bad units**

The Calculator	
Enter Hours	To get Minutes
1	60
Enter seconds	To get Minutes
60	1
Enter Sec./Unit	To get Units/Min.
60	1
Enter Minutes/Unit	To get Units/Min.
5	0.2
Enter Hours/Unit	To get Units/Min.
1	0.016666667

# Summary

- Measures of effectiveness:
  - FTY: First-time yield
  - Cpk: Capability index
  - Sigma performance level
- Measures of efficiency:
  - PCT: Process cycle time
  - OEE: Overall equipment effectiveness

## Goals Achieved in this Module

- Explain the value of measuring process performance
- Understand the various ways to measure process performance
- Calculation of Sigma level using DPMO method
- Automated Sigma level calculator

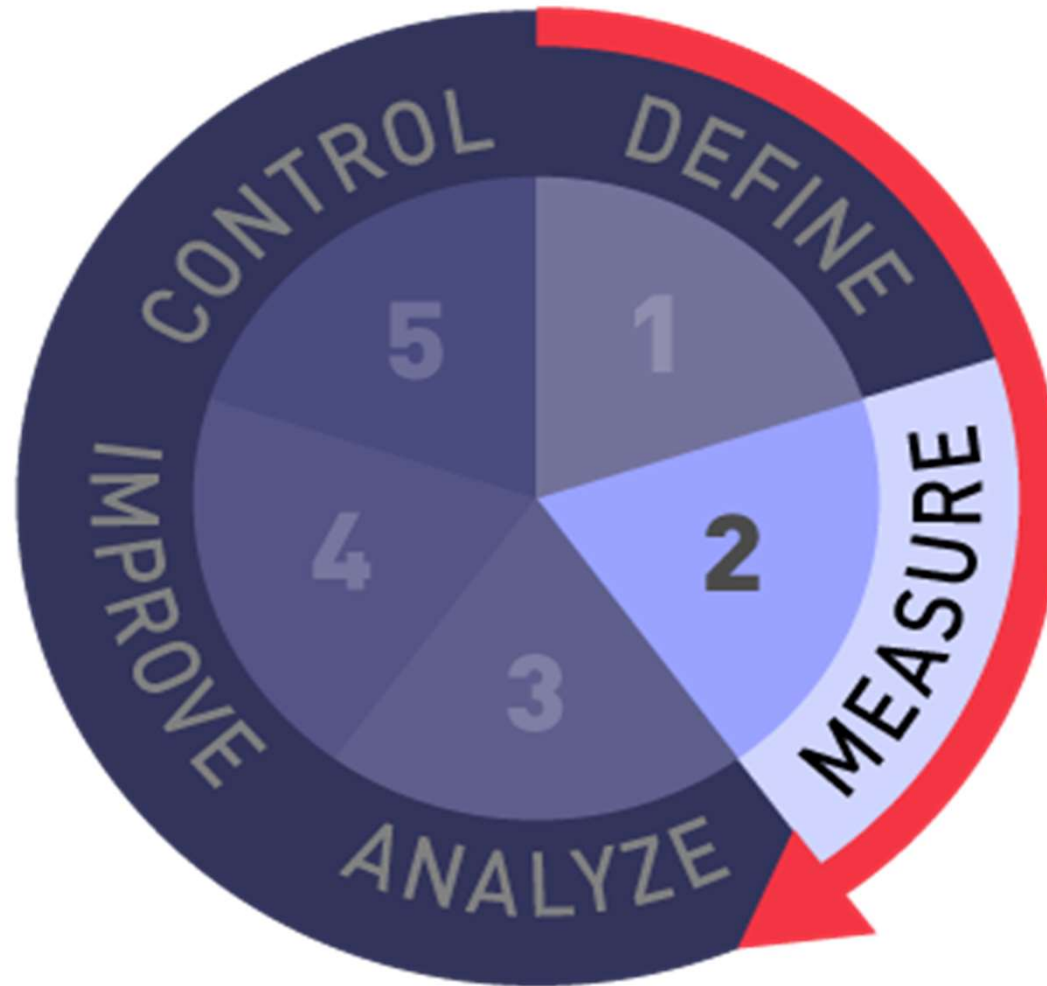


# Anexas Consultancy Services

## MEASURE: The Current Situation

Review

# You Were Here—MEASURE



# MEASURE Checklist

- By the end of the MEASURE phase, you should have answered the following questions and have data to demonstrate the following points—please check to be sure that you do:
  - What problems are occurring?
    - Under what conditions do these problems occur?
  - What specifically is the main problem or problems?
    - Focused problem statement
  - What time-related patterns did you see?
  - What special causes of variation were uncovered?
    - Have they been addressed?

## MEASURE Checklist, cont.

- By the end of the MEASURE phase, you should have answered the following questions and have data to demonstrate the following points—please check to be sure that you do, cont.:
  - What patterns appeared in the plots of the data?
  - What is the current process sigma for your process?
  - What did the process map of the process reveal?
  - Which bottlenecks appeared?
  - What are the current process parameters?
    - Cycle time, output, etc.

# Anexas Consultancy Services

**ANALYZE: Analyze  
to Identify Causes**

Overview

# You Are Here—ANALYZE



# ANALYZE: Analyze to Identify Causes

- Goal:
  - Identify root causes and confirm them with data
- Output:
  - A theory that has been tested and confirmed



# Approach to ANALYZE

**Develop a  
focused  
problem  
statement**

**Brainstorm  
potential  
causes**

**Organize  
potential  
causes**

**Collect data**

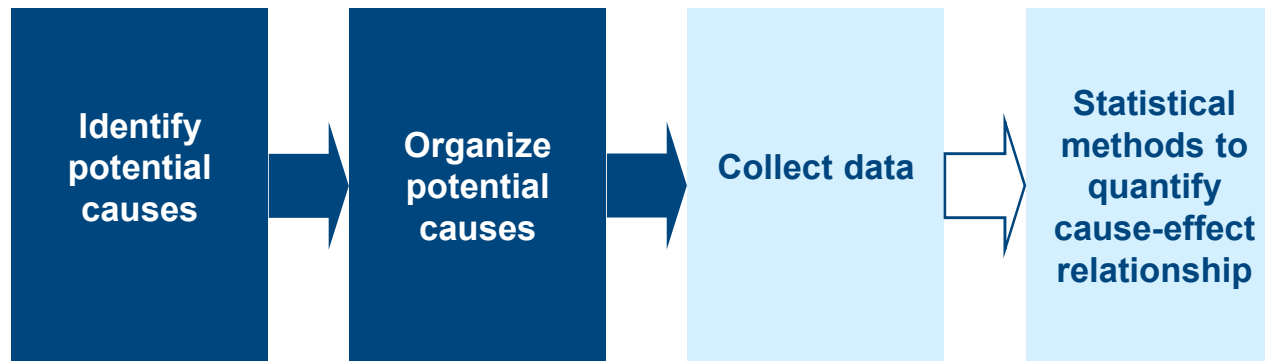
**Statistical  
methods to  
quantify  
cause-effect  
relationship**



# Anexas Consultancy Services

## Identifying Potential Causes

# You Are Here



# Goals

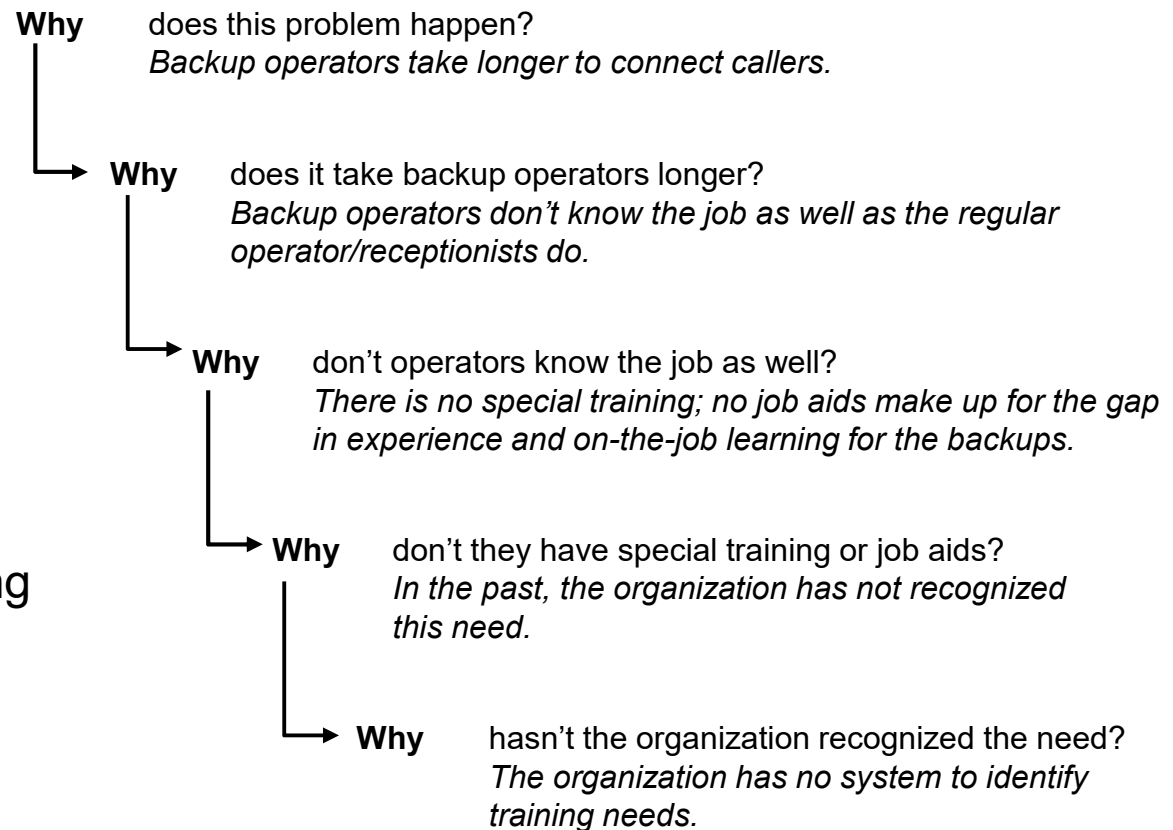
- Given a focused problem (the output of the MEASURE phase), construct a cause-and-effect diagram that demonstrates causal thinking
- Given a focused problem, create a tree diagram with at least four levels of depth for at least one path

# The Why-Why Technique

- To push for root causes, start with your focused problem and then ask “Why” several times

- Example:

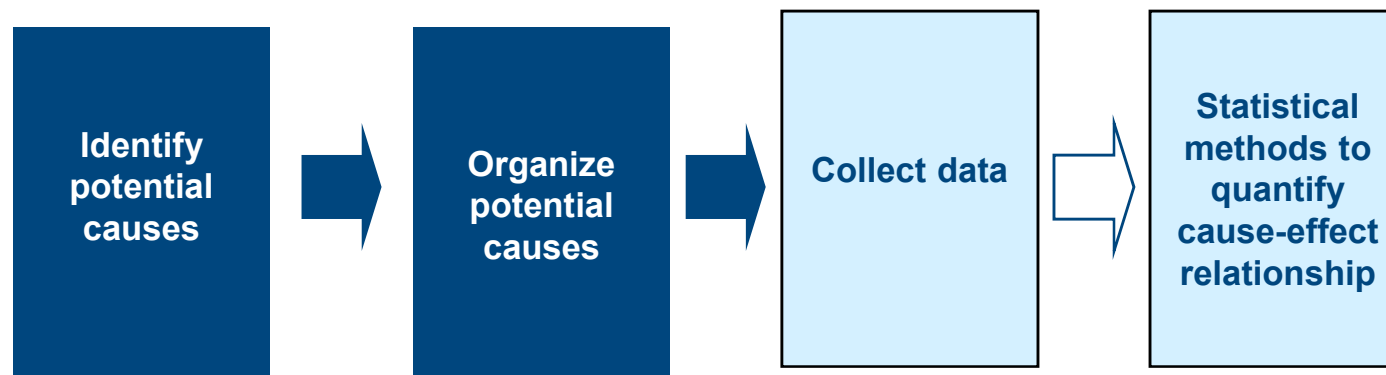
- Focused problem: Customers complain about waiting too long to get connected to staff during lunch hours



Verify each assumption on the spot—where the activity in question is taking place

# Anexas Consultancy Services

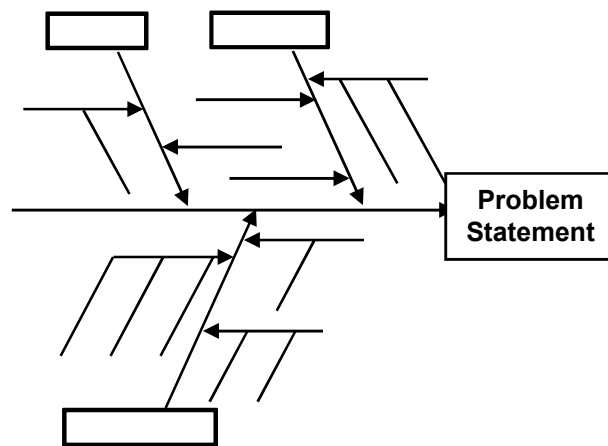
## Organizing and Prioritizing Potential Causes



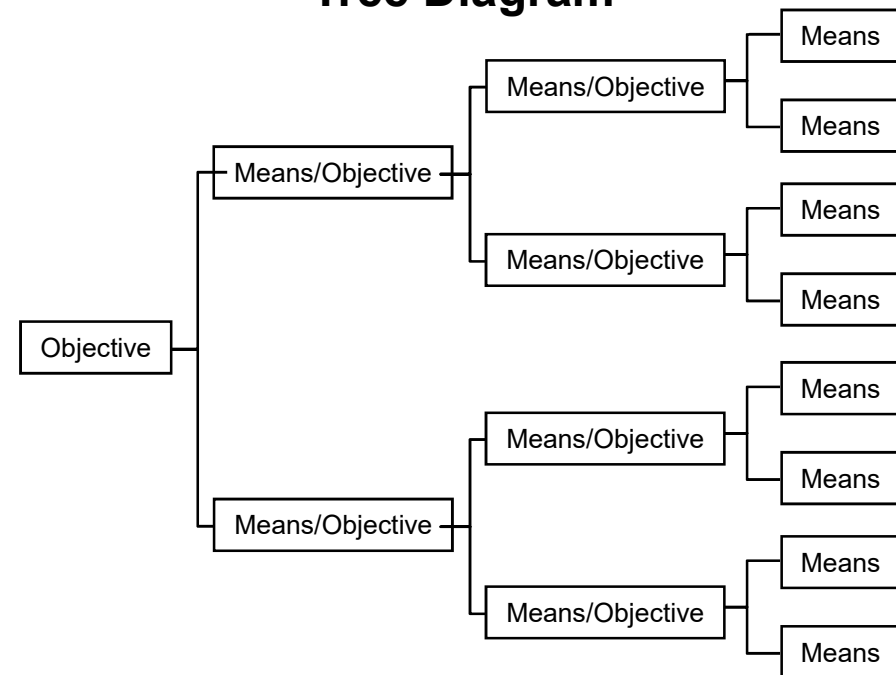
# Looking for Relationships

- Graphic displays can help you structure possible causes to find relationships that will shed new light on your problem

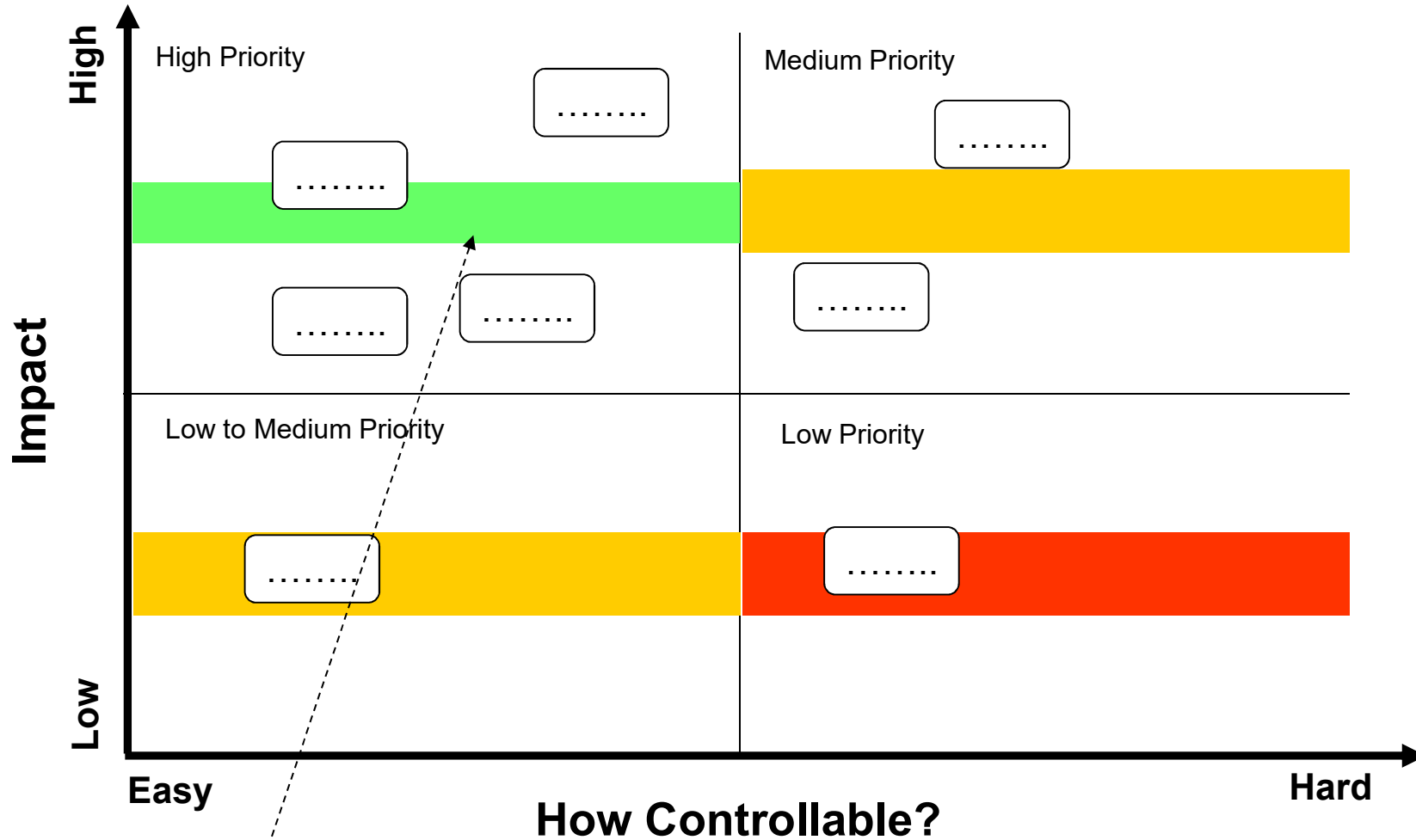
## Cause-and-Effect Diagram



## Tree Diagram



# Prioritizing Input Variables



Focus on low-effort high-impact items

# Identifying Potential Causes: Review

- Start with a narrow problem definition
- Brainstorm ideas
- Arrange ideas on a cause-and-effect or tree diagram
- Use the Effort × Impact diagram to narrow the scope of your investigation



# Anexas Consultancy Services

## Introduction to Hypothesis Testing

# Why Learn Hypothesis Testing?

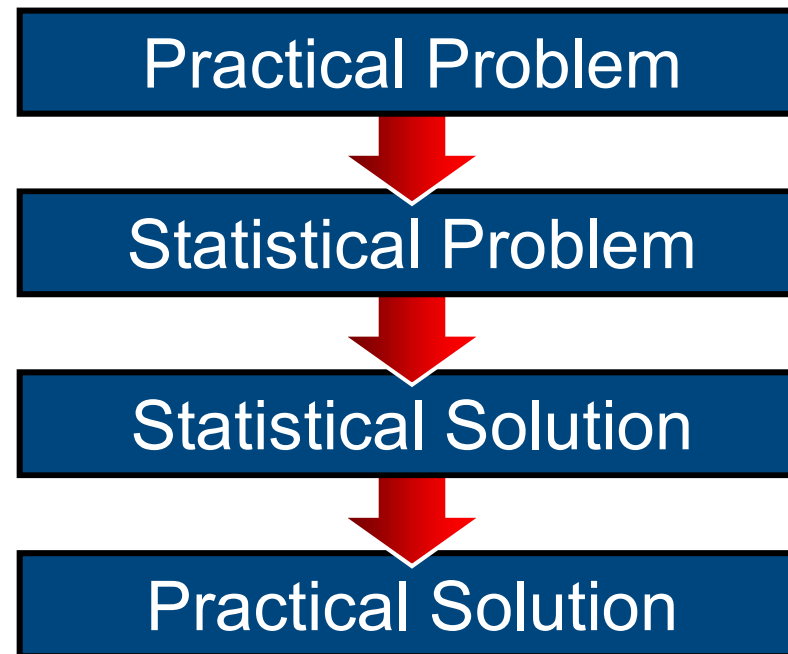
To identify sources of variability using historical or current data:

- Passive: a process is sampled or historic sample data is obtained
- Active: a modification is made to a process and then sample data is obtained

Provides objective solutions to questions which are traditionally answered subjectively

# What is Hypothesis Testing?

- A procedure for testing a claim about a population parameter
- Answers practical questions such as:
  - “Is there a real difference between \_\_\_\_\_ and \_\_\_\_\_ ?”



# What is Hypothesis Testing?

Example:

Practical Problem

Is there a real difference between production costs of Server 1 and Server 2?

Statistical Problem

$$H_o: \mu_T = \mu_A$$
$$H_a: \mu_T \neq \mu_A$$

Statistical Solution

T-Test of difference = 0 (vs. not =): T-Value = -0.88 p-Value = 0.390 DF = 17  
Fail to reject the null hypothesis

Practical Solution

There is no evidence of significant difference between production costs of Server 1 and Server 2

# What is Hypothesis Testing?

- In hypothesis testing, relatively small samples are used to answer questions about population parameters (inferential statistics)
- There is always a chance that the selected sample is not representative of the population; therefore, there is always a chance that the conclusion obtained is wrong
- With some assumptions, inferential statistics allows the estimation of the probability of getting an “odd” sample and quantifies the probability (p-value) of a wrong conclusion

# Parameters Versus Statistics

	Population Parameters	Sample Statistics
Mean	$\mu$	$\bar{x}$
Standard Deviation	$\sigma$	$s$
Proportion	$P$	$p$

- Population parameters (values) are fixed, but unknown
- Sample statistics are used to estimate or infer population values

Hypotheses are statements about population parameters, not sample statistics

# Hypothesis Tests

<b>Y</b>	<b>X</b>	<b>Hypothesis Test</b>
<b>Continuous / Variable Data</b>	<b>Attribute / Discrete Data</b>	<b>1-z, 1-t, 2-t, paired t, ANOVA</b>
<b>Attribute / Discrete Data</b>	<b>Attribute / Discrete Data</b>	<b>1-p, 2-p, Chi Square</b>
<b>Continuos / Variable Data</b>	<b>Continuos / Variable Data</b>	<b>Correlation, Regression, Multiple Regression</b>
<b>Attribute / Discrete Data</b>	<b>Continuous / Variable Data</b>	<b>Logistic Regression</b>

# Significance Level

Goal: show observed values are so unlikely to come from the same population, that  $H_0$  must be wrong

However, even if the values are unlikely there is still a chance that they may occur. The chance they may occur is  $\alpha$ .

This is called the significance level ( $\alpha$ )

There is an  $\alpha$  % chance that we are wrong when we say that Server 1 is more efficient than Server 2



# $\alpha$ (Alpha) - Simplified Perspective

Null Hypothesis ( $H_0$ ) assumed true

e.g., defendant assumed innocent

Prosecuting attorney must provide evidence beyond reasonable doubt that assumption is not true

Reasonable doubt =  $\alpha$

# Alpha ( $\alpha$ ) & Beta ( $\beta$ ) Risk

- $\alpha$ -risk
  - Risk of finding a difference when there really isn't one
  - Type I error or Producers' risk
- $\beta$ -risk
  - Risk of not finding a difference when there really is one
  - Type II error or Consumers' risk

# Truth Table: $\alpha$ and $\beta$ Risk

		Decision	
		Fail to reject $H_0$	Reject $H_0$
Truth	$H_0$ true	Correct Decision $CI = 1 - \alpha$	Type I Error ( $\alpha$ -Risk or <i>false positive</i> )
	$H_a$ true	Type II Error ( $\beta$ -Risk or <i>false negative</i> )	Correct Decision Power = $1 - \beta$

Producers' Risk

Consumers' Risk

# What is p - value?

- The probability of getting sample statistics like the one we observed if our null hypothesis is true
- The chance you will be wrong if you rejected null hypothesis
- Based on an assumed or reference distribution (Z, t, F, etc.)

# Decision Criteria

$p < \alpha$ , reject the null hypothesis

$p > \alpha$ , fail to reject the null hypothesis

# Anexas Consultancy Services

## Hypothesis Tests

# Hypothesis Tests

<b>Y</b>	<b>X</b>	<b>Hypothesis Test</b>
<b>Continuous / Variable Data</b>	<b>Attribute / Discrete Data</b>	<b>1-z, 1-t, 2-t, paired t, ANOVA</b>
<b>Attribute / Discrete Data</b>	<b>Attribute / Discrete Data</b>	<b>1-p, 2-p, Chi Square</b>
<b>Continuos / Variable Data</b>	<b>Continuos / Variable Data</b>	<b>Correlation, Regression, Multiple Regression</b>
<b>Attribute / Discrete Data</b>	<b>Continuous / Variable Data</b>	<b>Logistic Regression</b>

# Anexas Consultancy Services

## Hypothesis Testing of Mean



# Steps

## Steps in Hypothesis tests:

### 1. State the null hypothesis ( $H_0$ )

Null Hypothesis is:

All means are equal (1-z, 1-t, 2-t, paired t, ANOVA) [Cont- Att]

Y is independent of X (Regression) [Cont –Cont]

Y is not related to X (1 p, 2p, Chi Square) [Att-Att]

Y is not related to X (Binary Logistic Regression) [Att-Cont]

### 2. State the alternative hypothesis ( $H_a$ )

At least one mean is different(1-z, 1-t, 2-t, paired t, ANOVA)

Y is dependent on X (Regression)

Y is related to X (1 p, 2p, Chi Square)

Y is related to X (Binary Logistic Regression) [Att-Cont]

### 3. Choose alpha value ( $\alpha = .05$ ). Also known as level of significance. Confidence Level = $1-\alpha$

### 4. Collect data

# Steps

Steps in Hypothesis tests:

5. Choose appropriate hypothesis test

6. Get p value

7. If  $p$  is  $< 0.05$  , Reject  $H_0$

    If  $p$  is  $> 0.05$ , Accept  $H_0$

Remember :

If  $p$  is low  $H_0$  must go

If  $p$  is high,  $H_0$  must fly

# Why Learn Hypothesis Tests of Mean?

Make data driven decisions with defined confidence

Determine if a statistically significant difference of means exists between:

- A sample and a target
- Two independent samples
- Paired samples

# What are Hypothesis Tests of Mean?

## **Test Method for analyzing the differences between:**

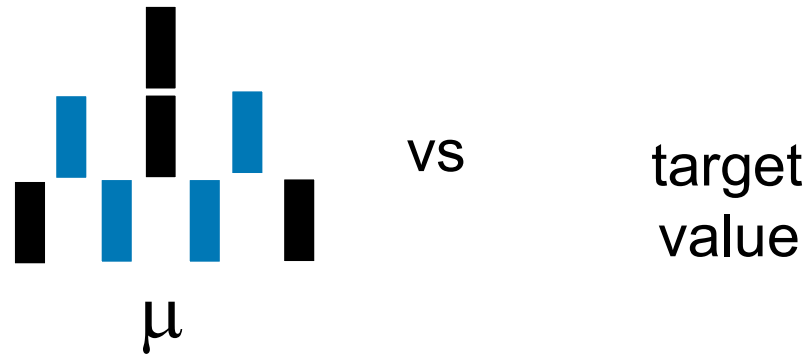
- 1 Sample Z a sample mean and a target value when population standard deviation is known
- 1 Sample t a sample mean and a target value when population standard deviation is not known
- 2 Sample t means obtained from two independent sample
- Paired t mean differences obtained from paired samples

Note: Above tests are used when the dependent variable (response) is continuous and the independent variable (factor) is discrete

# Anexas Consultancy Services

## 1 Sample z Test

# Single Mean Comparison



**$\sigma$  known**

Practical Question  
(example)

*“Is the population statistically different from the target value?”*



Statistical Question  
 $H_0: \mu = \text{target value}$   
 $H_a: \mu \neq \text{target value}$

# Business Process Example: Rising Transaction Costs

A financial institution is concerned about rising costs per teller transaction. Leadership of the institution wants to take appropriate action if the population average cost per teller transaction is greater than \$1.40.

A random sample of 45 costs per teller transaction produced an average value of \$1.45. It is known from previous experience that the population standard deviation of the transaction cost is approximately \$0.32.

Analyze the sample data from the file ***Tellercost.mtw*** and determine if we have evidence to show that the population mean cost per teller transaction cost is greater than \$1.40.

# Example: Rising Transaction Costs

## ■ **Practical Problem**

- ✓ Did the average cost per transaction increase?
- ✓ Is the average cost per transaction greater than \$1.40?

## ■ **Statistical Problem**

- ✓ Is there a shift in the mean cost per transaction from the historical average?
- ✓ Null hypothesis: Average cost is \$1.40
- ✓ Alternate hypothesis: Average cost is greater than \$1.40
- ✓ Is there evidence (at a significance level of 5%) to show that the average cost per transaction has increased? Otherwise we maintain the current belief - i.e., the null hypothesis



# Example: Rising Transaction Costs

- State the hypotheses and significance level

$$H_o: \mu = \$1.40$$

$$H_a: \mu > \$1.40$$

$$\alpha = 0.05$$

- What hypothesis test is appropriate?

These hypotheses deal with mean values

Only one factor for examination – rising transaction cost

Comparing population mean against a target value  
using one sample data

Data follows a normal distribution

$\sigma$  known, \$0.32

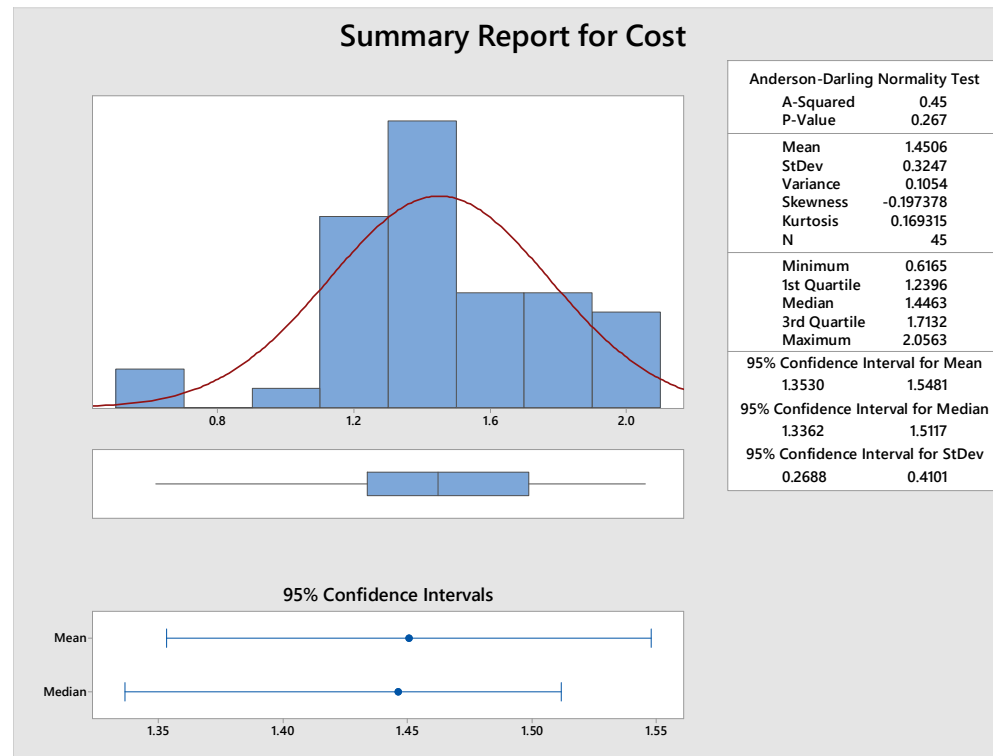
Use 1-Sample Z-Test

# Example: Rising Packing Costs

## Practical and Graphical

- Open the file ***Tellercost.mtw***
- What practical questions do you have about this data?
- Evaluate descriptive statistics

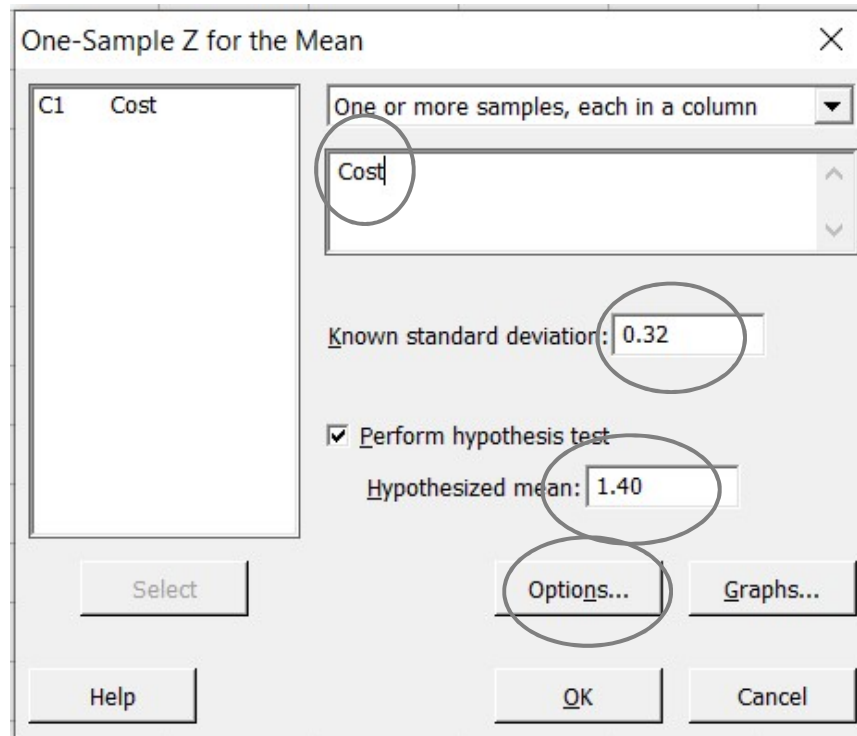
## Descriptive Statistics



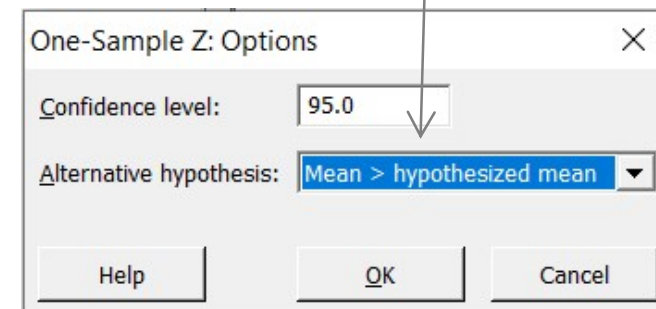
# Example: Rising Packing Costs

Tool Bar Menu > Stat > Basic Statistics > 1-Sample Z

## Analysis through Minitab



Choose *greater than* for Alternative  
 ( $H_a: \mu > \$1.40$ )



# Example: Rising Transaction Costs

## One-Sample Z: Cost

Test of  $\mu = 1.4$  vs  $\mu > 1.4$   
 The assumed sigma = 0.32

Variable	N	Mean	StDev	SE Mean
Cost	45	1.4506	0.3247	0.0477

Variable	95.0% Lower Bound	Z	P
Cost	1.3721	1.06	0.145

## Interpretation:

p-value = 0.145

Since p-value  $>$   $\alpha$ -value (0.05) fail to reject  $H_0$

Infer  $H_0$  true: not enough evidence that average teller transaction cost is greater than \$1.40

# 1-Sample Z test Exercise: Projector Lamps

A company that manufactures lamps for computer projectors, Sigma Lamps, Inc. (SLI), has received numerous complaints from customers concerning the quality of recently purchased lamps.

Specifically, the average life of new lamps is said to be less than previous purchases. Historical data shows that the measure of lamp life is approximately normally distributed with a mean of 1000 hours and standard deviation of 40 hours. Life information from a random sample of 25 lamps from a recent production batch is given in ***Lamps.mtw***.

Are the concerns of SLI's customers valid?

# 1-Sample Z-Test Exercise: Ohms

A customer believes that the resistors they purchased and installed in their instrument have higher average resistance values than required. They sent a representative sample of 10 resistors back for testing.

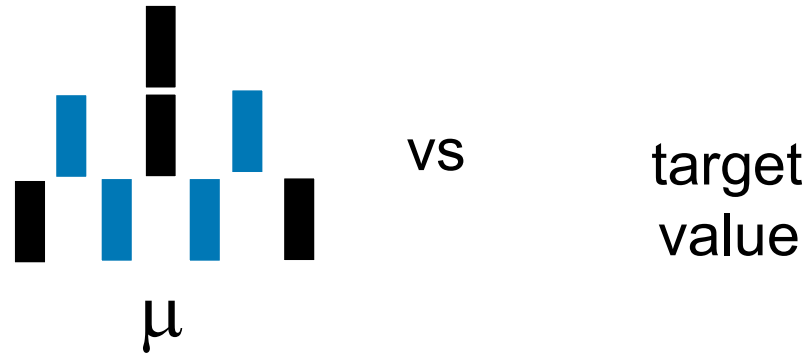
The resistor data (ohms) are included in file ***Ohms.mtw***

The resistors are supposed to be 60.5 ohms. Does the customer have a legitimate complaint? The population standard deviation ( $\sigma$ ) of resistors is 0.81 ohms.

# Anexas Consultancy Services

## 1 Sample t Test

# Single Mean Comparison



**$\sigma$  Not known**

Practical Question  
(example)

*“Is the population statistically different from the target value?”*



Statistical Question  
 $H_0: \mu = \text{target value}$   
 $H_a: \mu \neq \text{target value}$



# 1-Sample t-Test

- Hypothesis test about the unknown population mean using information from one sample
- Population standard deviation not known and distribution is normal

Note: Normality assumptions relaxed when the number of sample observations is large (generally true when sample size  $>30$ ).

# Business Process Example: Delinquent Mortgages

A bank holds 1432 delinquent mortgages on residential properties. The Black Belt investigating the loan process wants to know if the mean appraised value (current) of the properties is greater than \$175,000.

A preliminary random sample of 38 delinquent properties were appraised. The sample produced a mean appraised value of \$181,769.

Use the data in *mortgage.mtw* to determine if there is enough evidence that population mean appraised value is greater than \$175,000?

# Example: Delinquent Mortgages

- **Practical Problem**

- Is the mean appraised value of all 1432 properties higher than \$175,000 in spite of the sample mean value of \$181,769?

- **Statistical Problem**

- Is the population mean of appraised value greater than \$175,000?
- Null hypothesis: mean appraised value of properties = \$175,000
- Alternate hypothesis: population average value is greater than \$175,000
- Is there sufficient evidence to show that the population average of properties is greater than the \$175,000 at a significance level of 5%? Otherwise we will maintain the current belief - i.e., the null hypothesis

# Example: Delinquent Mortgages

- **State the hypotheses and significance level**

$$H_0: \mu = \$175,000$$

$$H_a: \mu > \$175,000$$

$$\alpha = 0.05$$

- **What hypothesis test is appropriate?**

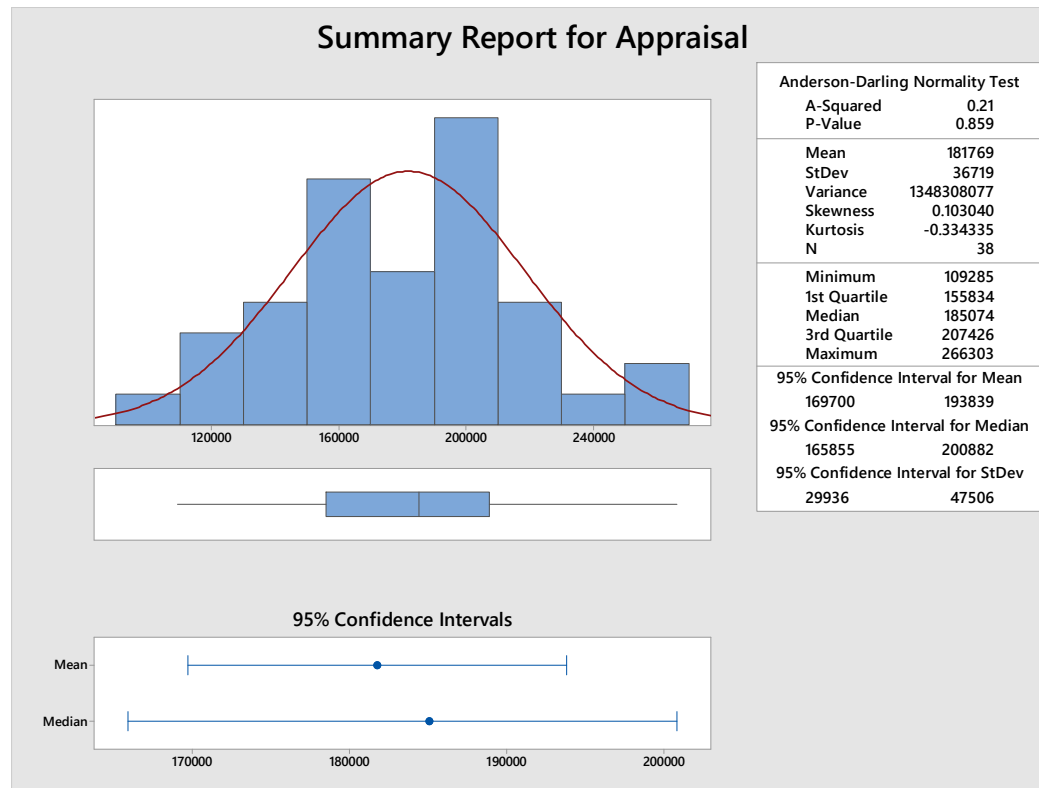
- These hypotheses deal with mean values
- Only one factor for examination - appraised value
- Comparing population mean against a target value using one sample data
- Data follows a normal distribution
- Population standard deviation NOT known
- Use 1-sample t-test

# Example: Delinquent Mortgages

## Practical and Graphical

- Open the file ***Mortgage.mtw***
- What practical questions do you have about this data?
- Evaluate descriptive statistics

## Descriptive Statistics

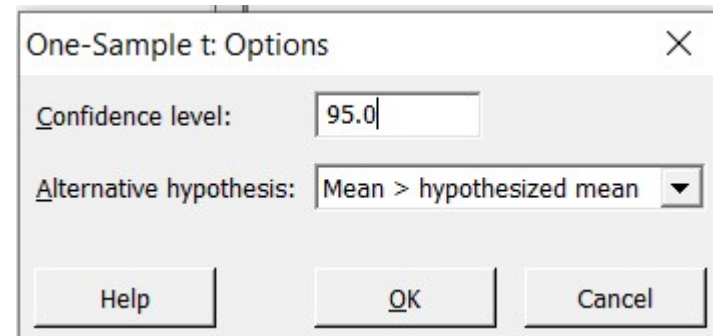
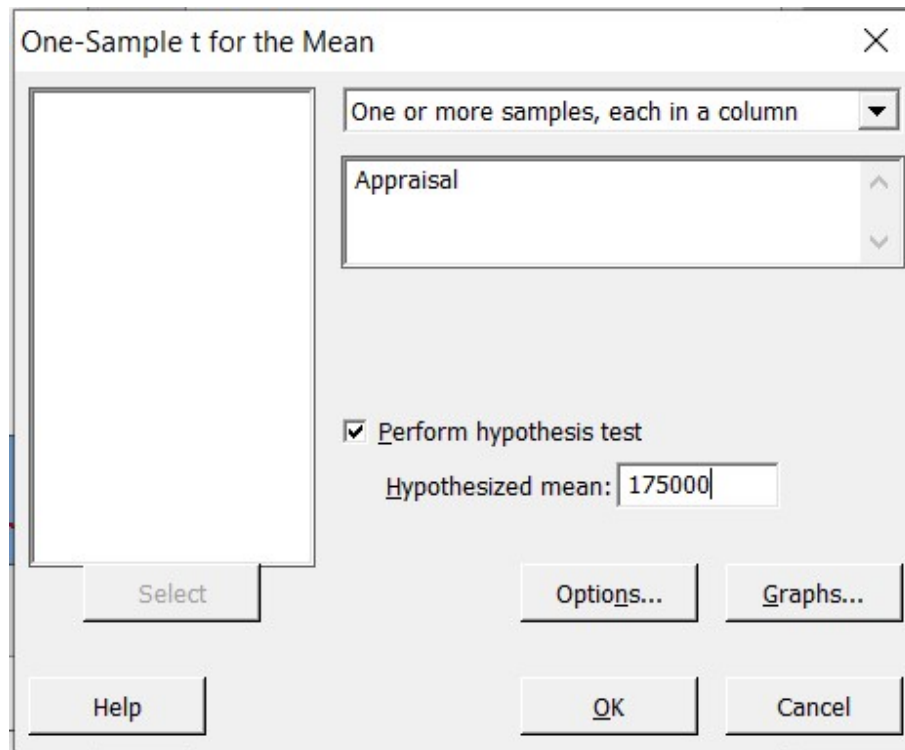


# Example: Delinquent Mortgages

Tool Bar Menu > Stat > Basic Statistics > 1-Sample t

## Analysis through Minitab

Choose *greater than* for  
Alternative ( $H_a: \mu > \$175,000$ )



# Example: Delinquent Mortgages

## One-Sample T: Appraisal

Test of  $\mu = 175000$  vs  $\mu > 175000$

Variable	N	Mean	StDev	SE Mean
Appraisal	38	181769	36719	5957

Variable	95.0% Lower Bound	T	P
Appraisal	171720	1.14	0.132

Interpretation:

- p-value = 0.132
- Since p-value  $>$   $\alpha$ -value (0.05) fail to reject  $H_0$
- Infer  $H_0$  true: not enough evidence that average appraised value of properties with delinquent mortgages is higher than \$175,000

# Industrial Process Exercise: Fuel Additive

An oil company has advertised that their new product, a fuel additive, can significantly enhance automobile fuel economy. They claim that on average automobiles can improve fuel economy by 2.3 miles per gallon. You are asked to investigate their claim.

As part of your investigation you have collected fuel economy information from 112 random samples of automobiles and found that on average the mileage enhancement for this sample was 2.03 miles per gallon (mpg) with a standard deviation of 1.125 mpg. You are concerned that the average value for your sample is less than the manufacturer's claim. The data is given in *mileage.mtw*.

Is the sample average of 2.03 mpg enough evidence to show that the manufacturer's claim is invalid?

Perform a statistical test at 5% significance level.



# Answer to exercise: Fuel Additive

One-Sample T: mpg change  
 Test of  $\mu = 2.3$  vs  $\mu < 2.3$

Variable	N	Mean	StDev	SE Mean
mpg change	112	2.031	1.125	0.106

Variable	95.0% Upper Bound	T	P
mpg change	2.207	- 2.53	0.006

## Interpretation:

- p-value is 0.006, suggesting that if the true average was still 2.3 mpg, only about 0.6% of the time we would expect to see a sample average of 2.31 mpg or less
- Since p-value  $<$   $\alpha$ -value (0.05), reject  $H_0$
- Infer  $H_a$  true: additive does not increase fuel economy by 2.3 mpg
- We have evidence against the manufacturer's claim

# Business Process

## 1-Sample t-Test Exercise

A rental car company is concerned that its annual repair costs per car on its fleet of full size cars are increasing. To get an understanding of where the costs are, a random sample of 28 full size cars were selected and its repair costs analyzed.

The sample data ***Carrepair.mtw*** contains repair costs per car for last year.

Test the hypothesis that the mean repair costs per car for last year is greater than \$1275.

# Industrial Process

## 1-Sample t-Test Exercise

A customer believes that the resistors they purchased and installed in their instrument have higher average resistance values than required. They sent a representative sample of 10 resistors back for testing.

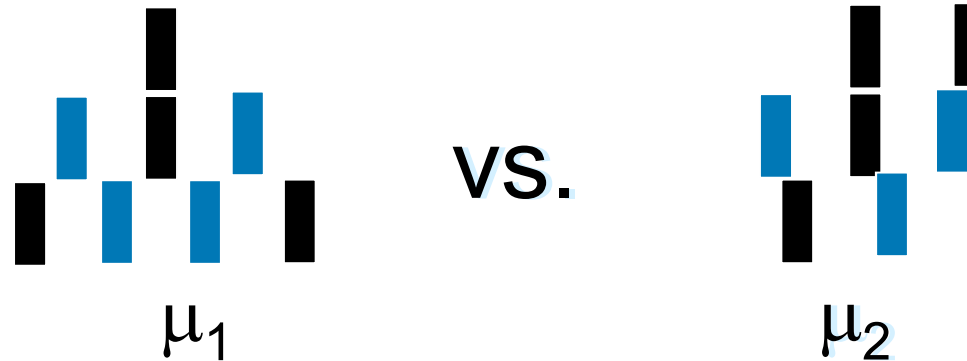
The resistor data (ohms) are included in file ***Ohms.mtw***.

The resistors are supposed to be 60.5 ohms. Does the customer have a legitimate complaint? The population standard deviation ( $\sigma$ ) of resistors is not known.

# Anexas Consultancy Services

## 2 Sample t Test

# Two Sample Comparison



Practical Question  
(example)

*“Are the two populations statistically different?”*

Statistical Question

$$H_o: \mu_1 = \mu_2$$

$$H_a: \mu_1 \neq \mu_2$$



## 2-Sample t-Test

- Hypothesis test about the difference between two population means using two samples
- Distributions are normal
- Two independent samples
  - Can be of different size

# Business Process Example: Teller vs. ATM Costs

**(A)** As part of an investigation to study the transaction costs of tellers versus ATMs, a bank has collected a random sample of 45 teller transaction costs and 53 ATM transaction costs.

The data is given in file *ATMTeller.mtw*.

Perform a hypothesis test to determine if average value teller transaction cost is higher than ATM transaction cost.

**(B)** Solve the same problem and perform a hypothesis test to determine if average value teller transaction cost is higher than ATM transaction costs by at least \$0.35.

## Example: Teller vs. ATM Costs (Part B Solution)

- Practical problem
  - Is average cost of teller transactions higher than average cost of ATM transactions by at least \$0.35?
- Statistical problem
  - Is the population mean for teller transaction cost higher than the population mean of ATM transaction costs by at least \$0.35?
  - Null hypothesis: difference between mean value of teller transaction costs and mean value of ATM transaction costs is equal to \$0.35
  - Alternate hypothesis: difference between mean value of teller transaction costs and mean value of ATM transaction costs is greater than \$0.35



## Example: Teller vs. ATM Costs

- State the hypotheses and significance level

$$H_o: \mu_{\text{Teller}} - \mu_{\text{ATM}} = \$0.35$$

$$H_a: \mu_{\text{Teller}} - \mu_{\text{ATM}} > \$0.35$$

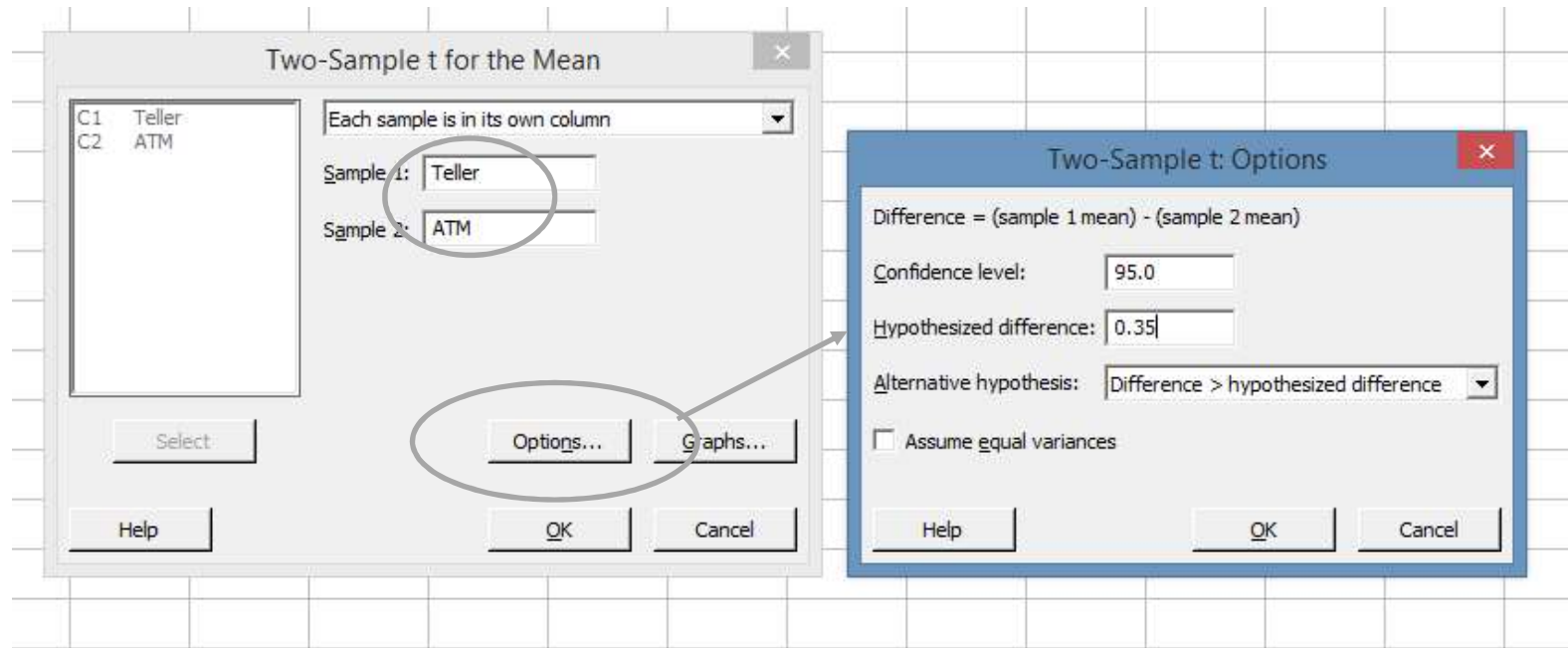
$$\alpha = 0.05$$

- What hypothesis test is appropriate?
  - These hypotheses deal with mean values
  - Only one factor for examination - transaction cost
  - Comparing population means based on two independent sets of sample data
  - Samples are normally distributed
  - Use 2-Sample t-Test

# Example: Teller vs. ATM Costs

Tool Bar Menu > Stat > Basic Statistics > 2-Sample t

## Analysis through Minitab



# Example: Teller vs. ATM Costs

Two-sample T for Teller vs ATM

	N	Mean	StDev	SE Mean
Teller	45	1.451	0.325	0.048
ATM	53	0.985	0.210	0.029

Difference = mu Teller - mu ATM

Estimate for difference: 0.4654

95% lower bound for difference: 0.3716

T-Test of difference = 0.35 (vs >): T-Value = 2.05 P-Value = 0.022 DF = 72

## Interpretation:

- p-value 0.022

- Since p-value <  $\alpha$ -risk (0.05), reject the null hypothesis

- The difference between Teller cost and ATM costs is greater than \$0.35

# Industrial Process Exercise

## Alloy Tensile Strength

Your Purchasing department is faced with a dilemma in choosing between two suppliers of alloy bars for your manufacturing operation.

Decisions must be made based on the average tensile strength and price of the supplied alloy bars. Your potential supplier B claims that the tensile strength of their bars is comparable to your current supplier A.

Moreover, bars from supplier B are cheaper than those from supplier A. If there is no difference between the tensile strength of alloy bars from the two companies you would consider switching to supplier B based on price.

# Industrial Process Example

## Alloy Tensile Strength

To help make decisions you have measured the tensile strength of 50 samples of alloy bars from each supplier. The results are given in the file ***Alloy.mtw***. Summary of sample statistics are as follows: Supplier A: average tensile strength = 42.07 ksi, standard deviation = 2.91 ksi; Supplier B: average tensile strength = 41.38 ksi, standard deviation = 2.41 ksi

Is there a difference in average tensile strength of alloy bars from Supplier A and B? Would you recommend switching to Supplier B? What other questions do you have about the decision making process?

Note: Other things being equal, higher tensile strength is preferred.

## Example: Alloy Tensile Strength

- Practical problem
  - Is there a difference in alloy tensile strength between suppliers?
  - If there is a difference, is it statistically significant?
- Statistical problem
  - Is the population mean for alloy tensile strength from Supplier A different from Supplier B?
  - Null hypothesis: average value of tensile strength for A and B are equal
  - Alternate hypothesis: average value of tensile strength for A and B are not equal
  - Is there evidence to show that the difference between the population average of tensile strength for A and B is not zero when tested at a significance level of 5%? Otherwise we will maintain the current belief - i.e., the null hypothesis

# Solution: Alloy Tensile Strength

Two-sample T for SupplyA vs SupplyB

	N	Mean	StDev	SE Mean
SupplyA	50	42.07	2.91	0.41
SupplyB	50	41.70	2.72	0.38

Difference = mu SupplyA - mu SupplyB

Estimate for difference: 0.373

95% CI for difference: (-0.745, 1.490)

T-Test of difference = 0 (vs not =): T-Value = 0.66 P-Value = 0.510 DF = 97

Interpretation:

p-value 0.510

If the difference between population averages was still zero, about 51% of the time expect to see a sample average difference as high as 0.373 ksi. In other words, if we reject the null hypothesis we would be wrong about 51%. This is more than the risk we are willing to take.

## Example: Alloy Tensile Strength

- Interpretation (continued):

Since p-value is higher than the  $\alpha$ -risk (0.05), fail to reject the null hypothesis in favor of the alternative

The difference between tensile strength of alloys from Supplier A and B is not statistically significant

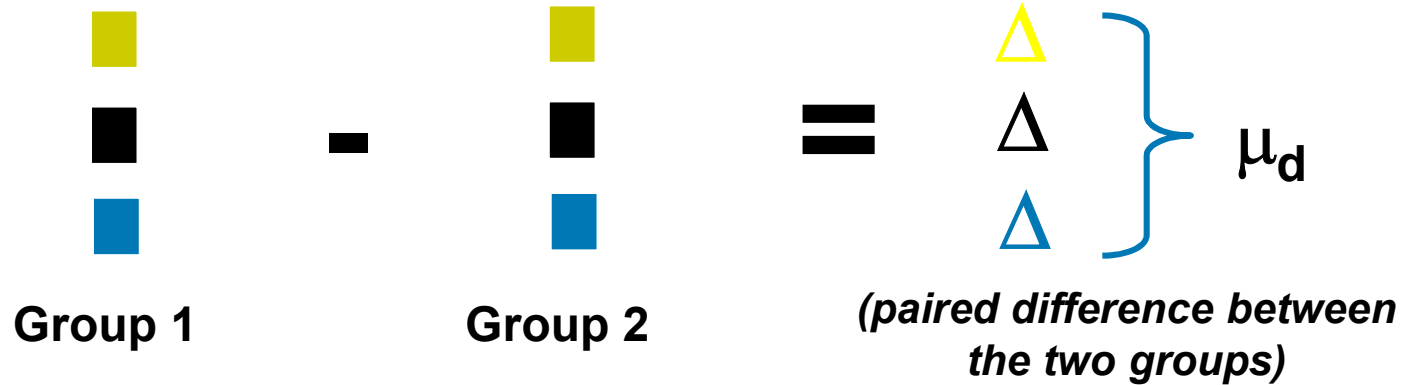
95% Confidence interval for difference between average alloy tensile strength from A and B is -0.745 and 1.490; since interval contains zero, insufficient evidence that the alloys are different



# Anexas Consultancy Services

## Paired t Test

# Paired Comparison



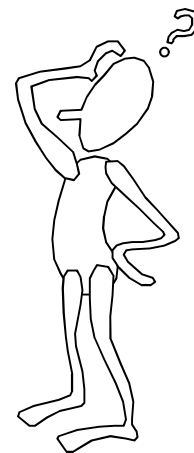
Practical Question  
(example)

*Is the difference between matched pairs greater than zero?"*

Statistical Question

$$H_0: \mu_d = 0$$

$$H_a: \mu_d > 0$$



# Paired t-Test

- Test for mean difference between pairs of samples
- Differences between matched pairs is normally distributed
- Each datum has a dependent mate
- Increased sensitivity over 2-Sample t-Test
- Effectively used to block out excess variability

# Paired t-Test - Procedure

- State the practical problem
- State the statistical problem
  - Generate the null and alternative hypotheses

For example,  $H_o: \mu_d = D_0$  or  $\mu_d \leq D_0$

vs

$H_a: \mu_d > D_0$

**$D_0$  represents hypothesized mean difference of pairs**

- Calculate the test statistic  $t_{\text{observed}}$
- $\bar{d}$  and  $s_d$  are sample mean and standard deviation of 'n' differences

$$t_{\text{obs}} = \frac{\bar{d} - D_0}{s_d / \sqrt{n}}$$

# Paired t-Test - Procedure

- Determine the critical value ( $t_{n-1, \alpha}$ ) of test statistic
- To test  $H_a: \mu_d > D_0$ , reject  $H_o$  if  $t_{\text{observed}} > t_{n-1, \alpha}$
- To test  $H_a: \mu_d < D_0$ , reject  $H_o$  if  $t_{\text{observed}} < - t_{n-1, \alpha}$
- To test  $H_a: \mu_d \neq D_0$ , reject  $H_o$  if  $t_{\text{observed}} < - t_{n-1, \alpha/2}$  or  $t_{\text{observed}} > t_{n-1, \alpha/2}$
- Reject the null hypothesis if p-value  $< \alpha$ -value
- Ensure that none of the assumptions for test are violated
- Translate statistical conclusion into practical solution

# Industrial Process Example: High Performance Tires

R&D department of Sigma Tires, Inc. has invented a new brand of tires (Brand X) that can potentially offer superior fuel economy. Based on tests performed under laboratory conditions the Engineering department believes that use of Brand X tires on the average improves the fuel economy by 0.3 mpg.

As part of the next phase of development, Sigma Tires has decided to do further field trials on a fleet of 42 taxis. The test to evaluate fuel economy obtained for Brand X tires as well as regular brand tires is as follows:

# Industrial Process Exercise: High Performance Tires

First, all 42 taxis were fitted with Brand X tires and driven over a prescribed course collecting fuel economy information. Then the same taxis were fitted with regular tires and driven once again over the same test course without changing drivers. The results obtained from the test are recorded in the file ***Tirebrandx.mtw***.

Sigma Tires would like to know if these tests are consistent with the Engineering department's previous finding that cars equipped with Brand X tires give better fuel economy than those equipped with regular brand tires?

# Solution: High Performance Tires

Paired T-Test and CI: BrandX, Regular  
Paired T for BrandX - Regular

	N	Mean	StDev	SE Mean
BrandX	42	14.420	2.621	0.404
Regular	42	14.072	2.758	0.426
Difference	42	0.3478	0.5327	0.0822

95% lower bound for mean difference: 0.2095

T-Test of mean difference = 0 (vs > 0): T-Value = 4.23 P-Value = 0.000

Interpretation:

- p-value 0.000
- p-value <  $\alpha$ -value (0.05), reject  $H_0$
- Infer  $H_a$ : Improvement in fuel economy due to Brand X compared to regular brand is statistically significant



## Example contd.: High Performance Tires

- Having seen an improvement, Engineering also wants to know if they can say that Brand X tires, on average, improve the fuel economy by 0.3 mpg?
- Practical problem
  - Is the improvement at least 0.3 mpg?
- Statistical problem
  - Is the difference in paired fuel economy greater than 0.3?
  - Null hypothesis: Average value of paired difference is equal to 0.3
  - Alternate hypothesis: average value of paired difference is greater than 0.3
  - Is there evidence to show that population average of paired difference between fuel economy for Brand X and Regular tires is greater than 0.3 when tested at a significance level of 5%? If not we will maintain the current belief - i.e., the null hypothesis

# Example: High Performance Tires

Question: Does Brand X improve the fuel economy by more than 0.3 mpg?

State the Hypotheses and Risk

$$H_0: \mu_d = 0.3$$

$$H_a: \mu_d > 0.3$$

$$\alpha = 0.05$$

Paired t - Options

**C**onfidence level: 95.0

**T**est mean: 0.3

**A**lternative: greater than

Help OK Cancel

# Example: High Performance Tires

Paired T-Test and CI: Brand X, Regular

Paired T for Brand X - Regular

	N	Mean	StDev	SE Mean
Brand X	42	14.420	2.621	0.404
Regular	42	14.072	2.758	0.426
Difference	42	0.3478	0.5327	0.0822

95% lower bound for mean difference: 0.2095

T-Test of mean difference = 0.3 (vs >0.3): T-Value = 0.58 P-Value = 0.282

Interpretation:

- p-value = 0.282
- p-value >  $\alpha$ -value (0.05), fail to reject  $H_0$
- Infer  $H_a$ : insufficient evidence to prove Brand X improves the fuel economy by more than 0.3 mpg

# Business Process Example: Comparing Analysts

A financial institution wants to compare the predictions of earnings per share (EPS) by two different financial analysts, George and Lisa. The two financial analysts' EPS forecasts for a random sample of 18 fortune 500 corporations are recorded in ***FaEPS.mtw***. Financial institution wants to know if there is a difference between the forecasts produced by George and Lisa.

Perform a hypothesis test at 5% significance level to determine if there is a statistically significant difference between the predictions by these analysts.

# Example: Comparing Analysts

- Practical problem
  - Is there a difference between the EPS forecasts of two analysts?
  - Is the average value of prediction by one analyst different from the other?
- Statistical problem
  - Is the difference in paired EPS predictions greater than zero?
  - Null hypothesis: mean value of paired difference is zero
  - Alternate hypothesis: mean value of paired difference is not equal to zero
  - Is there evidence to show that population average of paired difference between EPS forecasts by George and Lisa is not zero when tested at a significance level of 5%? If not, we will maintain the current belief - i.e., the null hypothesis

## Example: Comparing Analysts

- State the hypotheses and significance level

$$H_o: \mu_d = 0$$

$$H_a: \mu_d \neq 0$$

$$\alpha = 0.05$$

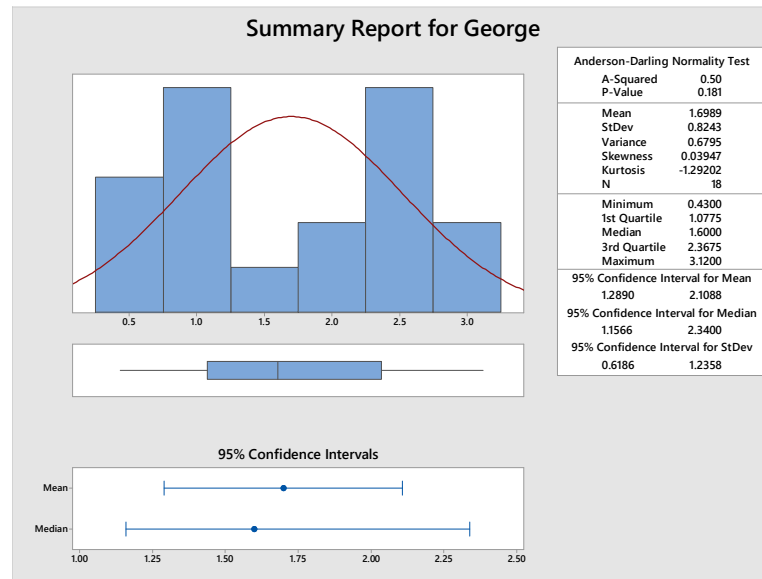
- What hypothesis test is appropriate?
  - These hypotheses deal with mean values
  - Only one factor for examination - EPS forecasts
  - Comparing mean differences of paired sample data
  - Difference between paired data follows a normal distribution
  - Use Paired t-Test

# Example: Comparing Analysts

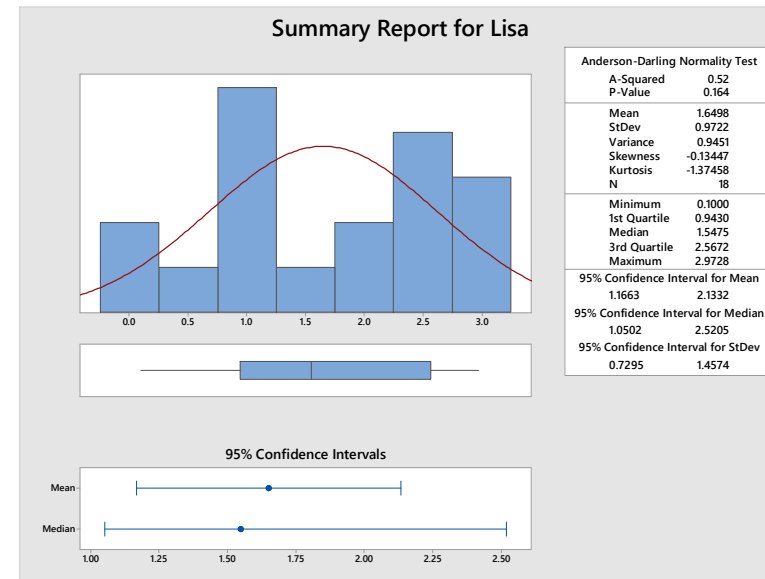
## Practical and Graphical

- Open *FAEps.mtw* in Minitab
- What practical questions do you have about this data?
- What assumptions are used in a Paired t-Test?

### Descriptive Statistics



### Descriptive Statistics

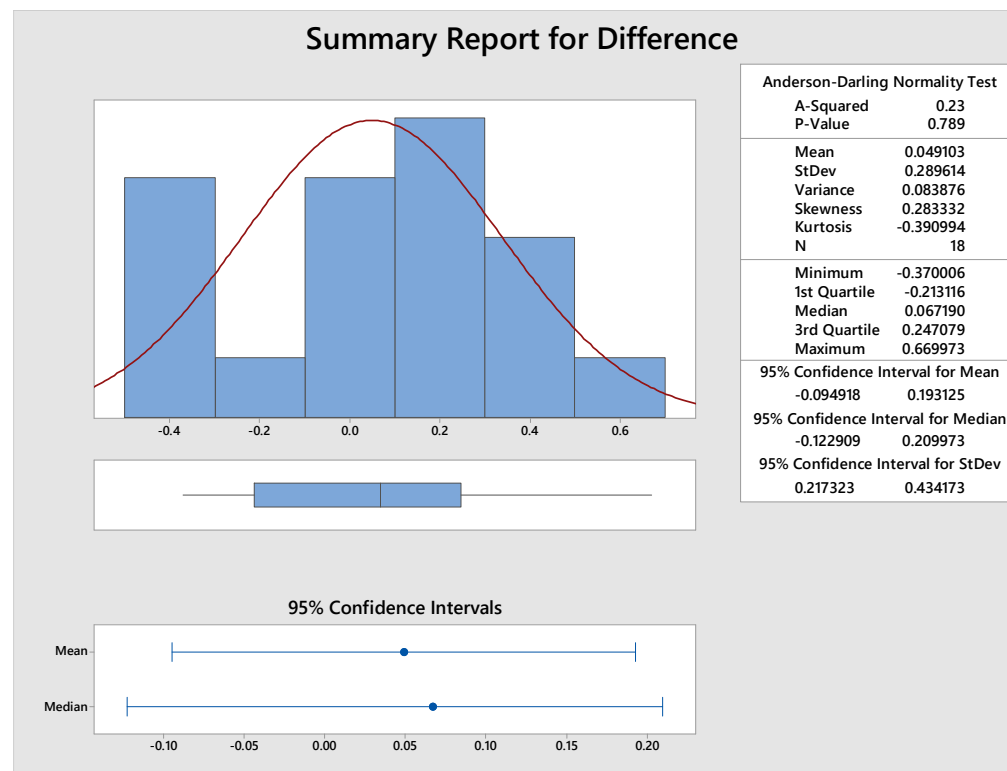


# Example: Comparing Analysts

## Practical and Graphical

- Review descriptive statistics of the difference between paired data

### Descriptive Statistics

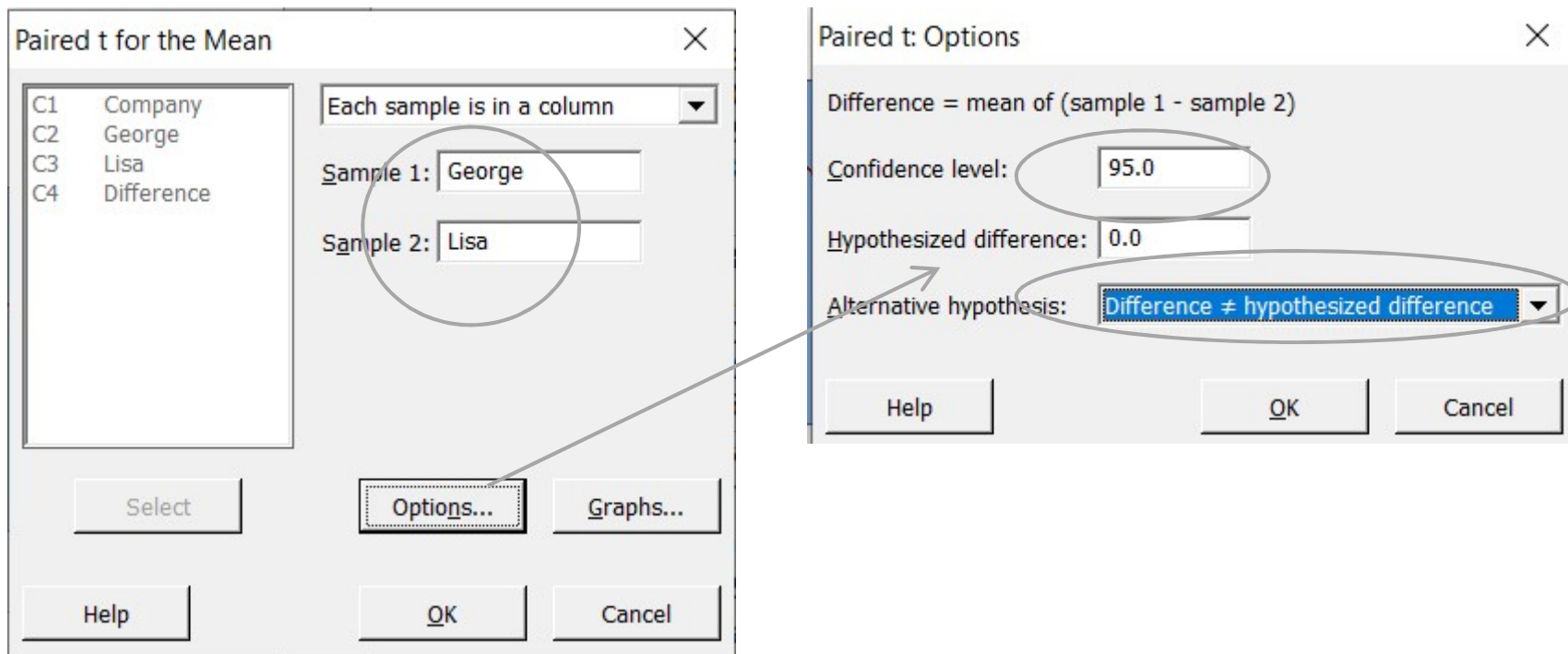




# Example: Comparing Analysts

Tool Bar Menu > Stat > Basic Statistics > Paired t

## Analysis through Minitab



# Example: Comparing Analysts

## Paired T-Test and CI: George, Lisa

Paired T for George - Lisa

	N	Mean	StDev	SE Mean
George	18	1.699	0.824	0.194
Lisa	18	1.650	0.972	0.229
Difference	18	0.0491	0.2896	0.0683

95% CI for mean difference: (-0.0949, 0.1931)

T-Test of mean difference = 0 (vs not = 0): T-Value = 0.72 P-Value = 0.482

Interpretation:

- p-value 0.482
- p-value >  $\alpha$ -value (0.05), fail to reject  $H_0$
- Infer  $H_0$ : Difference in EPS forecasts by Lisa and George are not statistically significant

## Paired t-Test Exercise

A customer believes that the resistors they purchased and installed in their instrument have higher average resistance values than ordered. They tested a representative sample of 10 resistors and data recorded in column1 (Customer) of file **CSresist.mtw**.

The sample of 10 resistors were sent back to a supplier testing facility. These resistors were tested at the supplier facility and recorded in column2 (Supplier of the file **CSresist.mtw**).

Is the customer's test method comparable to the testing facility?  
Assume the resistors were tested in the same order, such that 61.3 and 62.1 are the resistance for resistor1, etc.

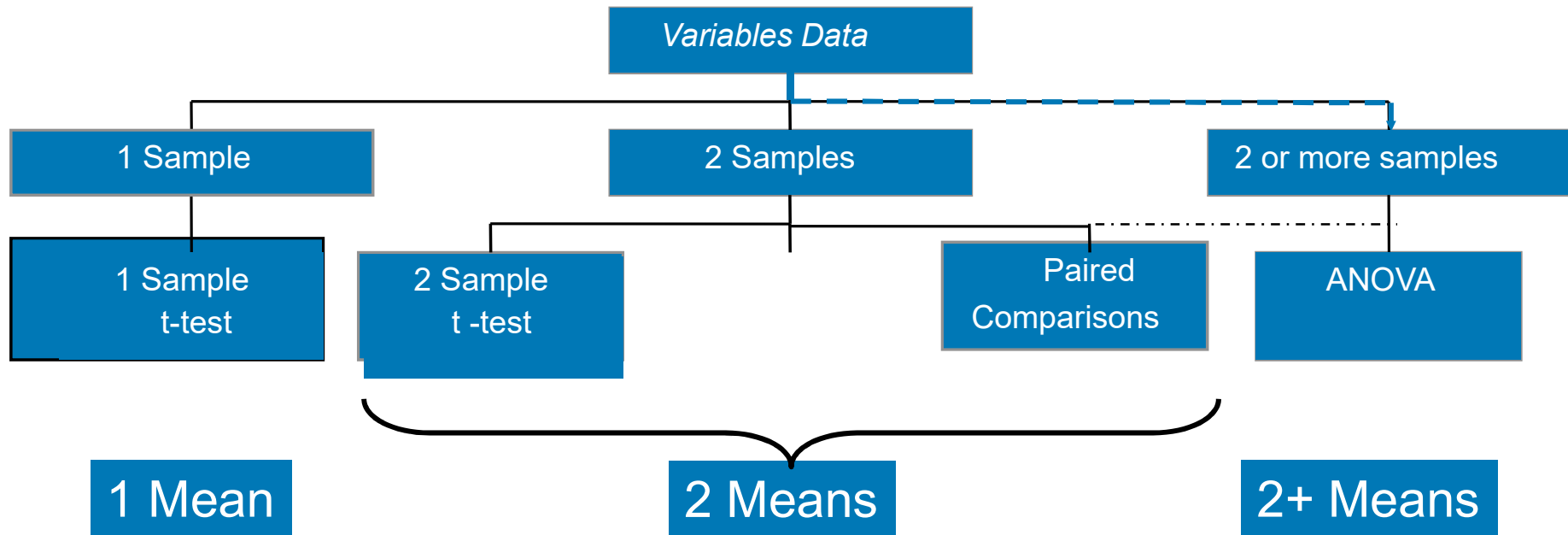
# Anexas Consultancy Services

Comparing Two or More  
Group Averages

ANOVA

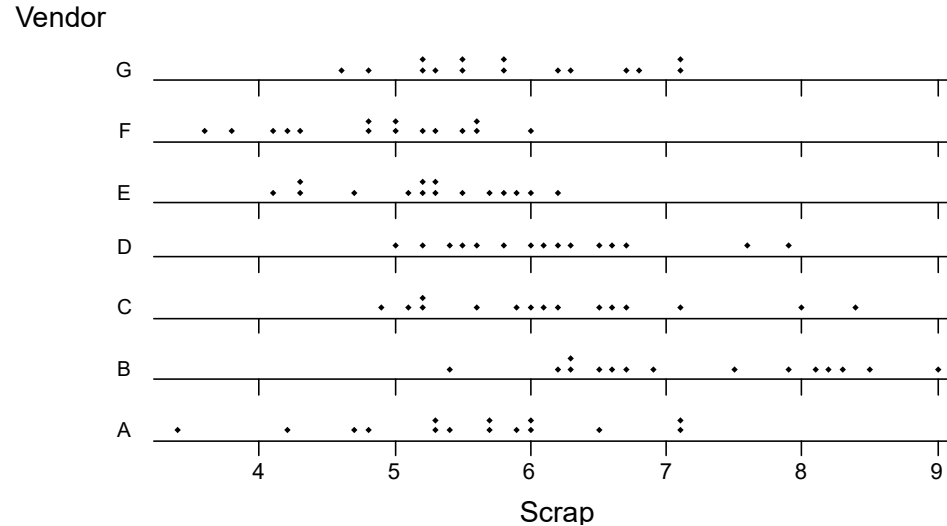
# When to Use ANOVA

## *Variables Road Map*



ANOVA is used to test two or more means

# Discussion: How Would You Compare Seven Groups?



An improvement team wants to compare seven packaging material vendors to see whether there are any significant differences in the average amount of scrap produced among the different materials:

- How would you make seven comparisons?
- Can you find the difference of seven items at the same time?

# Answers: How Would You Compare Seven Groups?

- Although a dot plot helps us compare seven groups visually, mathematically you can take the difference only between two numbers at a time, not among seven
- If you make pairwise comparisons, two at a time, you would end up doing 21 *t-tests*:
  - This is an inefficient use of time as well as data (to estimate the variance)
  - You also increase the chance of making a Type I error (on at least one of the comparisons)\*
- We must find a better way:
  - To compare the groups simultaneously
  - To use all the data efficiently

# Analyze Variances Instead of Averages

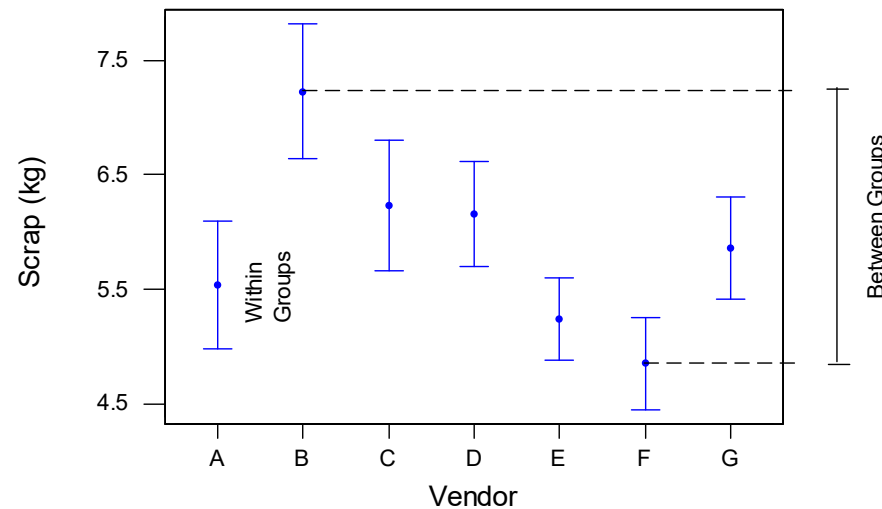
- There is a statistical test that uses variances to compare multiple group averages simultaneously:
  - ANOVA: Analysis of Variance
- Instead of comparing pairwise averages, it compares the variance **between** groups with the variance **within** groups:
  - The between group variance is obtained from the variance  $s^2$  of the **group averages**
  - The within group variance is obtained from the variance  $s^2$  among values within each group and then **pooled** (or averaged with appropriate degrees of freedom) **across the groups**
- If the variance between groups is the same as the variance within groups, we say there is no difference among the group averages

$$\frac{S_{\text{between}}^2}{S_{\text{within}}^2}$$



# Analyze Variances Instead of Averages, cont.

- Here is a plot to help you visualize “between” and “within” variance for seven groups (think of a Normal distribution centered over each line shown)



- In this diagram, the within group variation appears smaller than the between group variation, so the test will probably show that at least one of the groups is different from the rest

# Analysis of Variance (ANOVA)

- Obtain the variance between groups:  $S_B^2$
- Obtain the variance within groups:  $S_W^2$
- If they are about the same, conclude there is no significant difference between groups:
  - Take a ratio: if  $\frac{S_B^2}{S_W^2} = 1$  then say there is no difference between group averages
- The ratio of two variances = F-statistic
- We can get a P-value from the F distribution
- Minitab menu:
  - Stat > ANOVA > One-way

# ANOVA Hypothesis

- The null hypothesis:
    - $H_0$ : Average A = Average B = Average C = . . .
- (or)
- No difference among the group averages

(or)

$$S_{\text{Between}}^2 = S_{\text{Within}}^2$$

- The alternative hypothesis:
  - $H_a$ : At least one group average is significantly different from the others

## ANOVA Hypothesis, cont.

- If  $P \geq 0.05$ :
  - Do not reject the  $H_0$
  - There is not enough evidence to say there is a statistically significant difference between any two of the groups
  - If there is a true difference, either the variation is too large or the sample size is too small to detect it
- If  $P < 0.05$ :
  - Reject the  $H_0$ , and conclude the  $H_a$
  - At least one of the groups is statistically different from the others
  - Examine confidence intervals to find which of the groups are different (non overlapping)
  - Actions: evaluate importance or size of the difference (is it enough to warrant further action?)

# ANOVA

- Analysis of variance (ANOVA):
  - Used to **compare averages** of two or more groups:
    - Assumes variances of each group are the same
  - Also used to **compare variances** of two or more groups:
    - Called the **test for equal variances**
    - Use this test to check the assumption that variances are the same when comparing averages

# Prerequisites for ANOVA

- Every subgroup has a normal distribution
- Subgroups have statistically equal variances
- Residuals are independent and normally distributed about the mean

# Anexas Consultancy Services

## Hypothesis Testing Exercise

ANOVA

Normality Check

# ANOVA – Exercise

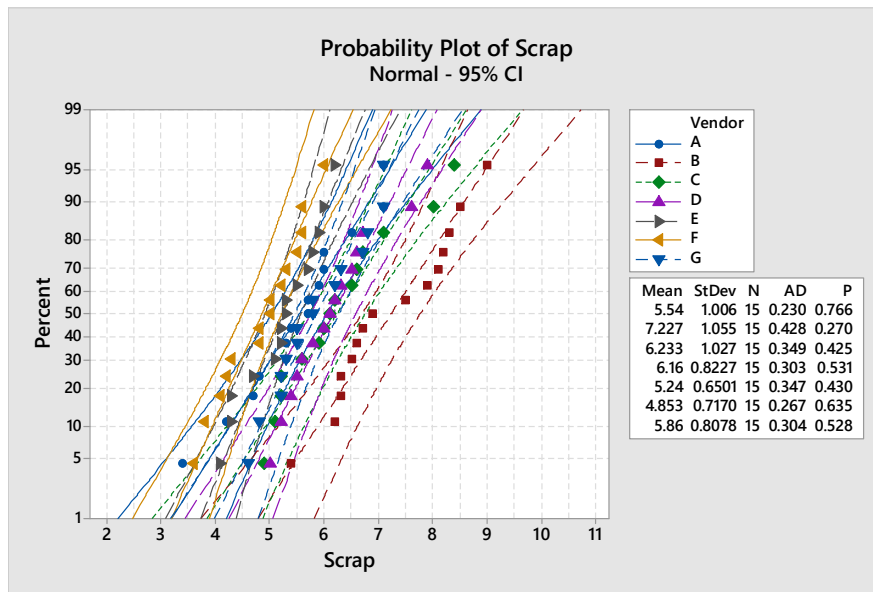
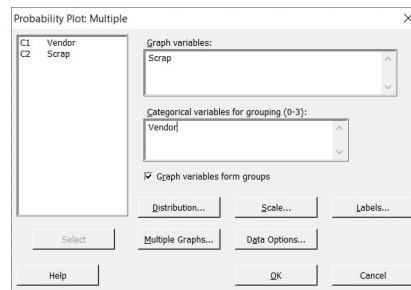
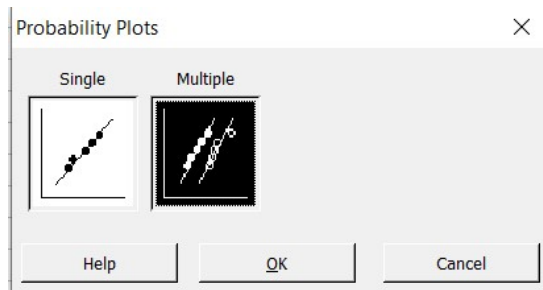
- Data: **ScrapAnova.MTW**
- Background: The data set contains the amount of scrap (kg) from 15 pieces of material from each of the seven vendors (identified as AG) for a total of 105 production runs on Process X
- An improvement team wants to compare seven packaging material vendors to see whether there are any significant differences in the average amount of scrap produced among the different materials:
  - How would you make seven comparisons?
  - Can you find the difference of seven items at the same time?



# Prerequisites for ANOVA

## Normality Check

Graph > Probability plot > (By Vendor)



P-values of all Scrap vendors is more than 0.05,

Data is considered as Normal.

# Anexas Consultancy Services

## Hypothesis Testing Exercise

ANOVA

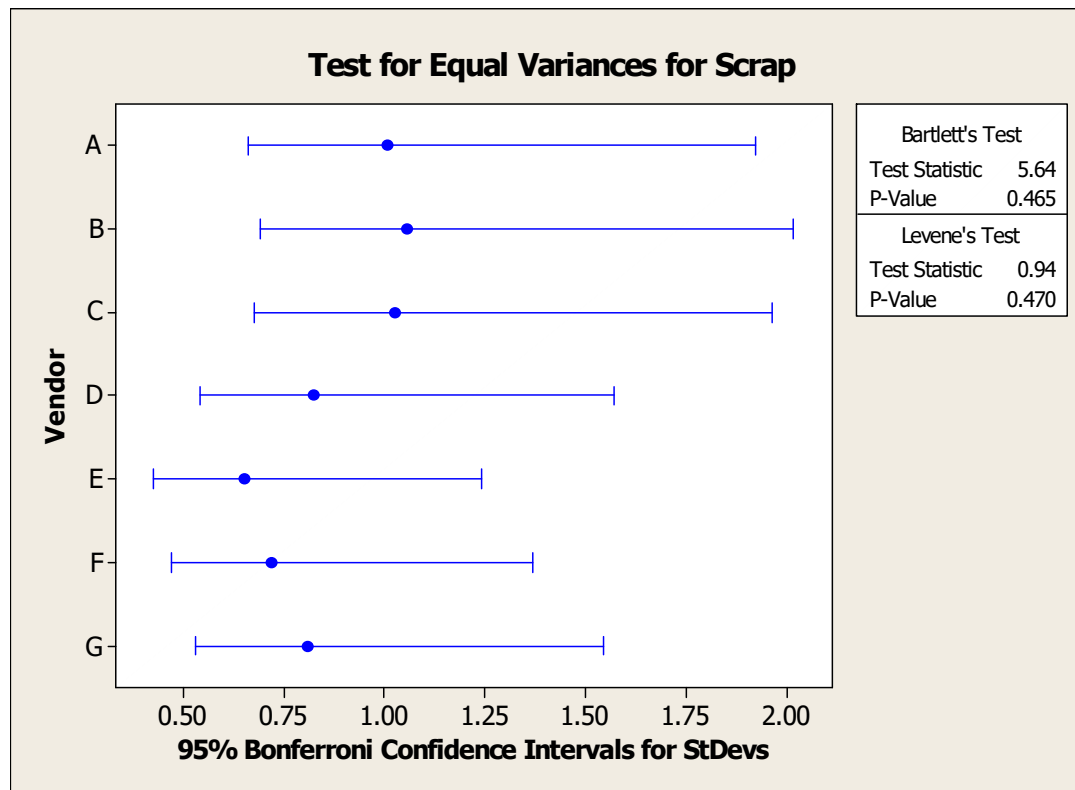
Test of Equal Variances

# Exercise: Test for Equal Variances

- **Objective:** Practice using Minitab to check whether the variances of several groups are the same
- **Time:** 5 minutes
- **Data:** *ScrapAnova.MTW*
- **Background:**
  - Recall the ANOVA test we did to determine whether material from seven vendors was associated with significantly different average scrap amounts
  - We made an assumption that the variances in the scrap were the same for each vendor when we did the ANOVA
- **Instructions:** Working alone:
  1. Check the assumption of same variances
  2. Be prepared to report your conclusions

# Answers: Test for Equal Variances

## Test for equal variances:



**Conclusion:** No significant difference in variances

# Anexas Consultancy Services

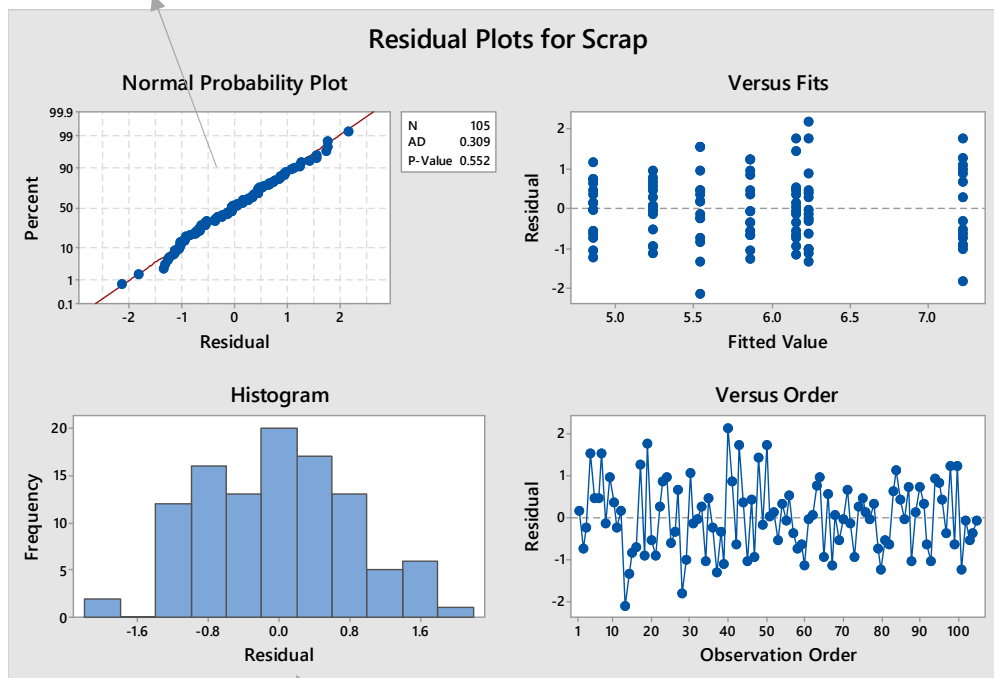
## Hypothesis Testing Exercise

ANOVA

Residuals for Violations

# Answers: Residuals for Violations

There is a straight line through nearly all the points, indicating the residuals are Normal. Examine the extreme points closely. Decide whether to leave them in or take them out.



Residuals vs. fits should not have patterns or extreme outliers. This plot should show a random pattern of residuals on both sides of 0. If a point lies far from the majority of points, it may be an outlier. There should not be any recognizable patterns in the residual plot. For instance, if the spread of residual values tend to increase as the fitted values increase, then this may violate the constant variance assumption.

2. Here again, with the exception of outliers, the residuals look Normal (good).

4. Residuals vs order should be scattered with no pattern. There seems to be a slight pattern (wave or cycle). What variable related to time may be causing this? Try to discover it and add it to the regression analysis. We'll demonstrate how to do this later.

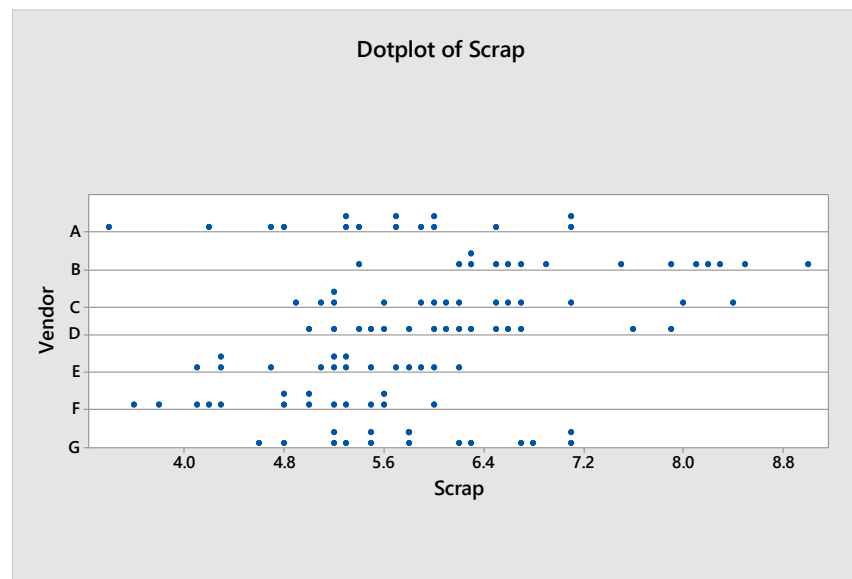
# Anexas Consultancy Services

## Hypothesis Testing Exercise

ANOVA

# Minitab Follow-Along: ANOVA

1. Make a stratified dot plot of the face amount by vendor:  
Graph > Dotplot > (By Vendor)





# Minitab Follow-Along: ANOVA, cont.

## 2. Get a summary of the data:

Stat > Basic Statistics > Display Descriptive Statistics >  
(Scrap by Vendor)

**Descriptive Statistics: Scrap by Vendor**

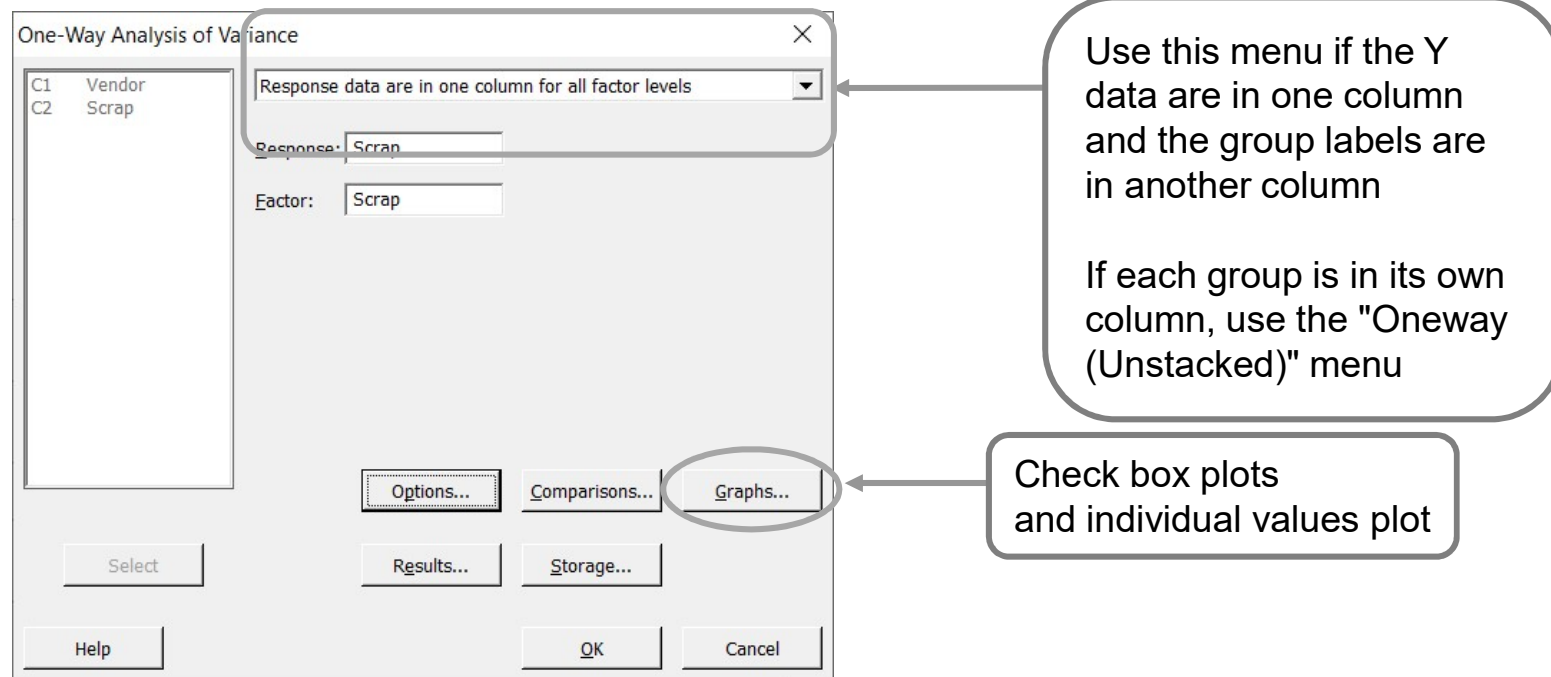
Variable	Vendor	N	Mean	Median	TrMean	StDev
Scrap	A	15	5.540	5.700	5.585	1.006
	B	15	7.227	6.900	7.231	1.055
	C	15	6.233	6.100	6.169	1.027
	D	15	6.160	6.100	6.115	0.823
	E	15	5.240	5.300	5.254	0.650
	F	15	4.853	5.000	4.862	0.717
	G	15	5.860	5.800	5.862	0.808

Variable	Vendor	SE Mean	Minimum	Maximum	Q1	Q3
Scrap	A	0.260	3.400	7.100	4.800	6.000
	B	0.272	5.400	9.000	6.300	8.200
	C	0.265	4.900	8.400	5.200	6.700
	D	0.212	5.000	7.900	5.500	6.600
	E	0.168	4.100	6.200	4.700	5.800
	F	0.185	3.600	6.000	4.200	5.500
	G	0.209	4.600	7.100	5.200	6.700

# Minitab Follow-Along: ANOVA, cont.

3. Do an analysis of variance (ANOVA) or F-test:  
Stat > ANOVA > Oneway



The screenshot shows the 'One-Way Analysis of Variance' dialog box in Minitab. The 'Response data are in one column for all factor levels' dropdown menu is highlighted with a callout box. The 'Response' field contains 'Scrap' and the 'Factor' field also contains 'Scrap'. The 'Graphs...' button is circled with another callout box. The dialog box includes a list of variables on the left (C1 Vendor, C2 Scrap), buttons for 'Options...', 'Comparisons...', 'Results...', 'Storage...', 'Select', 'Help', 'OK', and 'Cancel'.

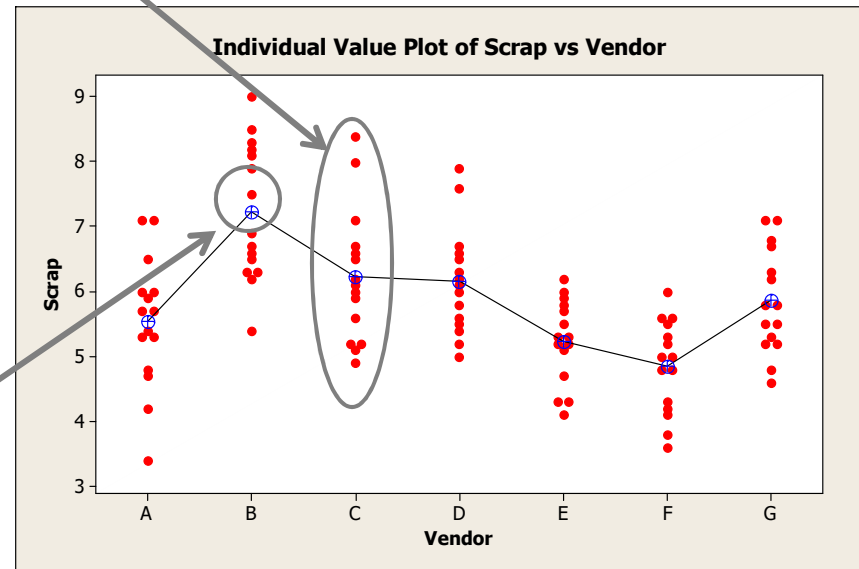
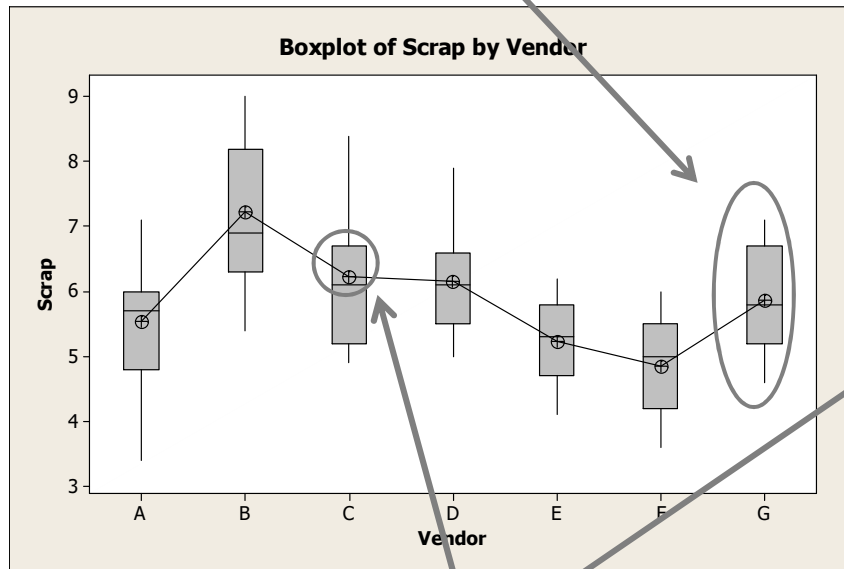
Use this menu if the Y data are in one column and the group labels are in another column

If each group is in its own column, use the "Oneway (Unstacked)" menu

Check box plots and individual values plot

# Minitab Follow-Along: ANOVA, cont.

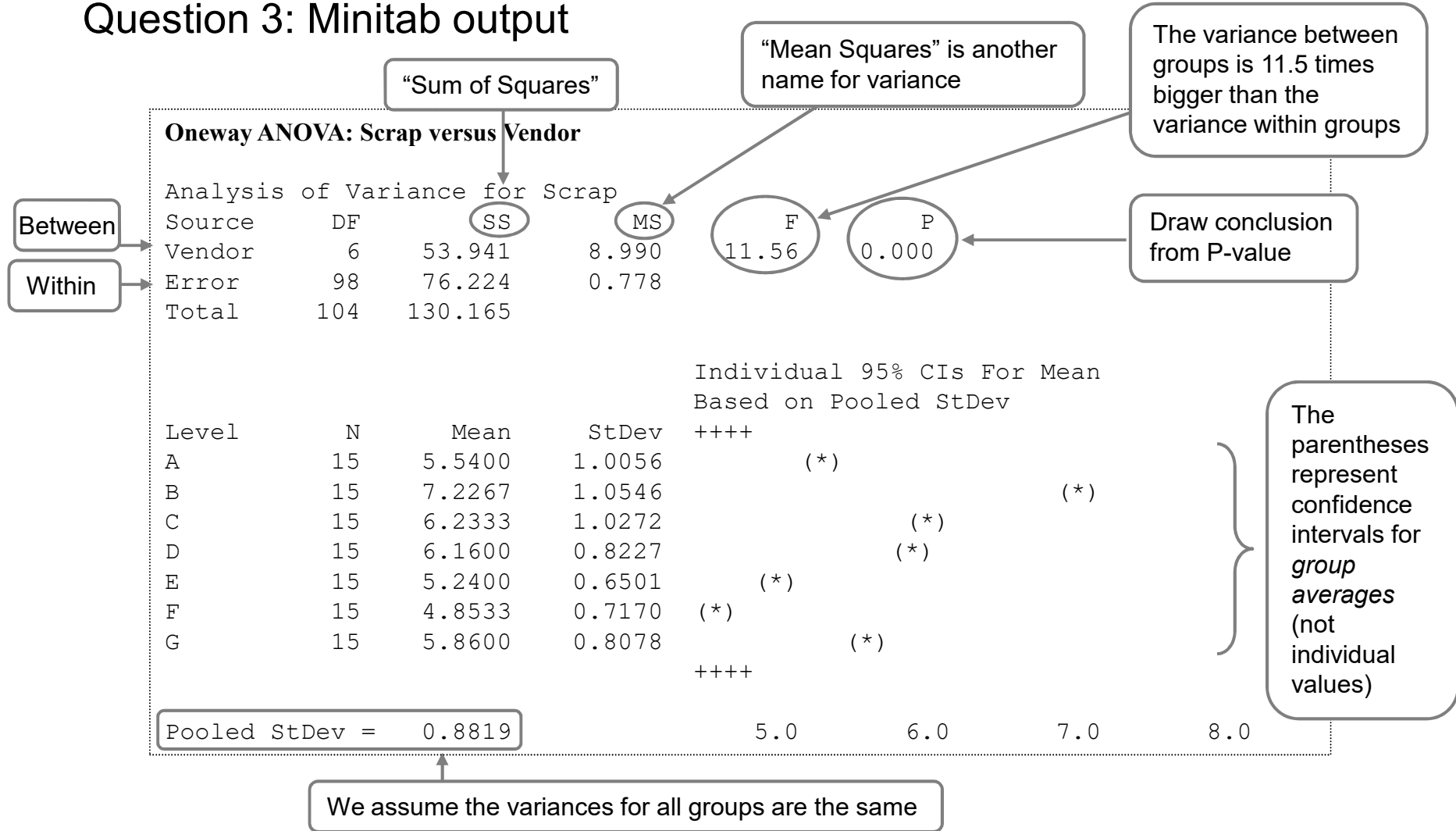
Extreme points provide a “visual” of the “within-vendor” variation



Mean values provide a “visual” of the “between-vendor” variation

# Minitab Follow-Along: ANOVA, cont.

## Question 3: Minitab output



# Minitab Follow-Along: ANOVA, cont.

## Conclusions:

- Because  $p < 0.05$ , reject the  $H_0$  and conclude that at least one group is significantly different from another
- Based on the confidence intervals:
  - Average scrap associated with Vendor B's material is significantly higher than all other vendors
  - Average scrap associated with Vendor F is significantly lower than that associated with Vendors B, C, D, and G, but not significantly lower than that associated with Vendors A or E
- The ANOVA shows the scrap will be significantly lower if you use materials from Vendors F, E, or A
- Use other criteria (e.g., cost, ease of use, or other aspects) to choose among these three materials

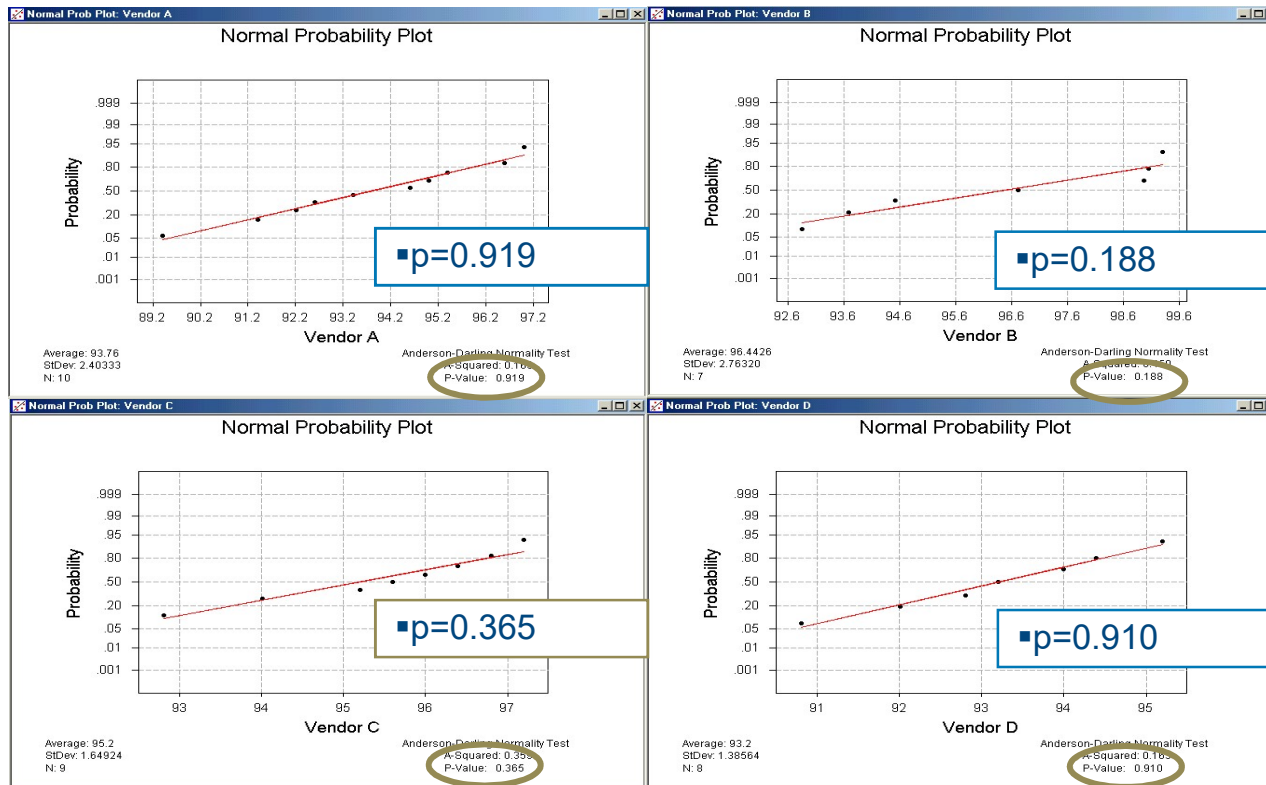
# Setting Up the Data in Minitab

Open worksheet *Vendor Yield.mtw*

Vendor Yield.MTW ***							
↓	C1	C2	C3	C4	C5-T	C6	
	Vendor A	Vendor B	Vendor C	Vendor D	Vendor	Yield	
1	91.4	99.3	92.8	94.4	Vendor A	91.4000	
2	94.6	93.7	96.4	92.8	Vendor A	94.6000	
3	92.6	99.1	96.0	90.8	Vendor A	92.6000	
4	95.0	99.0	94.0	93.2	Vendor A	95.0000	
5	92.2	92.8	92.8	95.2	Vendor A	92.2000	
6	97.0	96.7	95.6	93.2	Vendor A	97.0000	
7	89.4	94.5	96.8	92.0	Vendor A	89.4000	
8	95.4		97.2	94.0	Vendor A	95.4000	
9	93.4		95.2		Vendor A	93.4000	
10	96.6				Vendor A	96.6000	

Sorted data is in columns C1-C4. Stacked data is in column C5-T and C6

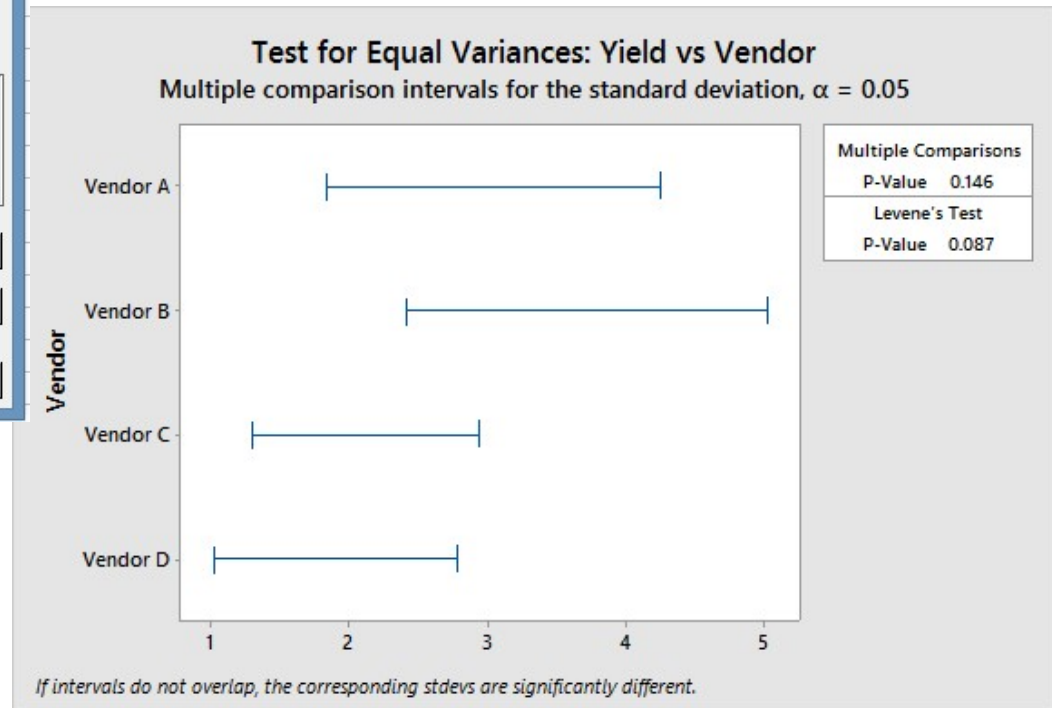
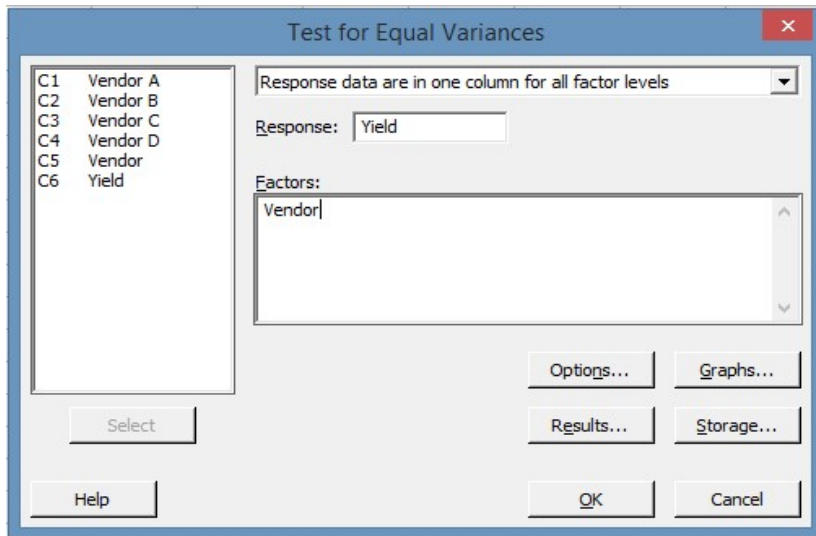
# Testing Data for Normality



All subgroups have a normal distribution

# Testing Data for Equal Variances

Tool Bar Menu > Stat > ANOVA > Test for Equal Variances

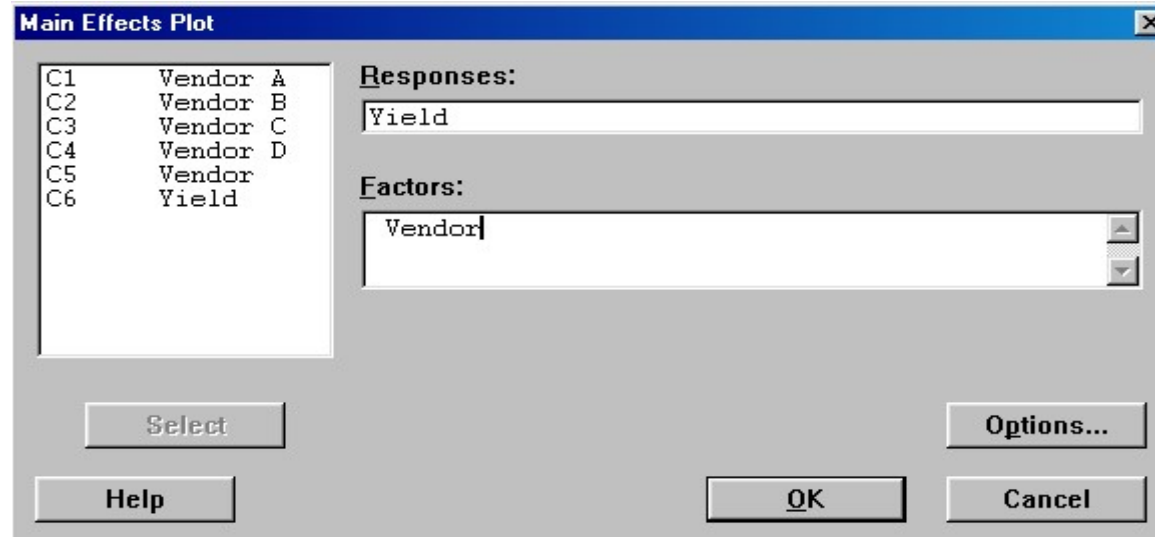


P=0.146 indicates variances are equal

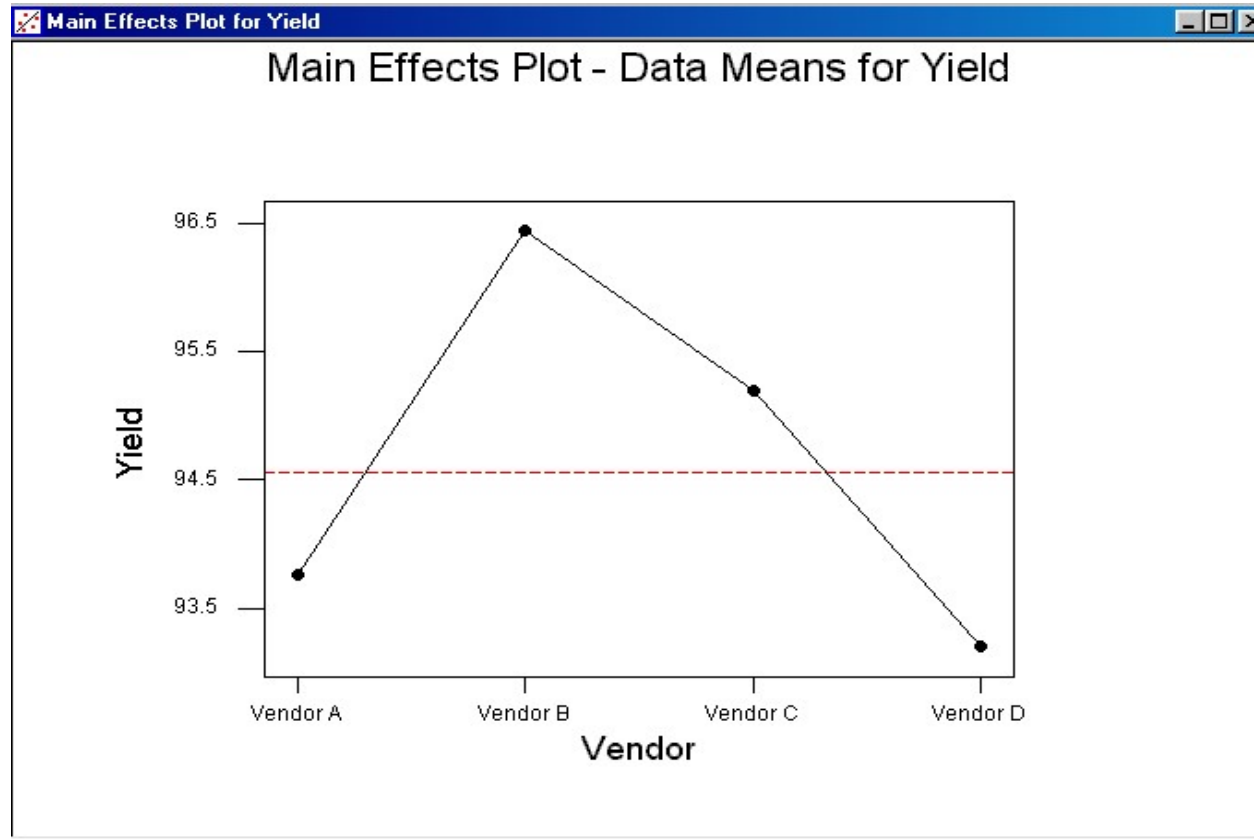


# Running a Main Effects Plot

Tool Bar Menu > Stat > ANOVA > Main Effects Plot



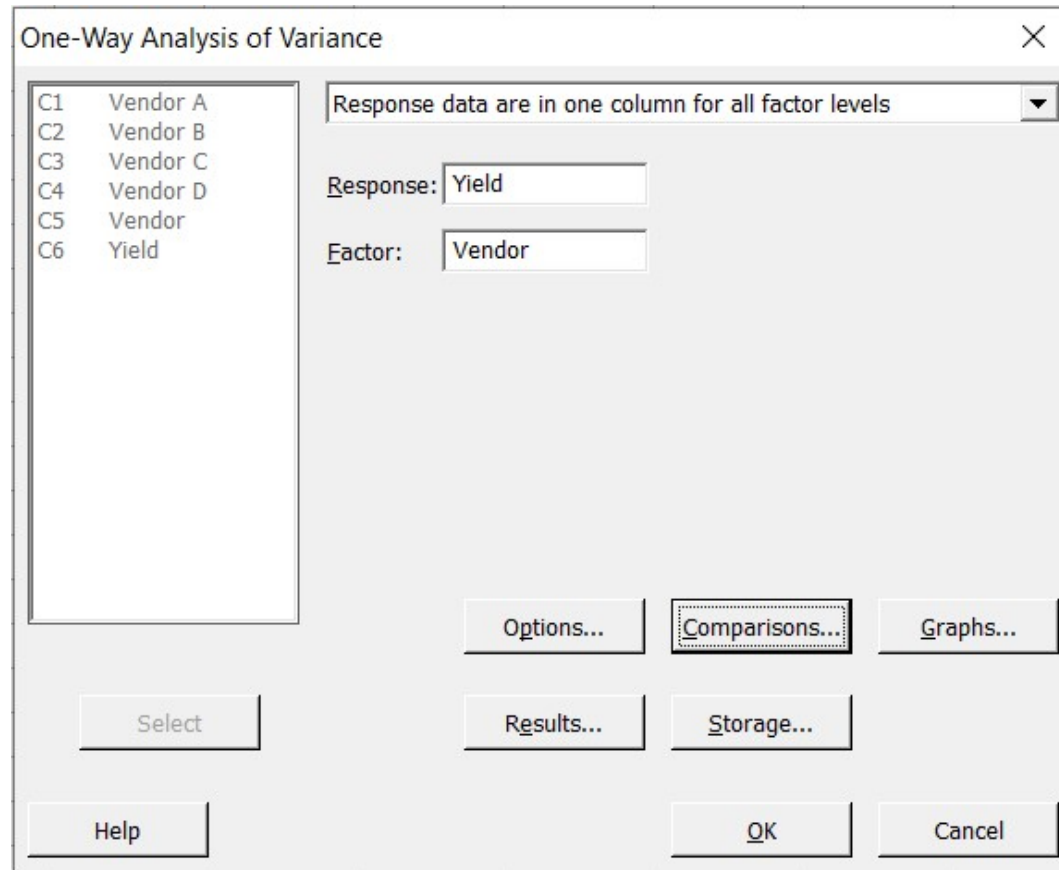
# The Main Effects Plot



The plot shows the output vs. the factor

# Running a One Way ANOVA

Tool Bar Menu > Stat > ANOVA > One Way...



# The One Way ANOVA

## One-way ANOVA: Yield versus Vendor

Method

Null hypothesis All means are equal  
 Alternative hypothesis At least one mean is different  
 Significance level  $\alpha = 0.05$

Equal variances were assumed for the analysis.

Factor Information

Factor	Levels	Values
Vendor	4	Vendor A, Vendor B, Vendor C, Vendor D

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Vendor	3	49.69	16.564	3.74	0.022
Error	30	133.00	4.433		
Total	33	182.69			

P < 0.05: source is significant!

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
2.10551	27.20%	19.92%	6.05%

Means

Vendor	N	Mean	StDev	95% CI
Vendor A	10	93.760	2.403	(92.400, 95.120)
Vendor B	7	96.44	2.76	(94.82, 98.07)
Vendor C	9	95.200	1.649	(93.767, 96.633)
Vendor D	8	93.200	1.386	(91.680, 94.720)

Pooled StDev = 2.10551

# Anexas Consultancy Services

Tests of Significance  
(Non- Parametric)

# Tests of Significance (Non- Parametric)

	<b>Test of Mean/ Median</b>	<b>Test of Variance</b>	<b>Test of Proportion</b>
<b>1 Sample</b>	<ul style="list-style-type: none"> <li>▪ 1 sample Z Test</li> <li>▪ 1 Sample t Test</li> <li>▪ 1 sample Sign</li> <li>▪ 1 Sample Wilcoxon</li> </ul>	Descriptive Statistics Bartlett's test Levene's test	1 Proportion Test
<b>2 Sample</b>	<ul style="list-style-type: none"> <li>▪ 2 Sample t Test</li> <li>▪ Mann-Whitney</li> <li>▪ Paired T Test</li> </ul>		2 Proportion Test
<b>2 or more Samples</b>	<ul style="list-style-type: none"> <li>▪ ANOVA</li> <li>▪ Mood's Median/</li> <li>▪ Kruskal Wallis Test</li> </ul>		Chi Square

# 1 Sample Sign Test - Overview

1 sample sign tests allow you to compare the median of just one sample against a known median value, such as an industry benchmark or well established historical mean.

## **Example:-**

A recruitment consultancy has recently implemented a new salary negotiation process and a project team is trying to verify that it has improved (increased) the salaries that are being achieved. The salaries of the first 20 placements made using the new negotiation process have been recorded and the project team want to compare these results against the benchmark.

# 1 Sample Sign Test

Because the sample of salaries is not Normally distributed and there is only one sample to compare against the benchmark.

The median salary of the sample is 60K and the historical salary median was 48K, and so it appears that there has been an increase in the median of 12K. however its quite a small sample and there is lots of variation within the sample and it is difficult to be sure.

The data file for this example can be found in ***Salaries.mpj***

In this example, the sample median is being tested against a historical benchmark (the test median). Enter 48 (the historical benchmark level)

Alternate Hypothesis: Less than / Not equal / Greater than):

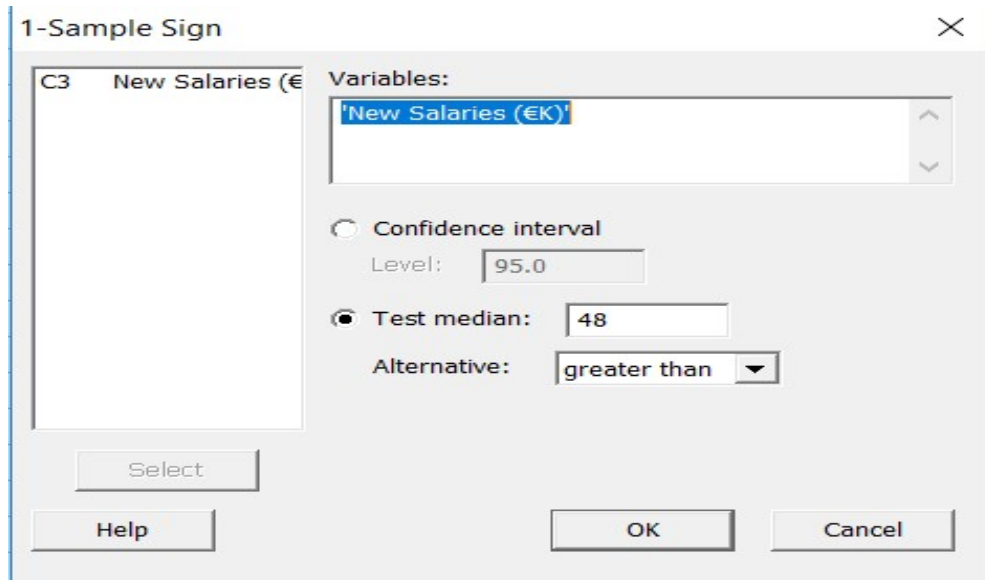
In this example because it appears that the sample median is greater than test median, the alternative greater than has been selected.



# 1 Sample Sign Test - Interpreting the Results

Stat > Nonparametrics > 1-Sample Sign

**Session Window Results:**



The first line provides a summary of the Null and Alternate Hypothesis, in a technical shorthand. Translated into day-to-day language, this line says: These are the results of a Sign Test for the sample median being equal to 48 (median = 48.00) versus it being greater than 48 (versus > 48.00)

**Sign Test for Median: New Salaries (€K)**

Sign test of median = 48.00 versus > 48.00

	N	Below	Equal	Above	P	Median
New Salaries (€K)	20	5	1	14	0.0318	60.00

# 1 Sample Sign Test – Interpreting the Results

## Sign Test for Median: New Salaries (€K)

Sign test of median = 48.00 versus > 48.00

	N	Below	Equal	Above	P	Median
New Salaries (€K)	20	5	1	14	0.0318	60.00

### How the Sign test works:

While the Sign test produces a p-value that can be interpreted in a similar way to other hypothesis tests, the mathematics behind the Sign Test are quite different.

The sign test works by classifying each result within the sample as either above, below or equal to the test median. If the null hypothesis were true, we would expect to see approximately half of the results above and half below, the test median

However in this case, the majority of the results (14 out of 20) were above the test median, and this was high enough for the test to indicate (with statistical confidence) that the sample median (60) is greater than test median (48)

It is interesting to note that one of the results was exactly equal to the test median as summarised in the session window output above. The row was 7 in the example data file.

# 1 Sample Sign Test – Interpreting the Results

## Sign Test for Median: New Salaries (€K)

Sign test of median = 48.00 versus > 48.00

	N	Below	Equal	Above	P	Median
New Salaries (€K)	20	5	1	14	0.0318	60.00

The p-value is 0.0318 (which is lower than the Alpha Level of 0.05), and so you can reject the null, and conclude (with 95% confidence) that the median salary in the sample is greater than the historical median of 48, So the new negotiation process does increase placement salaries.

The 1 Sample Sign test menu in Minitab does not offer any additional graphs. However using a Dot plot from the graph menu, the sample results can be visually compared to the historical benchmark of 48K. As can be seen, while some of the results are below 48, most of them are above and this is enough to conclude, statistically, that the median new salary is higher than 48K

# One-Sample Wilcoxon Test - Overview

Use the one-sample Wilcoxon (also called one-sample Wilcoxon signed rank) confidence interval and test procedures to make inferences about a population median based on data from a random sample.

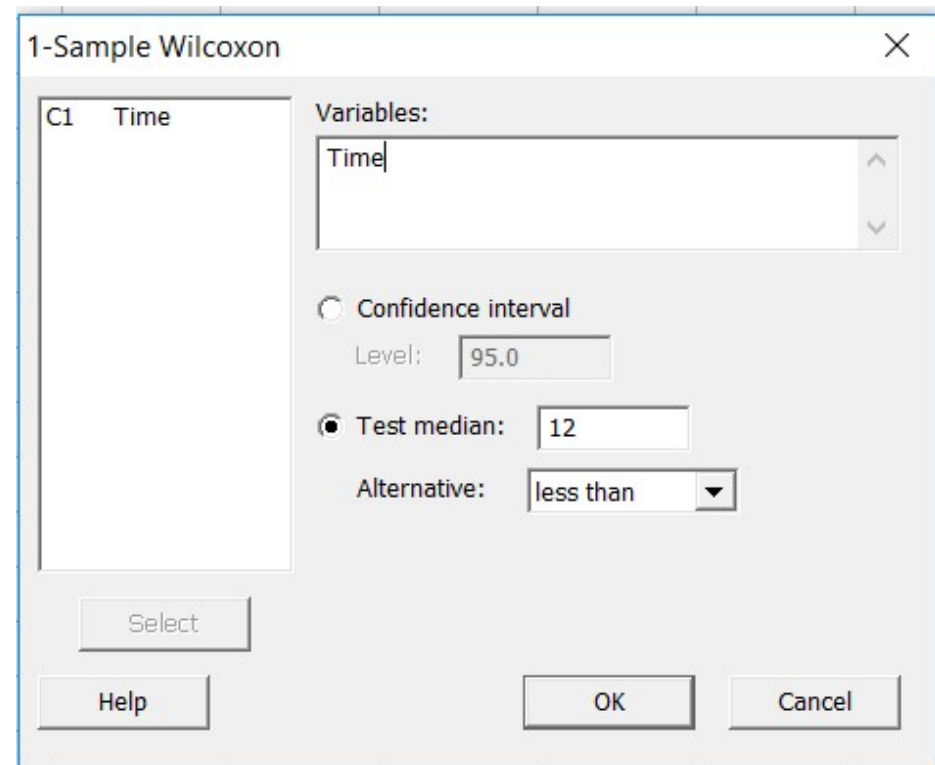
Use the 1-Sample Wilcoxon when you are unable to assume a distribution for the population from which the sample was drawn, but you can assume the distribution is symmetric . This is a nonparametric alternative to one-sample Z and one-sample t procedures.

# One-Sample Wilcoxon Test - Overview

**Example:** A chemist wants to see if a newly developed antacid relieves pain in less than 12 minutes.

Stat > Nonparametrics > One Sample Wilcoxon

The data file for this example  
can be found in :  
***Antacid.MTW***



# One-Sample Wilcoxon Test – Interpreting the Results

## Wilcoxon Signed Rank Test: Time

Test of median = 12.00 versus median < 12.00

	N	for Test	Wilcoxon Statistic	P	Estimated Median
Time	16	16	26.5	0.017	10.05

### Other results in the Session Window :

The Wilcoxon statistic is 26.5 and the Associated p-value is 0.017

### Session Window Results:

Minitab produces only session window results for this test as follows:

Based on the sample data, you want to know if the newly developed antacid relieves pain in less than 12 minutes.

The hypotheses are

Null Hypothesis:  $H_0$ : Median = 12.00 and  
Alternate Hypothesis:  $H_1$ : Median < 12.00

### Estimated Median:

The median of the observations for each treatment. These medians provide an estimate of the population medians for each treatment.

p-value is 0.017, Since, the p-value is less than 0.05, therefore you should reject  $H_0$  and conclude that the antacid relieves pain significantly faster than 12 minutes.

# Mann-Whitney Test - Overview

Use the two-sample Mann-Whitney (also called two-sample rank or two-sample Wilcoxon rank sum) confidence interval and test procedures to make inferences about the difference between two population medians based on data from two independent, random samples.

For example, you can determine whether

- The packing time of two packing machines is the same
- The time to relief is the same for two pain relievers

Assumptions:

- Samples are randomly drawn whose distributions have the same shape
- The two random samples are independent

The Mann-Whitney test is a nonparametric alternative to the two-sample t test with pooled sample variances.

# Mann-Whitney Test - Overview

**Example:** A state's highway department uses two brands of paint for striping roads. A highway official wants to know if a difference exists between the two brands of paint. To assess the problem, the official records the number of months that stripes applied with each brand of paint last on the highway.

The data file for this example can be found in : ***Srtipes.MTW***

## Stat > Nonparametrics > Mann-Whitney

Mann-Whitney

C1	Brand A
C2	Brand B

First Sample: 'Brand A'

Second Sample: 'Brand B'

Confidence level: 95.0

Alternative: not equal

Select

Help

OK

Cancel



# Mann-Whitney Test – Interpreting the Results

## Mann-Whitney Test and CI: Brand A, Brand B

	N	Median
Brand A	11	36.000
Brand B	10	37.600

```

Point estimate for  $\eta_1 - \eta_2$  is -1.850
95.5 Percent CI for  $\eta_1 - \eta_2$  is (-3.000, -0.901)
W = 76.5
Test of  $\eta_1 = \eta_2$  vs  $\eta_1 \neq \eta_2$  is significant at 0.0019
The test is significant at 0.0019 (adjusted for ties)
    
```

### Other results in the Session Window :

The Mann-Whitney statistic is 76.5 and the associated p-value is 0.0019.

### Confidence Interval:

Because you do not know the true value of the median, the confidence interval gives you a range of likely values based on the sample. In repeated sampling, the proportion of intervals that include the true value of the median is equal to 1 minus the chosen  $\alpha$ -level.

### Session Window Results:

Minitab produces only session window results for this test as follows:

Based on your sample, you want to know if the time that the paint stripes last on the highway is the same for the two brands

The hypotheses are

Null Hypothesis:  $H_0: \eta_1 = \eta_2$  and

Alternate Hypothesis:  $H_1: \eta_1 \neq \eta_2$

### Point Estimate:

The difference between the sample medians is a point estimate for the difference between population medians.

p-value is 0.0019. Because the p-value is less than 0.05, you should reject  $H_0$  and conclude that the median times are significantly different.

# Kruskal-Wallis Test - Overview

The Kruskal-Wallis test compares the medians of different samples of data, and can be used where the data samples are not Normally distributed and do not have any obvious outliers.

**Example:-** A project is looking at the time to deliver different products (INGOT and BILLET). The box plot below shows that the INGOT product appears to be delivered quicker than BILLET, and the team are keen to validate this conclusion before other tools (such as detailed process mapping) are used to find out why.

Because the INGOT results do not appear to be normally distributed (a histogram and Box plot both indicate a skewed distribution), a Kruskal-Wallis test is being used to compare the median values of the two samples.

# Kruskal-Wallis Test – Interpreting the Results

## Kruskal-Wallis Test: Time to deliver versus Product

Kruskal-Wallis Test on Time to deliver

Product	N	Median	Ave Rank	Z
BILLET	25	7.500	29.7	0.73
INGOT	30	6.750	26.6	-0.73
Overall	55		28.0	

H = 0.53 DF = 1 P = 0.467

H = 0.54 DF = 1 P = 0.463 (adjusted for ties)

### Session Window Results:

Minitab produces only session window results for this test as follows:

Firstly the sample sizes and medians of the samples are summarised. The difference in the median values is 0.75 days (7.5-6.75), but this should be considered in combination with the following:

The size of the samples (25 for BILLET and 30 for INGOT) appears relatively low.

The resolution of the data was to the nearest 0.5 days (see previous page).

From these reasons, a hypothesis test is essential in order to decide if the difference in medians is statistically significant, as described on the left.

# Kruskal-Wallis Test – Interpreting the Results

## Kruskal-Wallis Test: Time to deliver versus Product

Kruskal-Wallis Test on Time to deliver

Product	N	Median	Ave Rank	Z
BILLET	25	7.500	29.7	0.73
INGOT	30	6.750	26.6	-0.73
Overall	55		28.0	

H = 0.53 DF = 1 P = 0.467  
 H = 0.54 DF = 1 P = 0.463 (adjusted for ties)

## Analysing the p-value:

The p-value from this test is 0.463. since this is higher than 0.05, we cannot say with confidence that there is a difference in the medians of the two samples.

In other words, the median delivery times of the two internet products (that the two samples of data represent) could be the same as each other.

Note: the two p-values usually very similar, but if not, use the value that is “adjusted for ties”

## So, to summarise the results in day to day language:

Based on the data we have collected, we cannot say with confidence that there is a difference between the medians.

The difference of 0.75 hours between the sample medians could easily have occurred just by chance.

If there is a difference, more data will have to be collected to prove it.

# Mood's Median Test - Overview

The Mood's Median test compares the medians (central position) of different samples of data, where the samples are not Normally distributed and where there are obvious outliers in the data samples.

**Example:-** A project is looking at the time to deliver different products (INGOT and BILLET).

The data has been stratified into two groups INGOT and BILLETS, and the box plot below shows that the INGOT product appears to be delivered quicker than BILLET.

The team are keen to validate this before they set out to find and understand the root cause of this difference. Mood's Median test is being used because the Billet data appears to be skewed and also has some outliers (the asterisks)

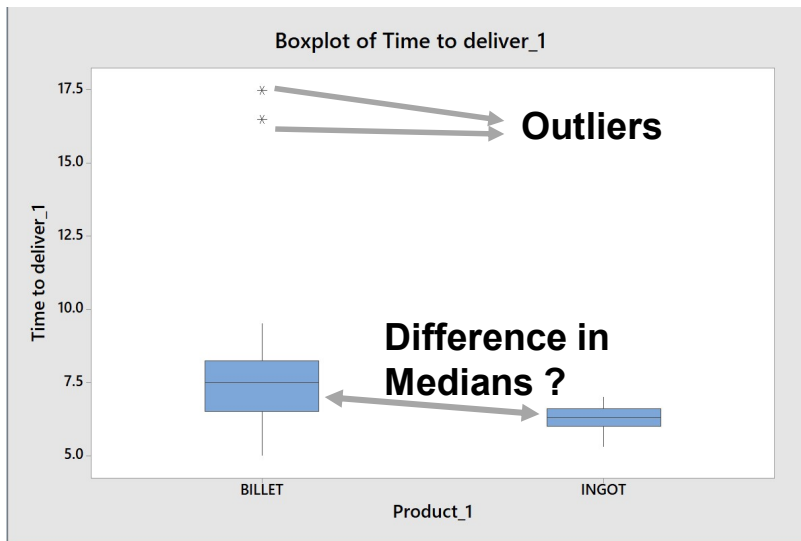
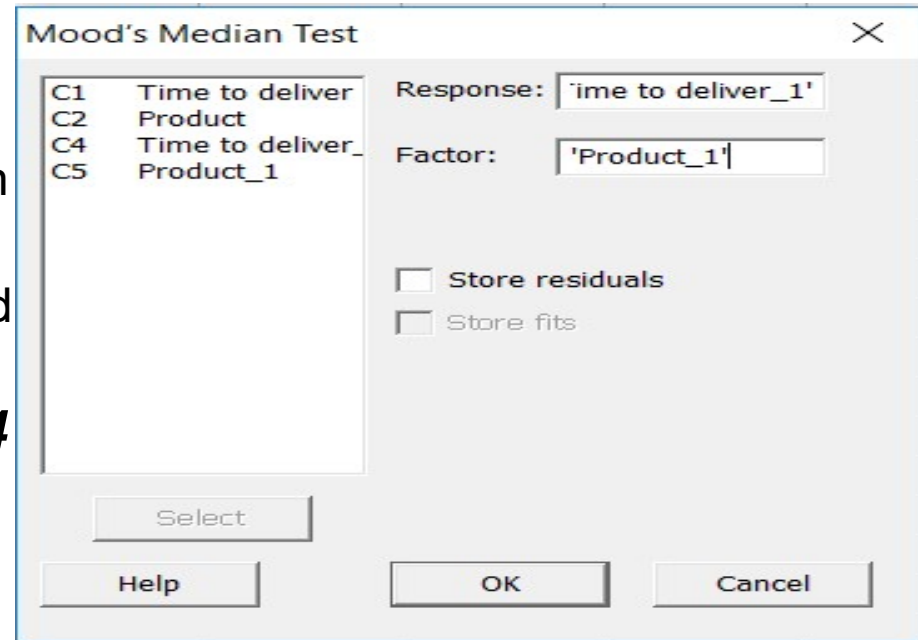
# Mood's Median Test - Overview

## Data format:

Data for the Mood's Median test must be stacked in one column, with the subgroup(factor) code alongside as shown on the right.

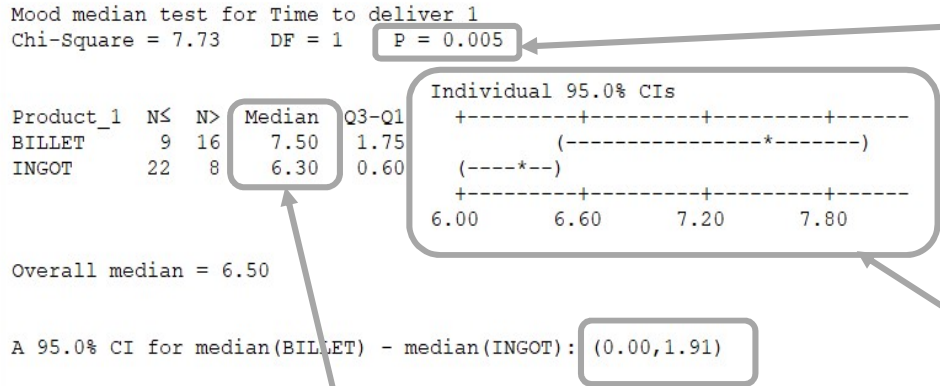
The data file for this example can be found in ***Time to deliver INGOT BILLETS.mpj*** and in **Columns C4**

Stat > Nonparametrics > Mood's Median



# Mood's Median Test – Interpreting the Results

## Mood Median Test: Time to deliver\_1 versus Product\_1



### Session Window Results:

Minitab produces only session window results for this test as follows:

The p-value for the test is 0.005, since this is less than Alpha Level of 0.05 we can say, with 95% confidence, that the medians of the subgroups are different.

A rough graph of the 95% Confidence Intervals (Cis) for the median of the subgroups is shown. Note that although the Cis are visually overlapping, the statistical conclusions is that they are different.

### Other results in the Session Window :

Based on the data subgroups, the difference between the subgroup sub-group medians is 1.2 Days(7.50-6.30)

The confidence interval for the difference provides more detail and confirms (with 95% confidence), that the difference in medians is somewhere between 1.91 and 0.00 Days.

### So, to summarise the results in day to day language:

We can be very confident that there is a difference in the median delivery time for INGOT & BILLETS.

The median delivery time of INGOTs is at least 0 days quicker than BILLETS but could be as much as 1.91 days quicker.

# Anexas Consultancy Services

## Proportions Tests



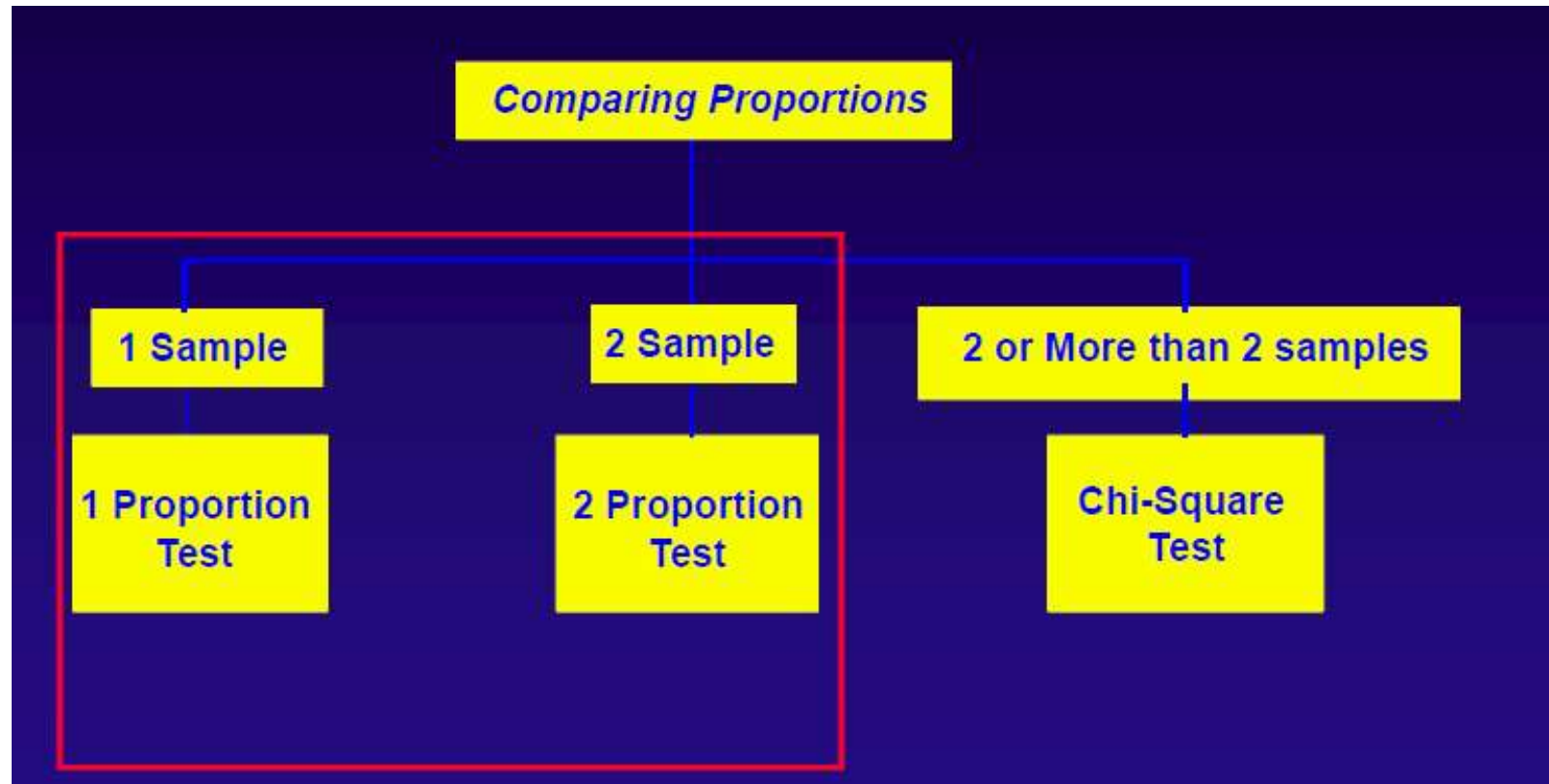
# Why Learn Hypothesis Tests of Proportion?

- Make data driven decisions with defined confidence
- Determine if a statistically significant difference of proportion exists between:
  - A sample and a target
  - Two independent samples
  - More than two independent samples

# What are Hypothesis Tests of Proportion?

Test	Method for analyzing the differences between:
1 Proportion	a sample proportion and a target value
2 Proportion	proportion obtained from two independent samples

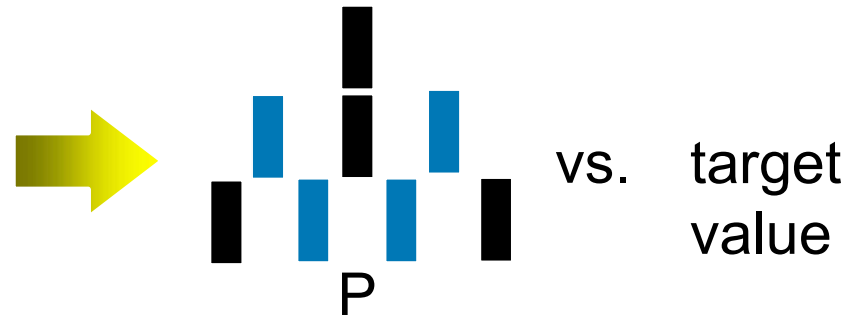
# Hypothesis Testing of Proportion - Roadmap



# Comparison of Proportion: 2 Scenarios

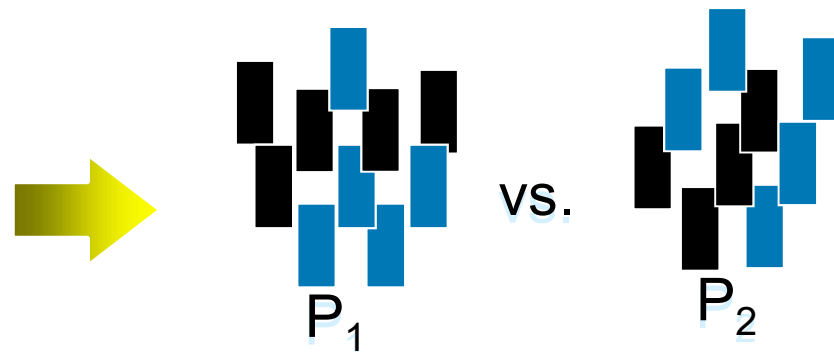
## 1) Single Proportion Comparison

One population proportion compared to a target value



## 2) Two Sample Comparison

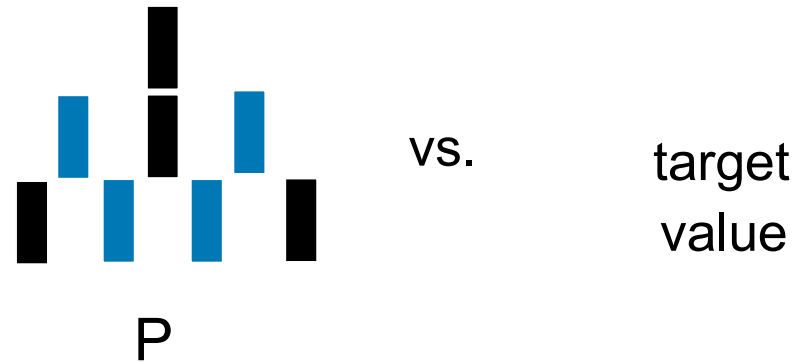
Proportions of two independent populations compared to each other



# Anexas Consultancy Services

## 1 Proportion Test

# Single Proportion Comparison



Practical Question  
(example)

“Is the population proportion statistically different from the target value?”

Statistical Question

$$H_0 : P = \text{target value}$$

$$H_a : P \neq \text{target value}$$

# 1 Proportion Test

- Hypothesis test about the population proportion using information from one sample

# Industrial Process Example

## Migraine Medicine

A pharmaceutical company has invented a new medicine for relieving migraine headaches. The company wants to test the hypothesis that the drug is effective more than 73% of the time.

A clinical study of 111 out of 143 adults suffering from migraine headaches reported relief after using the drug. Is this sufficient evidence that the drug is effective more than 73% of the time?

Use a 5% significance level.



# Industrial Process Example

## Migraine Medicine

- Practical Problem
  - Is the migraine medicine effective more than 73% of the time?
- Statistical Problem
  - Is population proportion of medicine effectiveness greater than or equal to 73%?
  - Null hypothesis: population proportion is 73%
  - Alternate hypothesis: population proportion is greater than 73%

# Example: Migraine Medicine

- State the hypotheses and significance level

$$H_o: P = 0.73$$

$$H_a: P > 0.73$$

$$\alpha = 0.05$$

- What hypothesis test is appropriate?
  - These hypotheses deal with proportions
  - Comparing population proportion against a target proportion using one sample data
  - Use 1 Proportion Test

# Example: Migraine Medicine

Tool Bar Menu > Stat > Basic Statistics > Proportion

## Analysis through Minitab

One-Sample Proportion

Summarized data

Number of events: 111

Number of trials: 143

Perform hypothesis test

Hypothesized proportion: 0.73

Select

Options...

Help

OK

Cancel

One-Sample Proportion: Options

Confidence level: 95.0

Alternative hypothesis: Proportion > hypothesized proportion

Method: Exact

Help

OK

Cancel

# Example: Migraine Medicine

## Test and CI for One Proportion

Test of  $p = 0.73$  vs  $p > 0.73$

Sample	X	N	Sample p	95.0% Lower Bound	Exact P-Value
1	111	143	0.776224	0.711327	0.124

- Interpretation:
  - p-value = 0.124
  - p-value  $>$   $\alpha$ -risk (0.05): fail to reject  $H_0$
  - Infer  $H_0$ : insufficient evidence that that the drug is more than 73% effective

# Business Process Example

## IPO Prospectus

A Black Belt is studying the effects of voluntary disclosure of earnings forecast in the Initial Public Offering (IPO) prospectus.

A random sample of 130 IPO prospectus revealed that 58 of them did not reveal their earnings forecast.

Test the hypothesis at 5% significance level that less than 50% of IPO prospectus do not disclose their earnings forecast.

# Example: IPO Prospectus

- Practical Problem
  - Is the percentage of IPO prospectus disclosing their earnings forecast less than 50%?
  
- Statistical Problem
  - Is population proportion of IPO prospectus revealing their earnings forecast less than 50%?
  - Null hypothesis: population proportion is 50%
  - Alternate hypothesis: population proportion is less than 50%

# Example: IPO Prospectus

State the hypotheses and significance level

$$H_o: P = 0.50$$

$$H_a: P < 0.50$$

$$\alpha = 0.05$$

What hypothesis test is appropriate?

These hypotheses deal with proportions

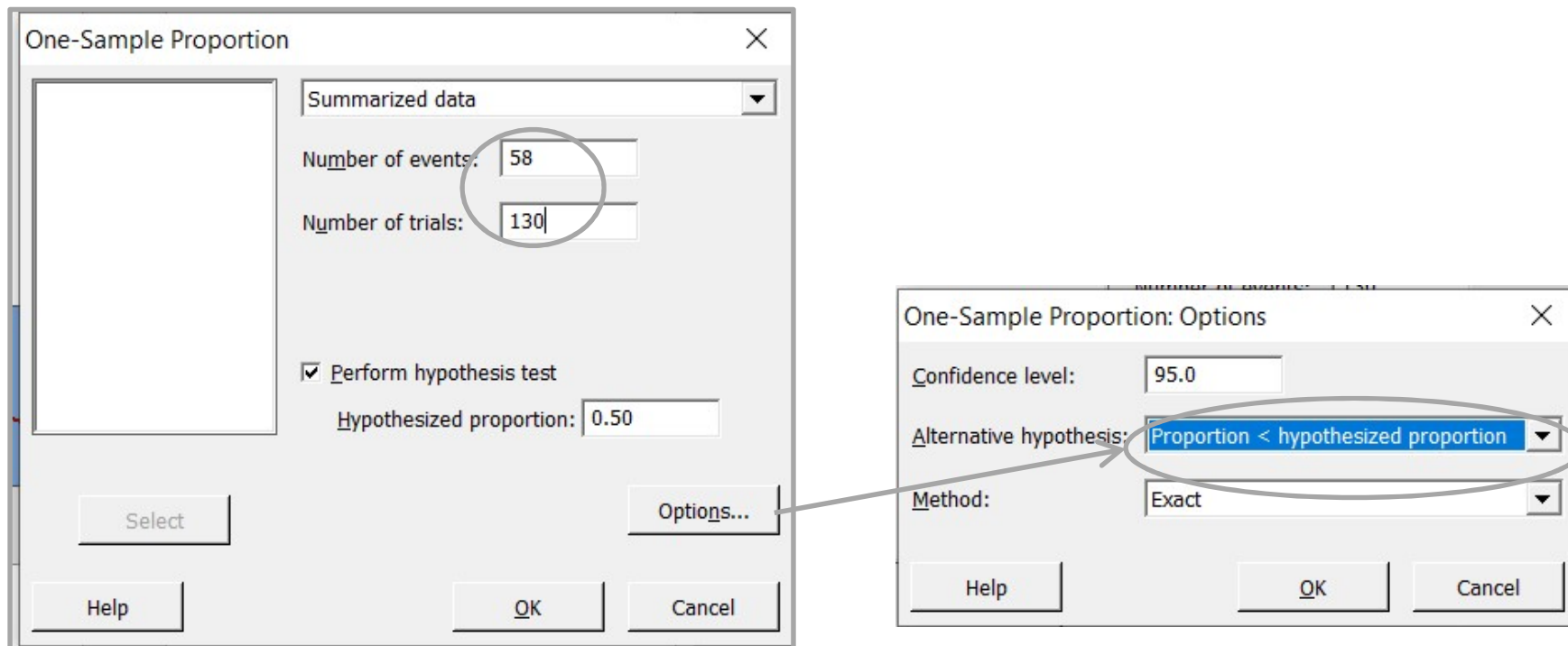
Comparing population proportion against a target proportion using one sample data

Use 1 Proportion Test

# Example: IPO Prospectus

Tool Bar Menu > Stat > Basic Statistics > 1 Proportion

## Analysis through Minitab





# Example: IPO Prospectus

## Test and CI for One Proportion

Test of  $p = 0.5$  vs  $p < 0.5$

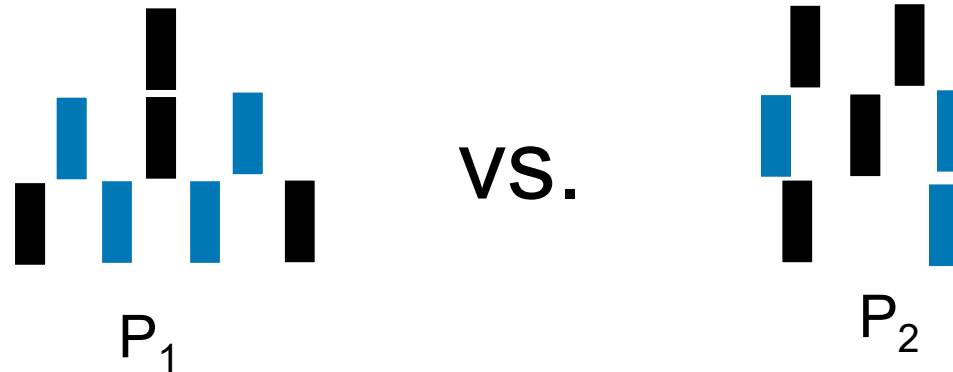
Sample	X	N	Sample p	95.0% Upper Bound	Exact P-Value
1	58	130	0.446154	0.522079	0.127

- Interpretation:
  - P-value = 0.127.
  - P-value  $>$   $\alpha$ -risk (0.05): Fail to reject  $H_0$
  - Infer  $H_0$ : insufficient evidence that only less than 50% of IPO prospectus disclose their earnings forecast

# Anexas Consultancy Services

## 2 Proportions Test

# Two Sample Proportion Comparison



Practical Question  
(example)

Are the two  
populations' proportions  
statistically different?

Statistical Question

$$H_0 : P_1 = P_2$$

$$H_a : P_1 \neq P_2$$

## 2 Proportion Test

- Hypothesis test about the difference between two population proportions using information from two samples
- Two sets of samples are statistically independent

# Industrial Process Example: Comparing Medicines

MedChoice, Inc. distributes two identical brands of medicine for relieving migraine headaches.

It is found from controlled studies that 145 out of 200 people suffering from migraines reported relief through use of Brand A whereas 101 out of 150 people reported relief through the use of Brand B.

The company wants to know if we can conclude at the 5% level of significance that the percentage of people getting relief through use of Brand A is higher than through Brand B?

# Example: Comparing Medicines

- Practical Problem
  - Is Brand A better than Brand B in providing relief from migraine headaches?
  
- Statistical Problem
  - Is population proportion of relief through Brand A greater than population proportion through Brand B?
  - Null hypothesis: population proportion for Brand A = population proportion for Brand B
  - Alternate hypothesis: population proportion for Brand A is greater than that of Brand B

# Example: Comparing Medicines

State the Hypotheses and Significance Level

$$H_o: P_A - P_B = 0$$

$$H_a: P_A - P_B > 0$$

$$\alpha = 0.05$$

- What Hypothesis Test is Appropriate?
  - These hypotheses deal with proportion values
  - Comparing population proportions using two sets of independent samples
  - Use 2 Proportion Test

# Example: Comparing Medicines

Tool Bar Menu > Stat > Basic Statistics > 2 Proportion

## Analysis through Minitab

Two-Sample Proportion

Summarized data

	Sample 1	Sample 2
Number of events:	145	101
Number of trials:	200	150

Select Options... Help OK Cancel

Two-Sample Proportion: Options

Difference = (sample 1 proportion) - (sample 2 proportion)

Confidence level: 95.0

Hypothesized difference: 0.0

Alternative hypothesis: Difference > hypothesized difference

Test method: Estimate the proportions separately

Help OK Cancel



# Example: Comparing Medicines

Test and CI for Two Proportions

Sample	X	N	Sample p
1	145	200	0.725000
2	101	150	0.673333

Difference = p (1) - p (2)

Estimate for difference: 0.0516667

95% lower bound for difference: -0.0299692

Test for difference = 0 (vs > 0): Z = 1.04 P-Value = 0.149

Fisher's exact test: P-Value = 0.176

## What is the Interpretation?

p-value = 0.149

p-value (0.149) >  $\alpha$ -risk (0.05); fail to reject  $H_0$

Infer  $H_0$ : insufficient evidence that brand A is more effective than brand B

# Anexas Consultancy Services

Hypothesis Testing  
Chi-Square Tests

# Why Learn Chi-Square Tools?

Make data driven decisions with defined confidence

Determine if

- Two attribute variables are related

- A population fits a certain probability model (distribution)



# What Are Chi-Square Tools?

## Chi-Square Goodness-of-Fit Test

To test if a particular distribution (model) is a good fit for a population

## Chi-Square Test for Association

To test if a relationship between two attribute variables exists

$$\chi^2 = \sum_{j=1}^g \frac{(f_o - f_e)^2}{f_e}$$

▪Chi-Square Statistic

Both of these tools use the Chi-Square distribution, where  $f_o$  and  $f_e$  are the observed and expected frequencies, respectively.

# Anexas Consultancy Services

## Test for Association

# Business Process Example: Black Belt Projects

A sample of Black Belts was asked to rate both their six sigma project performance and the average weekly hours spent with the Project Champion discussing project details. The results are shown in the following table. Test at the 5% level the null hypothesis of no association between the two sets of ratings.

Data is given in *Chi1.mtw*

Time with Champion ▪ <u>HOURS</u> <u>High</u>	PROJECT PERFORMANCE		
	<u>Low</u>	<u>Medium</u>	
▪ < 0.1	17	21	12
▪ 0.1 - 1	31	53	21
▪ > 1	17	42	71

# Example: Black Belt Projects

- Practical problem
  - Does the performance of Black Belt projects depend on time spent with Project Champions?
- Statistical problem
  - Is there an association between project performance and time spent with Champion?
  - Null hypothesis: project performance is independent of the time spent with Champion
  - Alternate hypothesis: project performance is dependent of the time spent with Champion
- What hypothesis test is appropriate?
  - These hypotheses deal with relationship between two attribute variables
  - Use Chi-Square Test for Association

# Example: Black Belt Projects

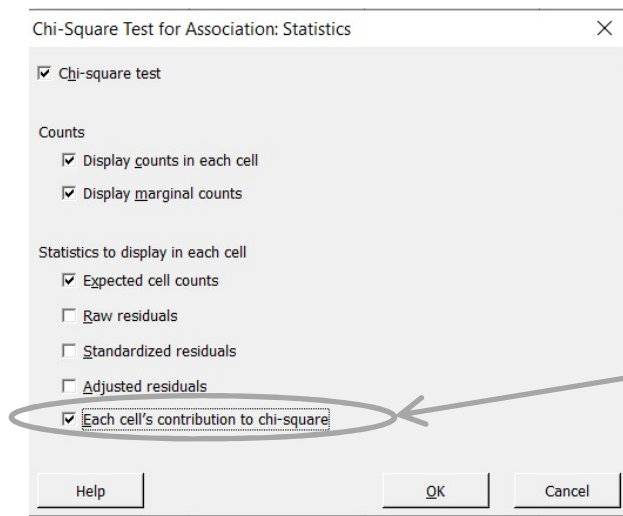
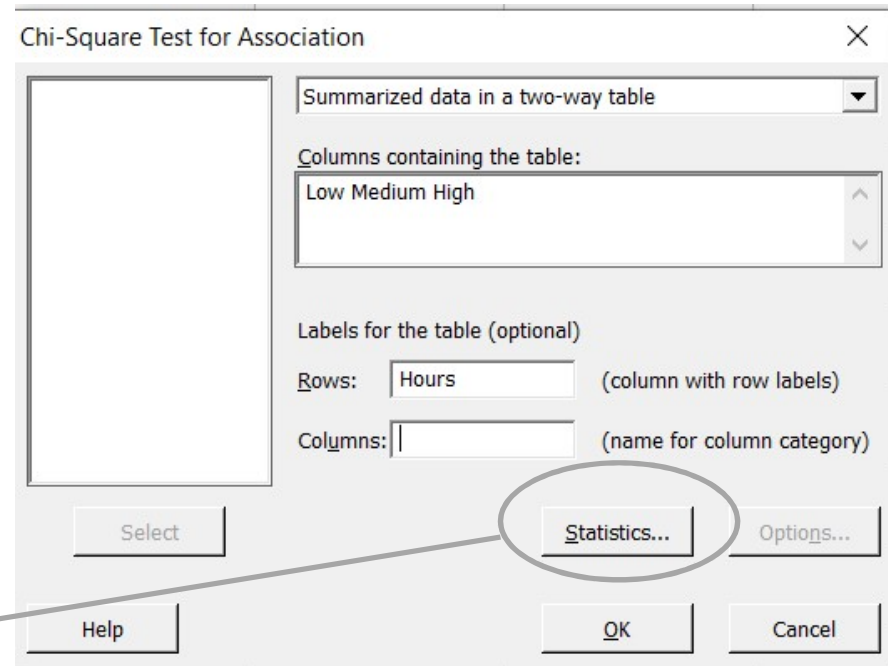
1. Open data file Chi1.mtw

C1-T	C2	C3	C4
Hours	Low	Medium	High
<0.1	17	21	12
0.1-1.0	31	53	21
>1	17	42	71

Tool Bar Menu > **Stat** > **Tables** > **Chi-Square Test**

2. *Stat > Tables > Chi-Square Test for Association*

3. Fill in the dialog as shown below:



4. Click **Statistics** and Enable **Each cells' contribution to chi-square**



# Example: Black Belt Projects

Chi-Square Test: Low, Medium, High

Rows: Hours Columns: Worksheet columns

	Low	Medium	High	All
<0.1	17 11.40	21 20.35	12 18.25	50
0.1-1.0	31 23.95	53 42.74	21 38.32	105
>1	17 29.65	42 52.91	71 47.44	130
All	65	116	104	285

Cell Contents: Count  
Expected count

Pearson Chi-Square = 36.622, DF = 4, P-Value = 0.000  
Likelihood Ratio Chi-Square = 37.348, DF = 4, P-Value = 0.000

- Interpret output
- What is  $\chi^2$ (calc)
- What is the p-value?
- What is its interpretation?

# Example: Black Belt Projects

- Interpretation:
  - p-value = 0.000
  - p-value <  $\alpha$ -risk (0.01): reject  $H_0$
  - Infer  $H_a$ : sufficient evidence that Black Belt project performance and time spent with Champion are dependent

# Test for Association Exercise

- **Objective:** Practice doing a Chi-square test in Minitab and interpret the results
- Data:** *Shipments.MTW*
- **Background:**
  - We have been tracking time taken to ship subassemblies between two manufacturing facilities by three carriers
    - Shipping records have been examined for 12 months
  - Deliveries are expected to be completed within 24 hours
  - The results for all shipments are summarized on the next page

# Test for Association Exercise, cont.

Carrier	Within 24 Hours	24-36 Hours	36+ Hours
A	1810	679	551
B	609	219	237
C	176	85	58

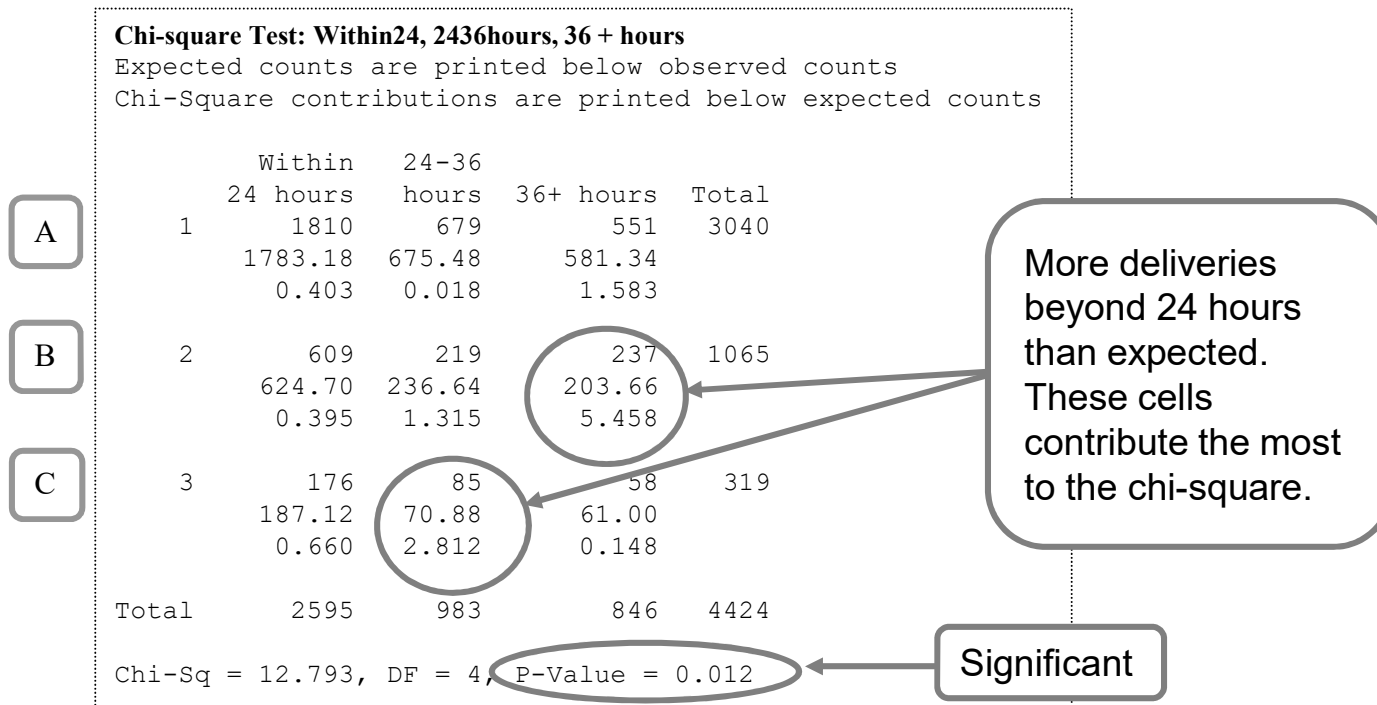
# Test for Association Exercise, cont.

- **Instructions:**

1. Use the summarized table data in the worksheet
2. Do a Chi-square test and interpret it
3. Be prepared to report the P-value, your conclusions, and recommended next steps

# Test for Association Exercise, Answers

- **Minitab output**



**Conclusion:** The carriers differ: Carrier C has more deliveries than expected between 24 and 36 hours and Carrier B has more deliveries than expected in 36+ hours

# Test for Association Exercise

Use the following data to decide if the outcome of a surgical procedure depends on the hospital used

Condition	Hosp A	Hosp B	Hosp C	Hosp D	Hosp E
no improvement	13	5	8	21	43
partial functional restoration	18	10	36	56	29
complete functional restoration	16	16	35	51	10

# Anexas Consultancy Services

Hypothesis Testing  
Correlation and Regression



# Hypothesis Tests

<b>Y</b>	<b>X</b>	<b>Hypothesis Test</b>
<b>Continuous / Variable Data</b>	<b>Attribute / Discrete Data</b>	<b>1-z, 1-t, 2-t, paired t, ANOVA</b>
<b>Attribute / Discrete Data</b>	<b>Attribute / Discrete Data</b>	<b>1-p, 2-p, Chi Square</b>
<b>Continuos / Variable Data</b>	<b>Continuos / Variable Data</b>	<b>Correlation, Regression, Multiple Regression</b>
<b>Attribute / Discrete Data</b>	<b>Continuous / Variable Data</b>	<b>Logistic Regression</b>

# Why Learn Correlation and Regression?

- Explore the existence of relationship between variables with the aid of data
- Screen variables and determine which variable(s) has the biggest impact on the response(s) variable
- Describe the nature of relationship with the help of an equation and use it for prediction

# Anexas Consultancy Services

**Correlation**

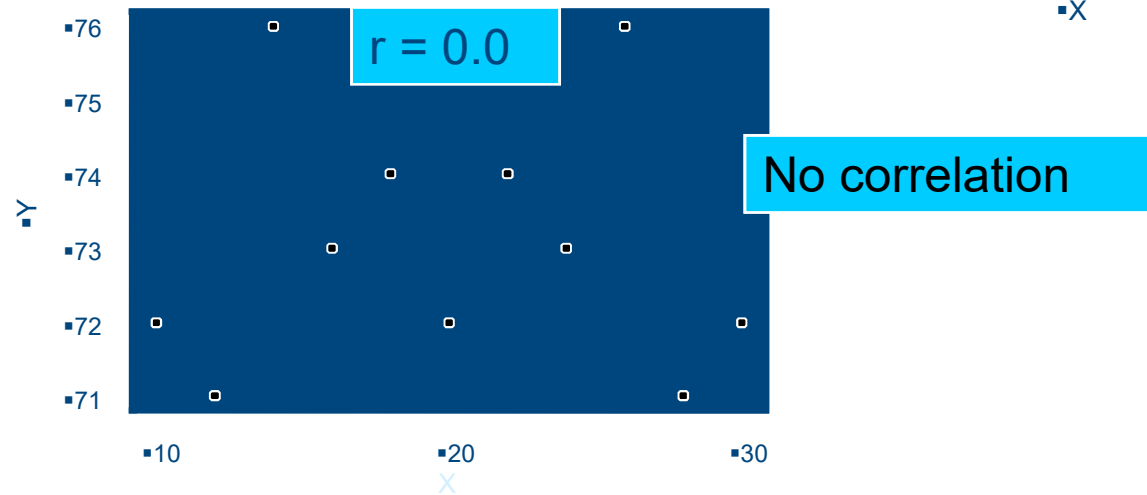
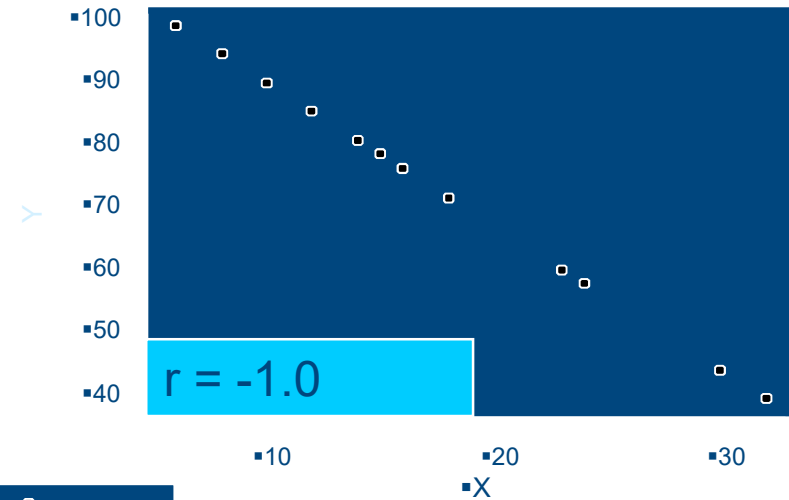
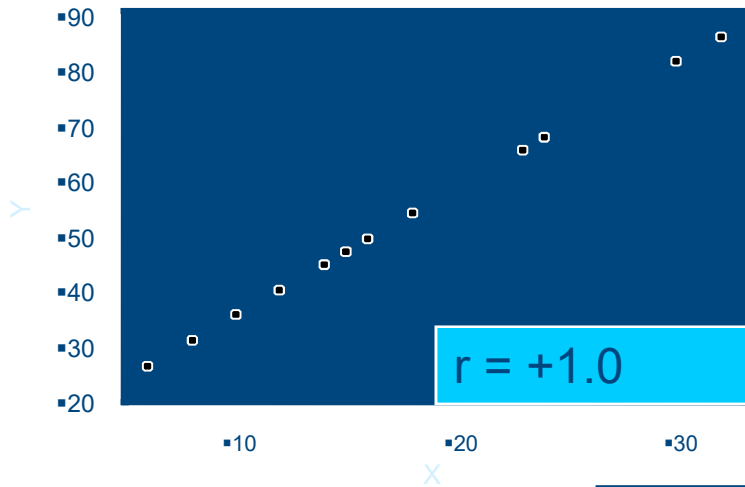
# What is Correlation?

- Correlation is a measure of the strength of association between two quantitative variables  
(Ex: Pressure and Yield)
- Measures the degree of linearity between two variables assumed to be completely independent of each other

Correlation coefficient or *Pearson* correlation coefficient is a way of measuring the strength of correlation

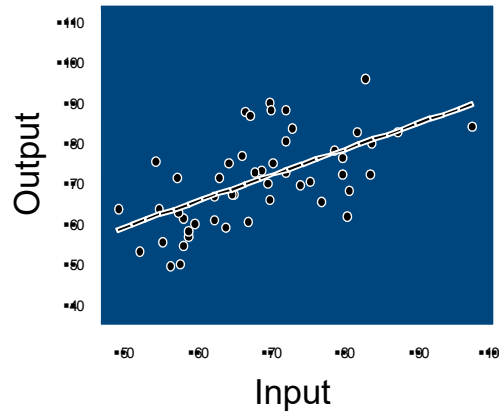


# Correlation Coefficient



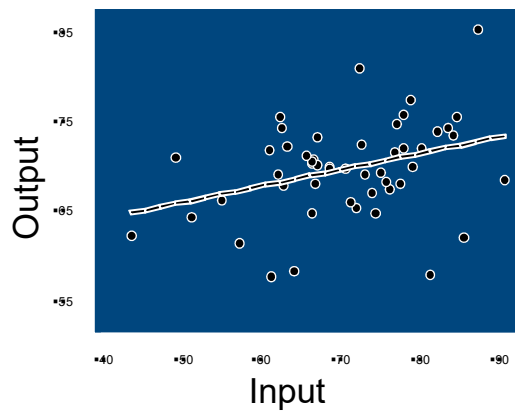
# Strength and Direction of “+” Correlation

Moderate positive correlation



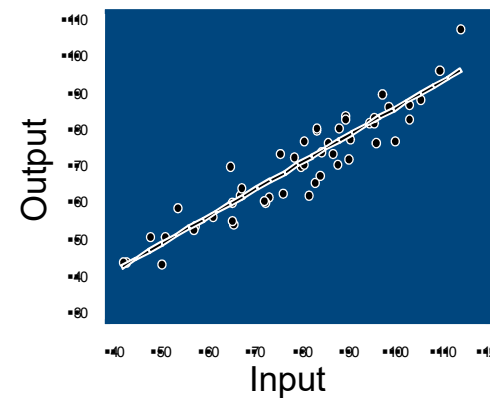
$Y=25.7595+0.645418X$   
R Squared=0.369

Weak positive correlation



$Y=56.6537+0.181987X$   
R Squared=0.115

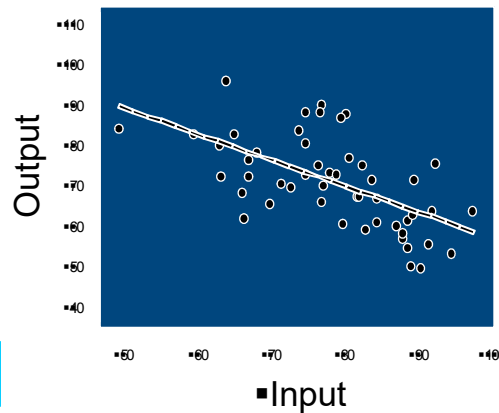
Strong positive correlation



$Y=9.77271+0.745022X$   
R Squared=0.876

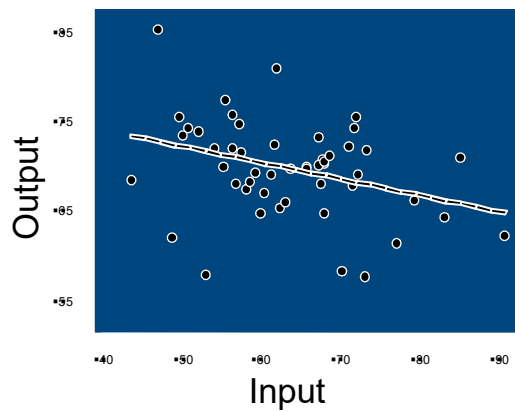
# Strength and Direction of “-” Correlation

Moderate negative correlation



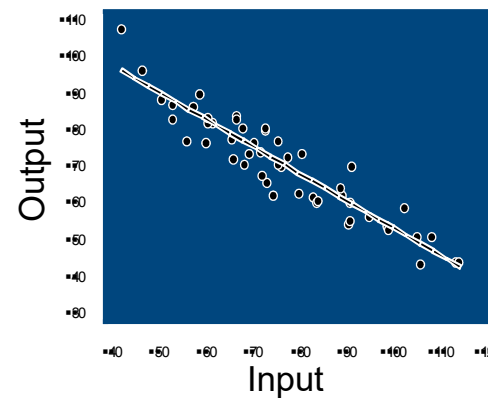
$Y=90.3013-0.645418X$   
R Squared=0.369

Weak negative correlation



$Y=74.8524-0.181987X$   
R Squared=0.115

Strong negative correlation

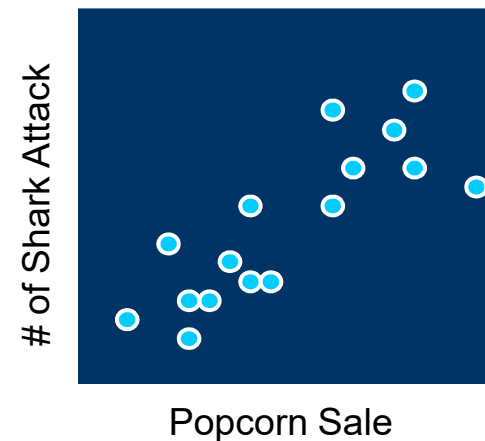
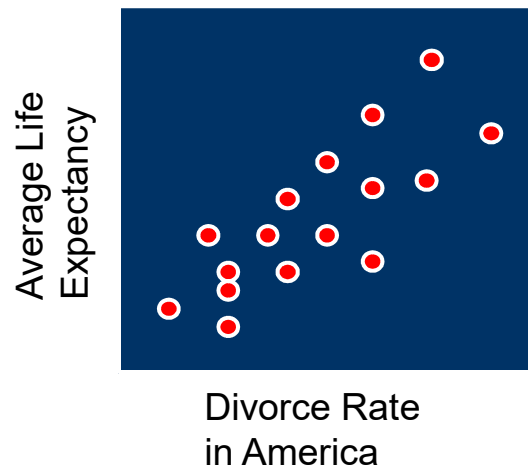


$Y=99.1754-0.745022X$   
R Squared=0.876

# Correlation vs. Causation

Data shows that average life expectancy of Americans increased when the divorce rate went up!

Is there a correlation between grass height and hair length?



Correlation does not imply causation! A third variable may be 'lurking' that causes both x and y to vary



# Business Process Example: Cereal Sales

A market research analyst for a certain brand of cereal is interested in finding out if there is a relationship between the sales generated and shelf space used to display the cereal. As a result she conducted a study and collected data from 12 different stores selling this brand of cereal.

Shelf Space	Sales
574	960
635	1709
533	651
560	831
628	1460
615	1473
540	751
587	1238
656	1924
594	1370
622	1609
567	1120

The data contains sales \$ generated for a certain month and the shelf space dedicated to the product.

What would you do?

What questions might you ask?

Data in *Sales.mtw*

# Example: Cereal Sales

- Practical Problem
  - Is there a relationship between sales \$ from cereal and the shelf space used to display the cereal?
  - If there a relationship, how strong is that relationship?
- Statistical Problem
  - Are the variables 'Sales' and 'Shelf Space' correlated?
  - Null hypothesis: Sales and Shelf space are not correlated
  - Alternate hypothesis: Sales and Shelf space are correlated

## Example: Cereal Sales

State the Hypotheses and Significance Level

$$H_o: \rho = 0$$

$$H_a: \rho \neq 0$$

$$\alpha = 0.01$$

Notice that the hypotheses are about a population parameter

What Hypothesis Test is Appropriate?

These hypotheses deal with correlation coefficient

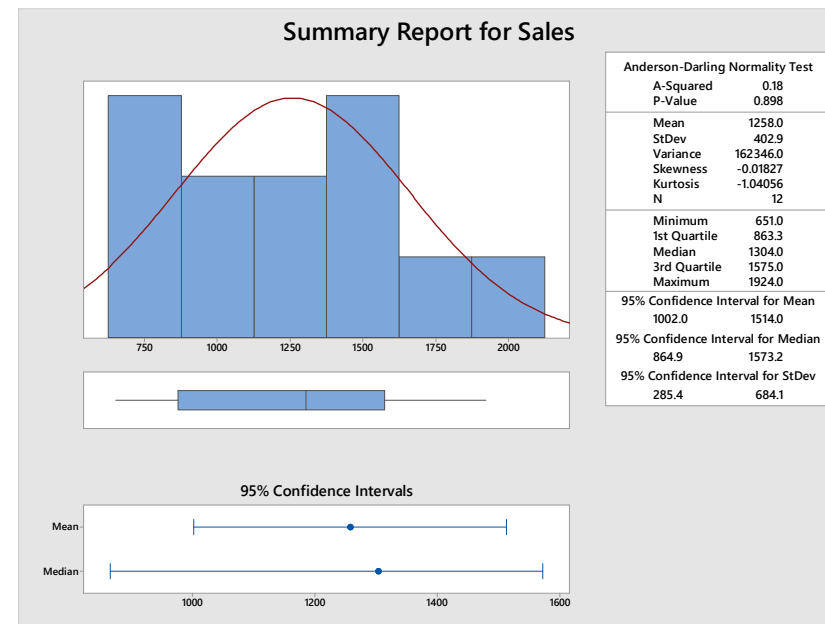
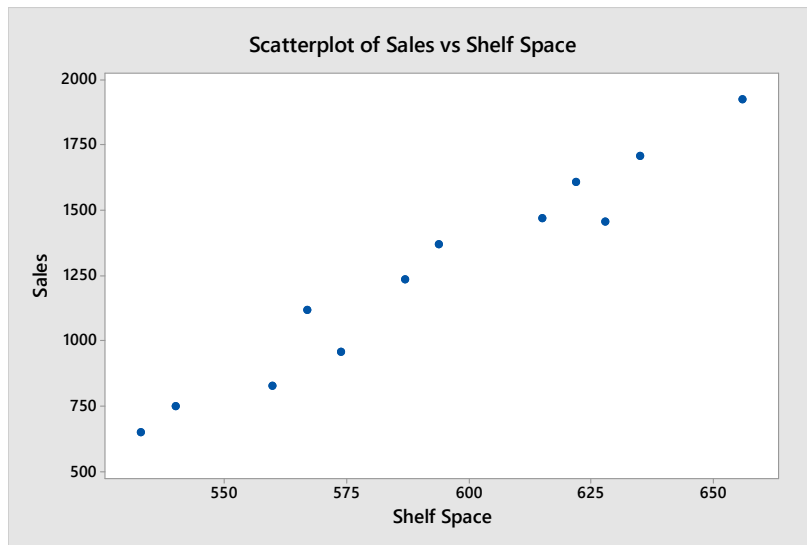
Make decisions based on Pearson correlation coefficient and 'p-value'

# Example: Cereal Sales

Tool Bar Menu > Stat > Basic Statistics > Graphical Summary

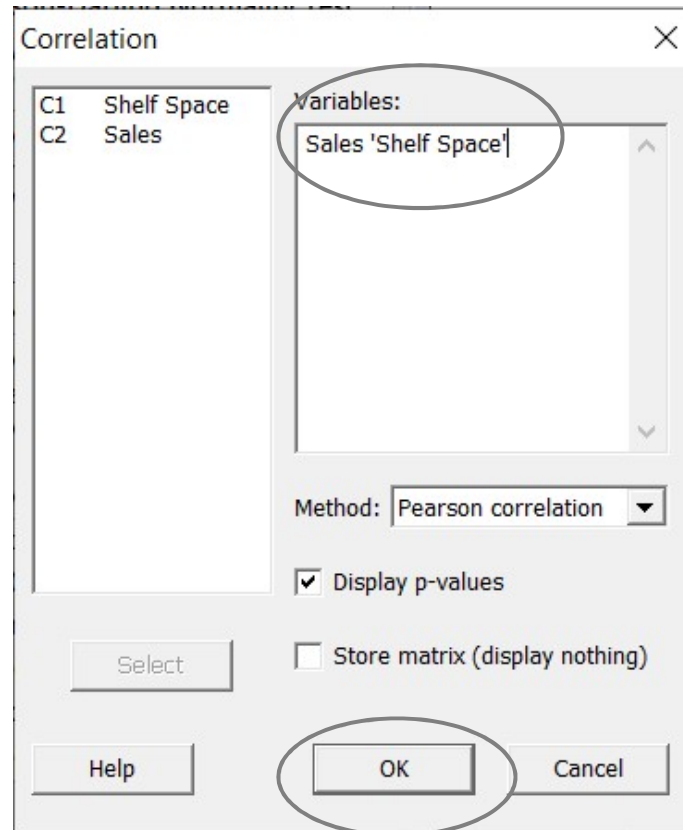
- Practical and Graphical:
  - Practical questions about the data?
  - Plot the data using different techniques

*Graph > Scatter Plot*



# Example: Cereal Sales

Tool Bar Menu > Stat > Basic Statistics > Correlation



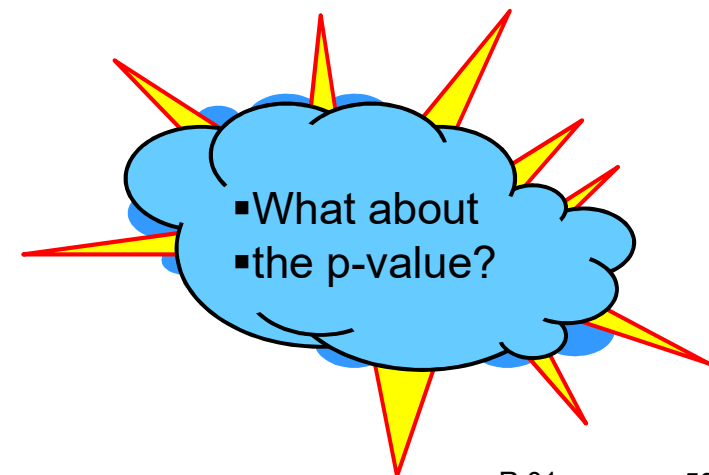
# Example: Cereal Sales

Correlations: Shelf Space, Sales

Pearson correlation of Shelf Space and Sales = 0.978

p-value = 0.000

- What is the Decision?
  - Pearson correlation or correlation coefficient for the sample,  $r = 0.978$
  - Does that mean ' $\rho$ ' is greater than zero? Or could it be that  $r = 0.978$  due to chance variation while ' $\rho$ ' is still zero?
  - Answer this question using table next page



## Example: Cereal Sales

- What is the statistical interpretation?
  - p-value (0.000) <  $\alpha$ -risk (0.01): reject the null hypothesis
  - Infer  $H_a$ : sufficient evidence that there is a correlation between sales \$ and shelf space

# Anexas Consultancy Services

Regression



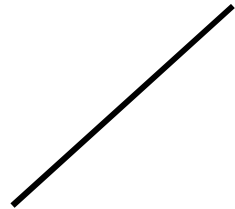
# Correlation and Regression

- Correlation tells how much linear association exists between two variables
- The Regression process creates a line that best resembles the relationship between process input and output and also provides an equation describing the nature of relationship.

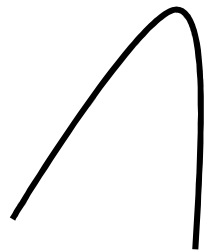
## Different types of Mathematical Models

- The regression process can fit several different types of line, since the linear relationship won't be applicable to all situations. The alternatives are

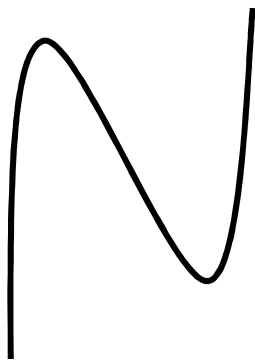
# Different types of Mathematical Models



- ❖ **Linear:** A simple, common relationship that has the simplest mathematical model:  $Y = m(x) + c$
- ❖ Where  $m$  and  $c$  are constants



- ❖ **Quadratic:** A more complex mathematical model that includes  $X^2$  term. This can be used to model process relationships that rise and then fall again.



- ❖ **Cubic:** A rarer situation, where the process relationship rises, falls, then rises again (or vice versa).

# Correlation and Regression

- Correlation tells how much linear association exists between two variables
- Regression provides an equation describing the nature of relationship

Correlations: Shelf Space, Sales

Pearson correlation of Shelf Space and Sales = 0.978

p-value = 0.000

Regression Analysis: Sales versus Shelf Space

The regression equation is  $\text{Sales} = -4711 + 10.1 \text{ Shelf Space}$

# Regression Terminology

- Response Variable
  - This is the uncontrolled variable - also known as dependent variable, output variable or Y variable
- Regressor Variable
  - Response depends on these variables - also known as independent variables, input variables, or X variables
- Noise Variable
  - Input variables (X) that are not controlled in the experiment
- Regression Equation
  - Equation that describes the relationship between independent variables and dependent variable
- Residuals
  - Difference between predicted response values and observed response values

# Regression Objectives

- Determination of a Model
  - Explore the existence of relationship
- Prediction
  - Describe the nature of relationship using an equation and use the equation for prediction
- Estimation
  - To assess the accuracy of prediction achieved by the regression equation
- Determination of KPIV
  - Screen variables and determine which variable has the biggest impact on the response variable

# Types of Regression

## Simple Linear Regression

Single regressor (x) variable such as  $x_1$  and model linear with respect to coefficients

Example 1:  $y = a_0 + a_1x + \text{error}$

Example 2:  $y = a_0 + a_1x + a_2 x^2 + a_3 x^3 + \text{error}$

Note: 'Linear' refers to the coefficients  $a_0, a_1, a_2$ , etc. It implies that each term containing a coefficient is added to the model. In example 2, the relationship between x and y are cubic polynomial in nature, but the model is linear with respect to the coefficients.

# Types of Regression

## Multiple Linear Regression

Multiple regressor (x) variables such as  $x_1, x_2, x_3$  and model linear with respect to coefficients

Example:  $y = a_0 + a_1 x_1 + a_2 x_2 + a_3 x_3 + \text{error}$

## Simple Non-Linear Regression

Single regressor (x) variable such as x and model non-linear with respect to coefficients

Example:  $y = a_0 + a_1 (1 - e^{-a_2 x}) + \text{error}$

## Multiple Non-Linear Regression

Multiple regressor (x) variables such as  $x_1, x_2, x_3$  and model non-linear with respect to coefficients

Example:  $y = (a_0 + a_1 x_1) / a_2 x_2 + a_3 x_3 + \text{error}$

# Anexas Consultancy Services

## Simple Linear Regression



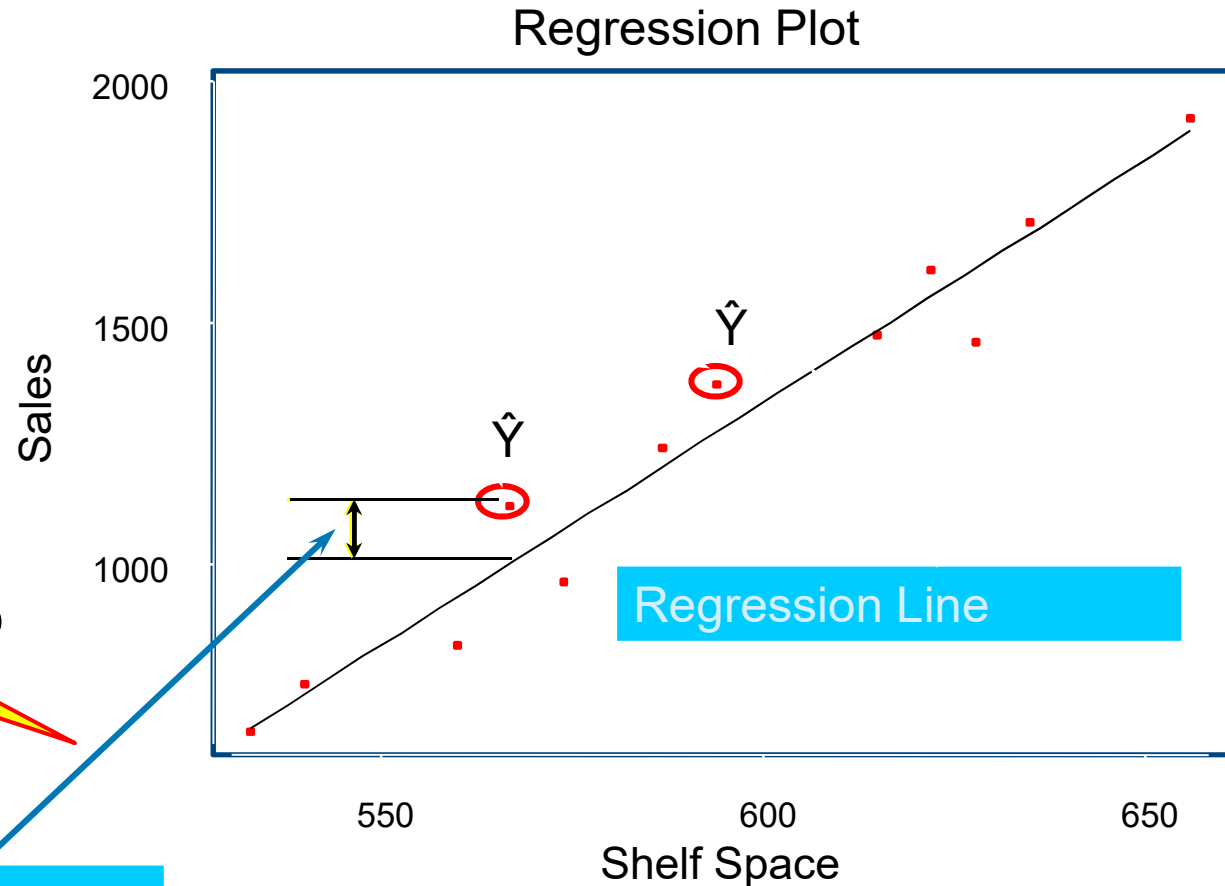
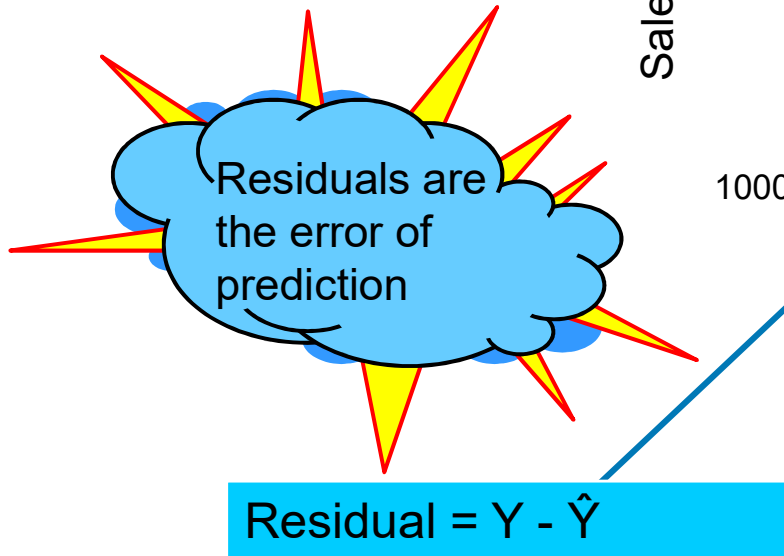
# Simple Linear Regression

- Use one independent variable ( $x$ ) to explain the variation in dependent variable ( $y$ )
  - Example 1: use shelf space to explain variation sales \$
  - Example 2: amount of fertilizer applied to explain the yield of crop
- Method of Least Squares
  - Use the 'Method of Least Squares' to find the best fitting regression line

# Method of Least Squares

Objective:

Find a line that will minimize sum of squares of residuals



# Business Process Example: Cereal Sales

A market research analyst for a certain brand of cereal is interested in predicting the sales generated from information on shelf space used to display the cereal. As a result she conducted a study and collected data from 12 different stores selling this brand of cereal

Shelf Space, Sq in	Sales, \$
574	960
635	1779
533	651
560	831
628	1460
615	1370
540	851
587	1220
656	1889
594	1370
622	1609
567	1120

- The data contains sales \$ generated for a certain month and the shelf space dedicated to the product
- How will we create a simple linear regression model for the two variables?
- Predict the sales \$ using the regression equation when shelf space is 615 sq. in.

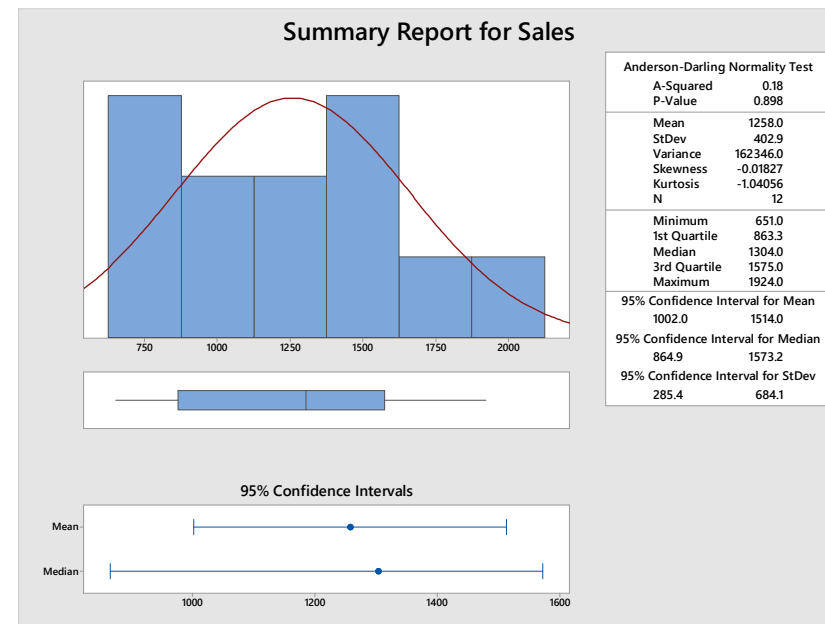
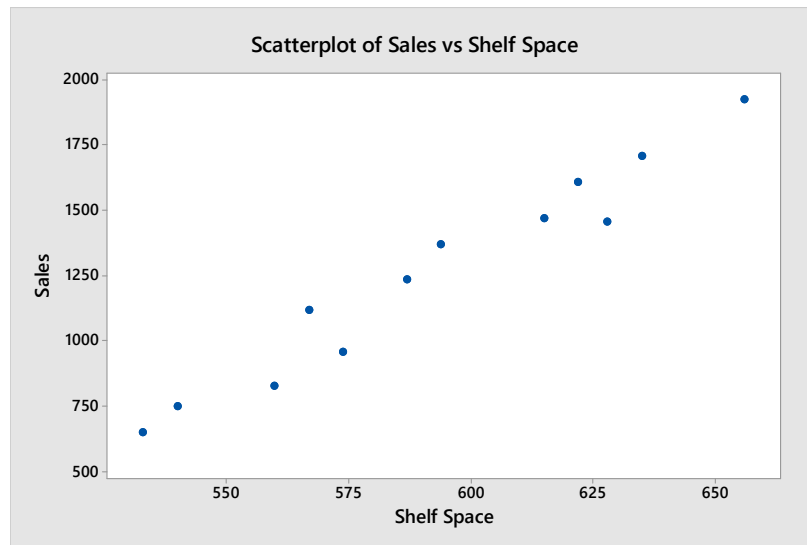
Data in Sales.mtw

# Example: Cereal Sales

Tool Bar Menu > Stat > Basic Statistics > Graphical Summery

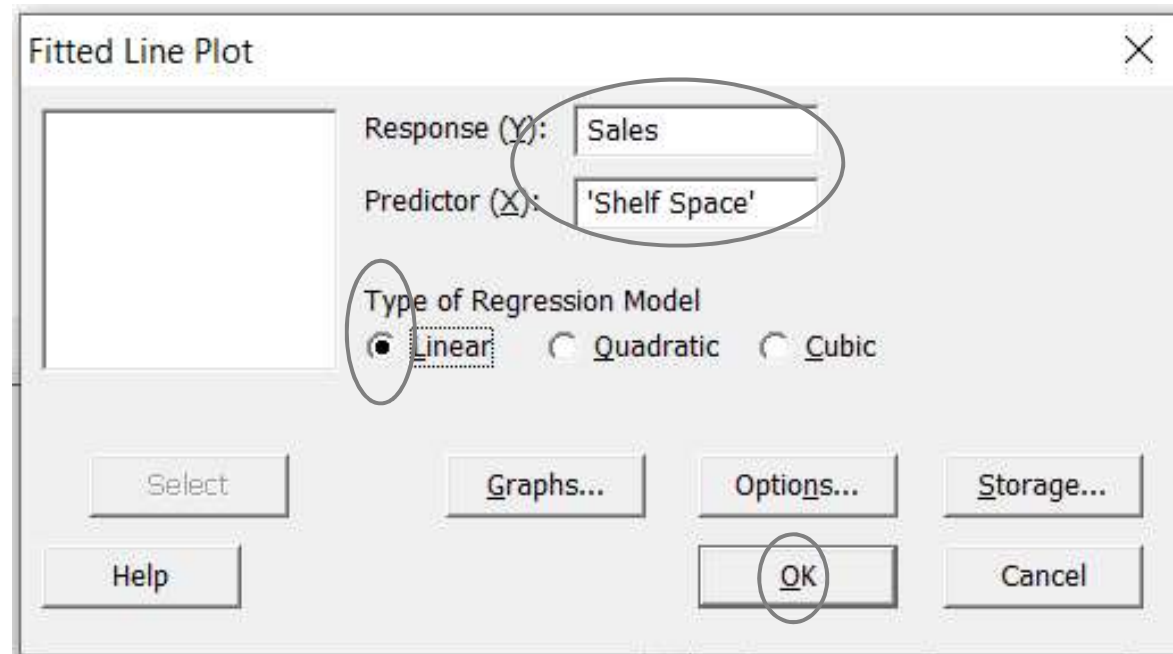
- Practical and Graphical:
  - Practical questions about the data?
  - Plot the data using different techniques

Graph > Scatter Plot



# Example: Cereal Sales

- Tool Bar Menu > Stat > Regression > Fitted Line Plot



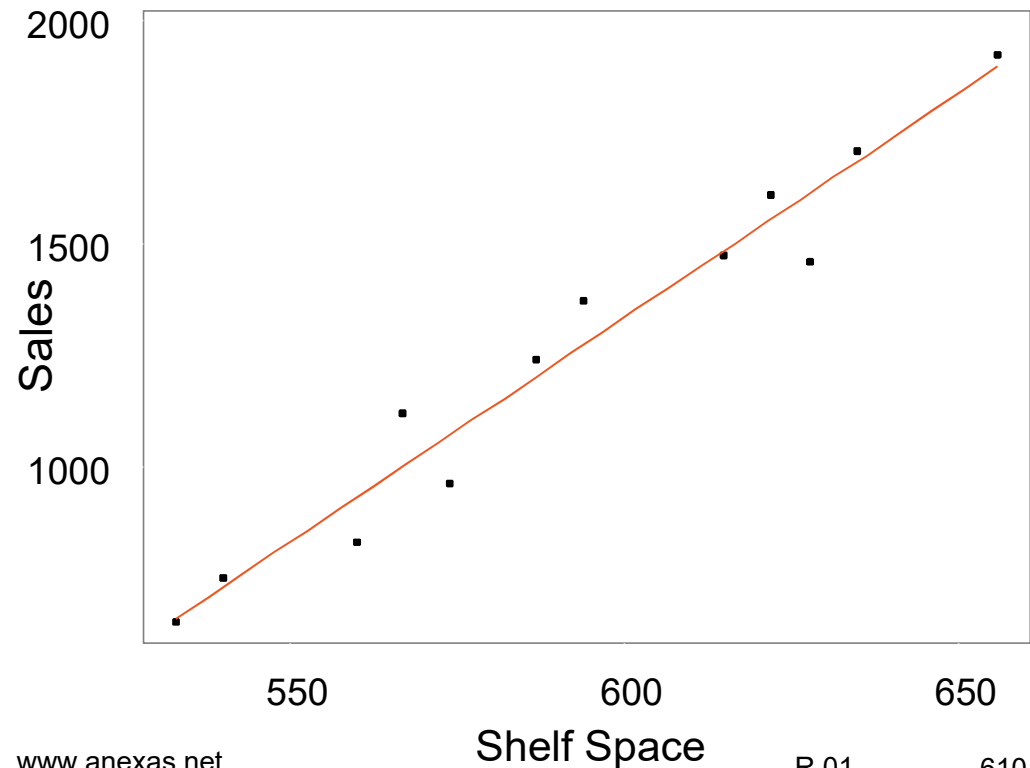
# Example: Cereal Sales

The regression equation is  
 $\text{Sales} = -4710.51 + 10.0720 \text{ Shelf Space}$   
 $S = 87.2641$   $R\text{-Sq} = 95.7\%$   $R\text{-Sq}(\text{adj}) = 95.3\%$

- Also from previous, correlation coefficient,  $r = 0.978$
- What do these numbers mean?

## Regression Plot

$\text{Sales} = -4710.51 + 10.0720 \text{ Shelf Space}$   
 $S = 87.2641$   $R\text{-Sq} = 95.7\%$   $R\text{-Sq}(\text{adj}) = 95.3\%$



# Example: Cereal Sales

## Session Output from Minitab

Regression Analysis: Sales versus Shelf Space

The regression equation is

$$\text{Sales} = -4710.51 + 10.0720 \text{ Shelf Space}$$

S = 87.2641    **R-Sq = 95.7 %**    R-Sq(adj) = 95.3 %

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	1	1709656	1709656	224.511	<b>0.000</b>
Error	10	76150	7615		
Total	11	1785806			

Regression is significant



# What About R-squared?

- R-squared is a measure describing the quality of regression
- Measures the proportion of variation that is explained by the regression model
- $R^2 = \frac{SS_{\text{regression}}}{SS_{\text{total}}} = \frac{(SS_{\text{total}} - SS_{\text{error}})}{SS_{\text{total}}} = 1 - [SS_{\text{error}} / SS_{\text{total}}]$

Source	DF	SS	MS	F	P
Regression	1	1709656	1709656	224.511	0.000
Error	10	76150	7615		
Total	11	1785806			

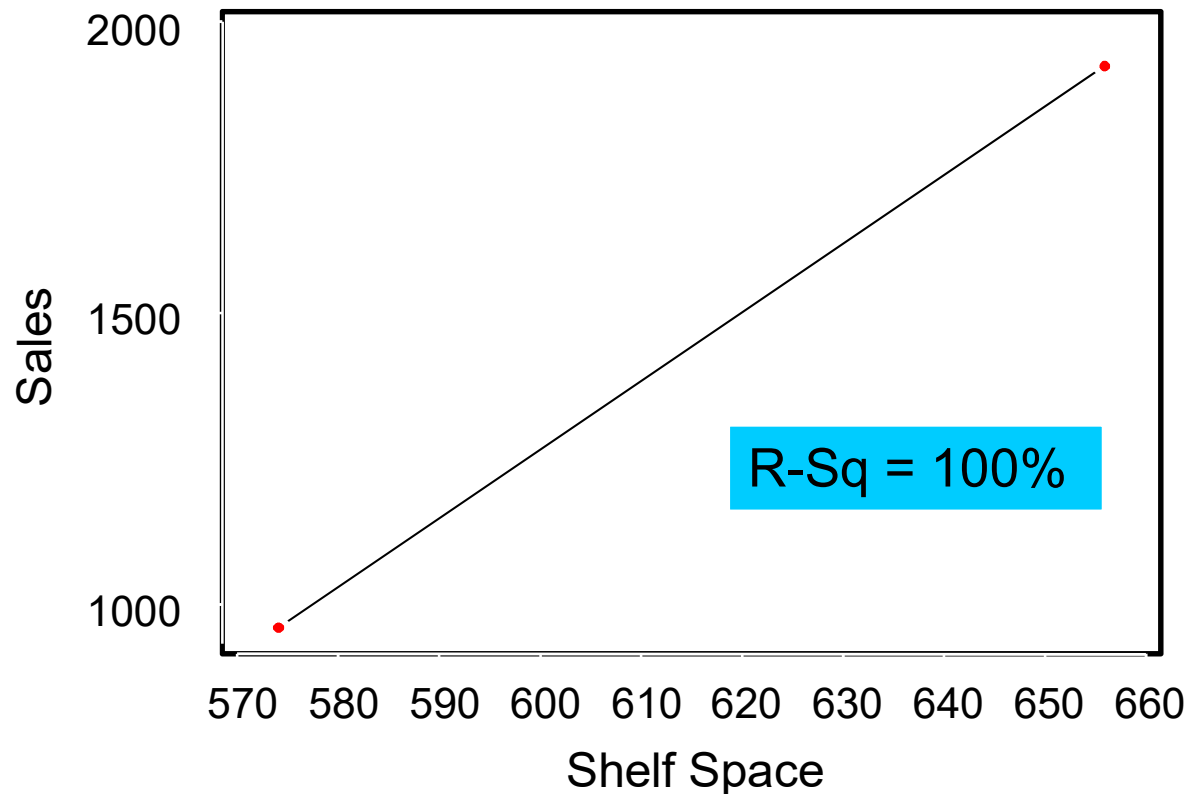
$$R^2 = 1709656 / 1785806 = 95.74\%$$

95.7% of variation in sales can be explained by variation in shelf space



# What About R-Sq?

- What is the R-squared on a regression with two data points?
- Does that mean a model with two data points is better?



# Example: Cereal Sales

Tool Bar Menu > Stat > Regression > Fitted Line Plot

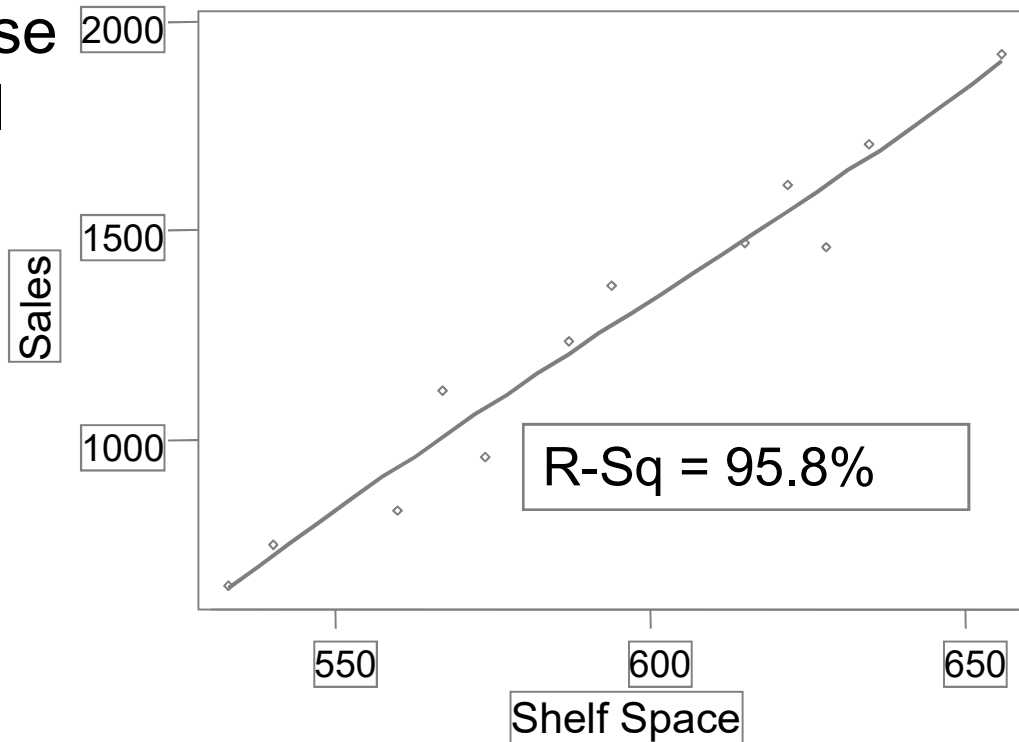
## Regression Plot

$$\text{Sales} = -32708.1 + 151.576 \text{ Shelf Space}$$

$$- 0.237788 \text{ Shelf Space}^{**2} + 0.0001329 \text{ Shelf Space}^{**3}$$

S = 97.2444    R-Sq = 95.8 %    R-Sq(adj) = 94.2 %

- What is the R-squared if we choose a 'cubic' polynomial regression?



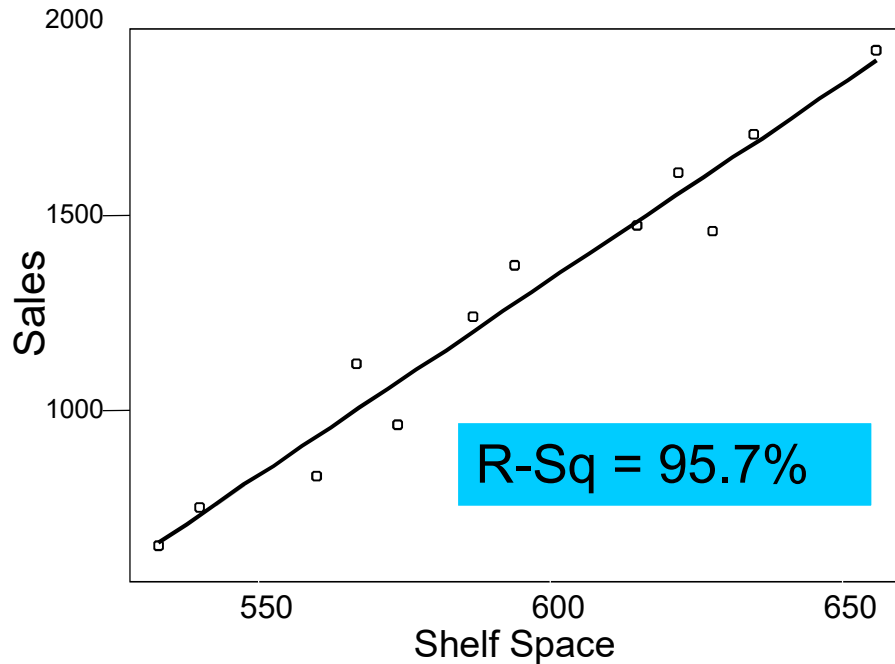
# Example: Cereal Sales

- Which model is better? Linear or Cubic model?

Regression Plot

$$\text{Sales} = -4710.51 + 10.0720 \text{ Shelf Space}$$

S = 87.2641    R-Sq = 95.7 %    R-Sq(adj) = 95.3 %

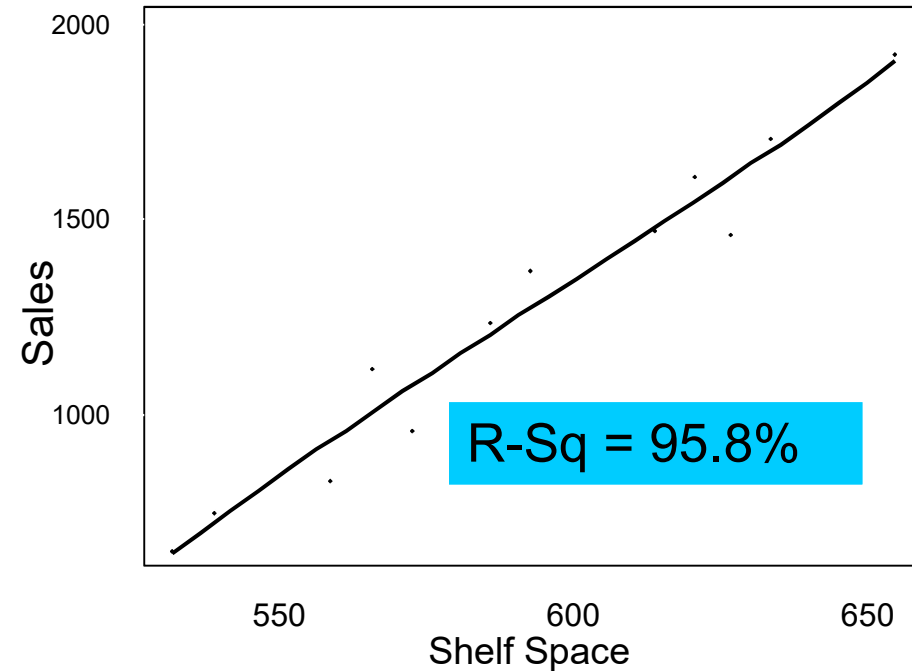


Regression Plot

$$\text{Sales} = -32708.1 + 151.576 \text{ Shelf Space}$$

$$- 0.237788 \text{ Shelf Space}^{**2} + 0.0001329 \text{ Shelf Space}^{**3}$$

S = 97.2444    R-Sq = 95.8 %    R-Sq(adj) = 94.2 %



- R-Squared gets bigger as we add more and more terms!
- So should we keep adding terms?

# What is R-Sq (adj)?

- More realistic measurement and is a modified measure of R-squared
- Takes into account of number of terms in the model and number of data points
- $$\text{Adj } R^2 = 1 - \frac{[\text{SS}_{\text{error}} / (n-p)]}{[\text{SS}_{\text{total}} / (n-1)]} = 1 - \frac{n-1}{n-p} (1 - R^2)$$
where  $n$  = number of data points and  $p$  = number of terms in the model
- Becomes smaller when added terms provide little new information and as the number of model terms gets closer to the total sample size

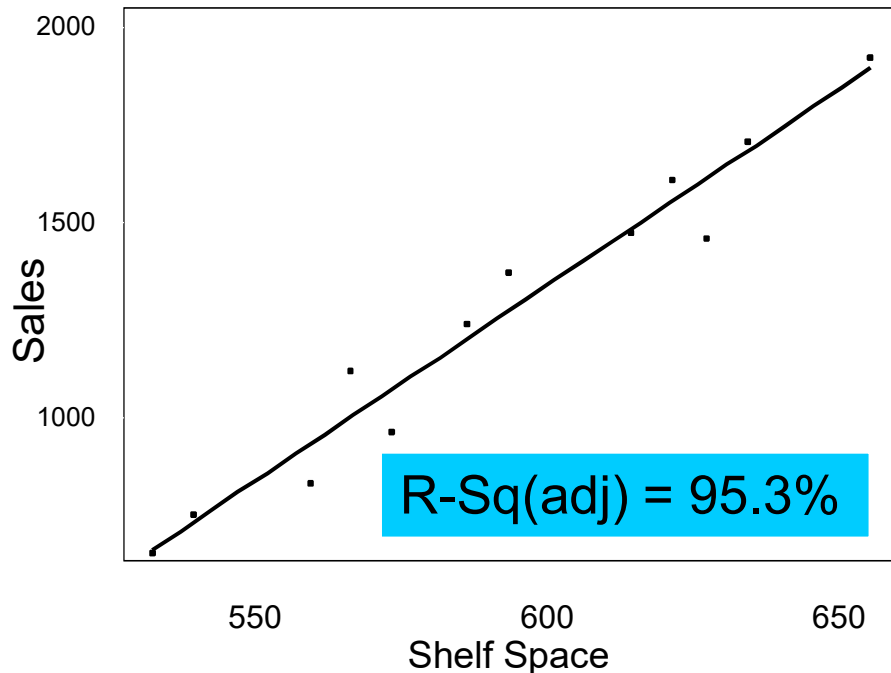
# Example: Cereal Sales

- Which model is better? Linear or Cubic model?

Regression Plot

$$\text{Sales} = -4710.51 + 10.0720 \text{ Shelf Space}$$

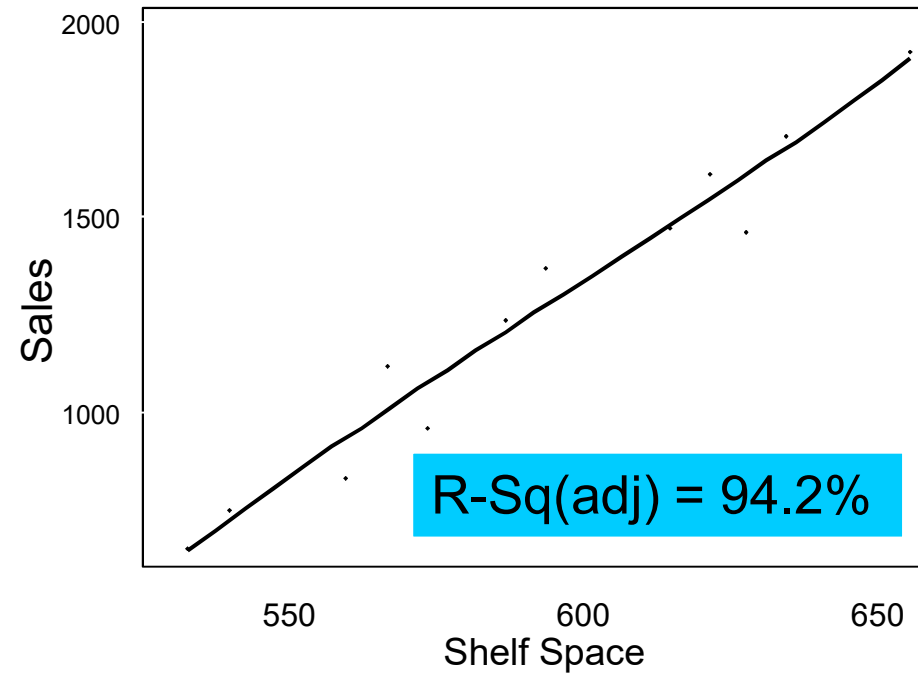
S = 87.2641    R-Sq = 95.7 %    R-Sq(adj) = 95.3 %



Regression Plot

$$\begin{aligned} \text{Sales} = & -32708.1 + 151.576 \text{ Shelf Space} \\ & - 0.237788 \text{ Shelf Space}^{**2} + 0.0001329 \text{ Shelf Space}^{**3} \end{aligned}$$

S = 97.2444    R-Sq = 95.8 %    R-Sq(adj) = 94.2 %



Linear model is better since the additional terms in cubic model did not add value. How about a quadratic model?

## Example: Cereal Sales

The regression equation is

$$\text{Sales} = -4710.51 + 10.0720 \text{ Shelf Space}$$

$$S = 87.2641 \quad R\text{-Sq} = 95.7 \% \quad R\text{-Sq}(\text{adj}) = 95.3\%$$

Predict 'Sales' for 615 'Shelf Space' in the above equation

- Substitute the value for 'Shelf Space' in the above equation
- $\text{Sales} = -4710.51 + 10.072 (615) = \$1483.77$
- What about the uncertainty around this prediction?  
Is sales expected to be exactly \$1483.77?

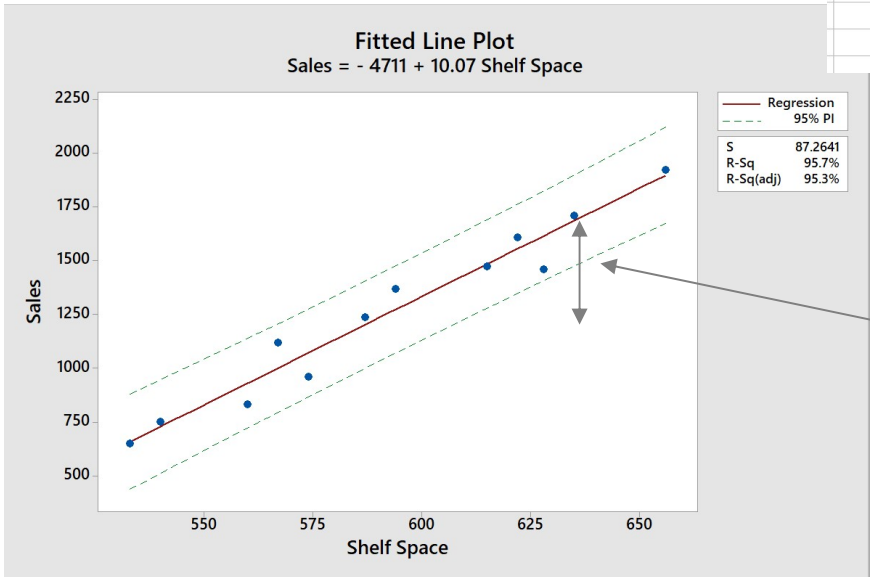
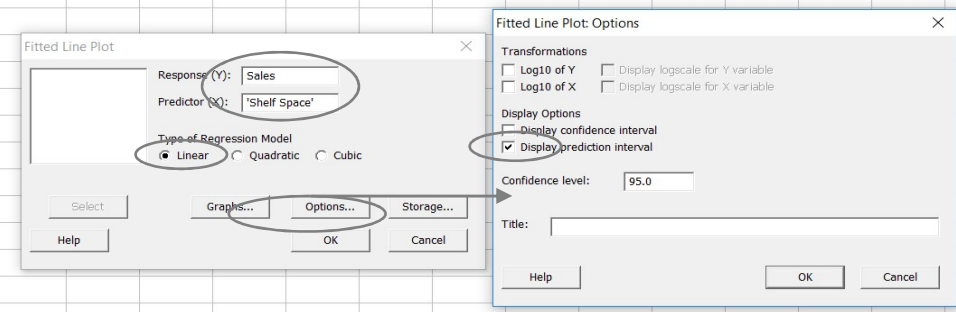
# Example: Cereal Sales

Tool Bar Menu > Stat > Regression > Fitted Line Plot

## Fitted Line Plot

$$\text{Sales} = -4711 + 10.0 \text{ Shelf Space}$$

S = 87.2641    R-Sq = 95.7 %    R-Sq(adj) = 95.3 %

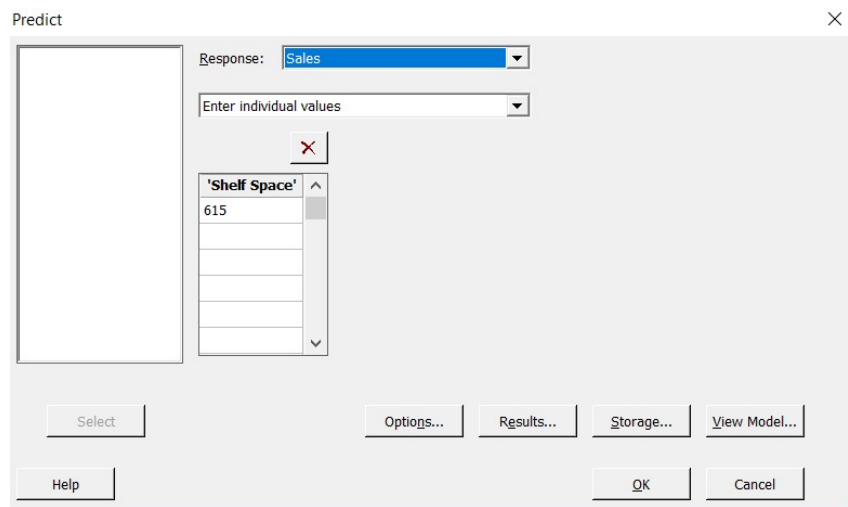


Prediction uncertainty for individual values

- Regression
- - 95% PI

# Example: Cereal Sales

Tool Bar Menu > Stat > Regression > Regression > Predict



- We are 95% certain that sales will be between \$1278.6 and \$1688.9 when shelf space is 615 sq. in.
- In the example, the actual value was \$1370

## Prediction for Sales

Regression Equation

$$\text{Sales} = -4711 + 10.072 \text{ Shelf Space}$$

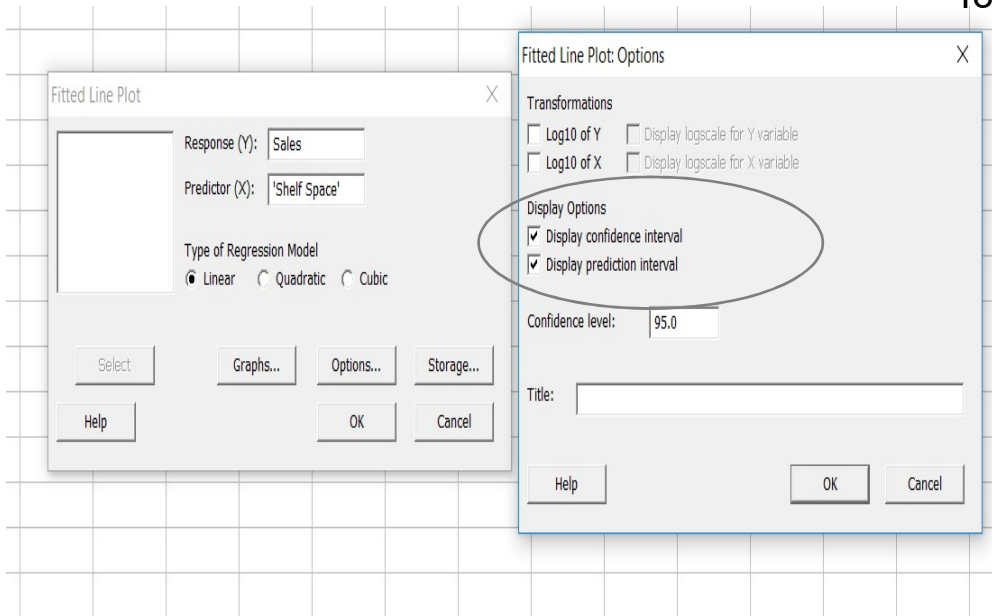
Variable	Setting
Shelf Space	615

Fit	SE Fit	95% CI	95% PI
1483.78	29.3538	(1418.38, 1549.19)	(1278.64, 1688.92)



# Example: Cereal Sales

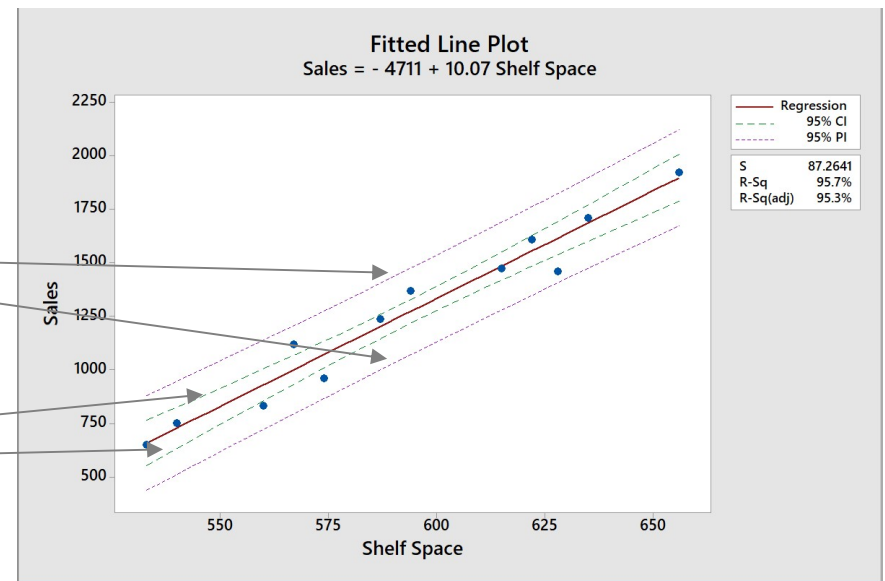
Tool Bar Menu > Stat > Regression > Fitted Line Plot



## Fitted Line Plot

$$\text{Sales} = -4711 + 10.07 \text{ Shelf Space}$$

S = 87.2641    R-Sq = 95.7 %    R-Sq(adj) = 95.3 %



95% prediction interval a single response

95% confidence interval for the mean response

# Assumptions for Regression

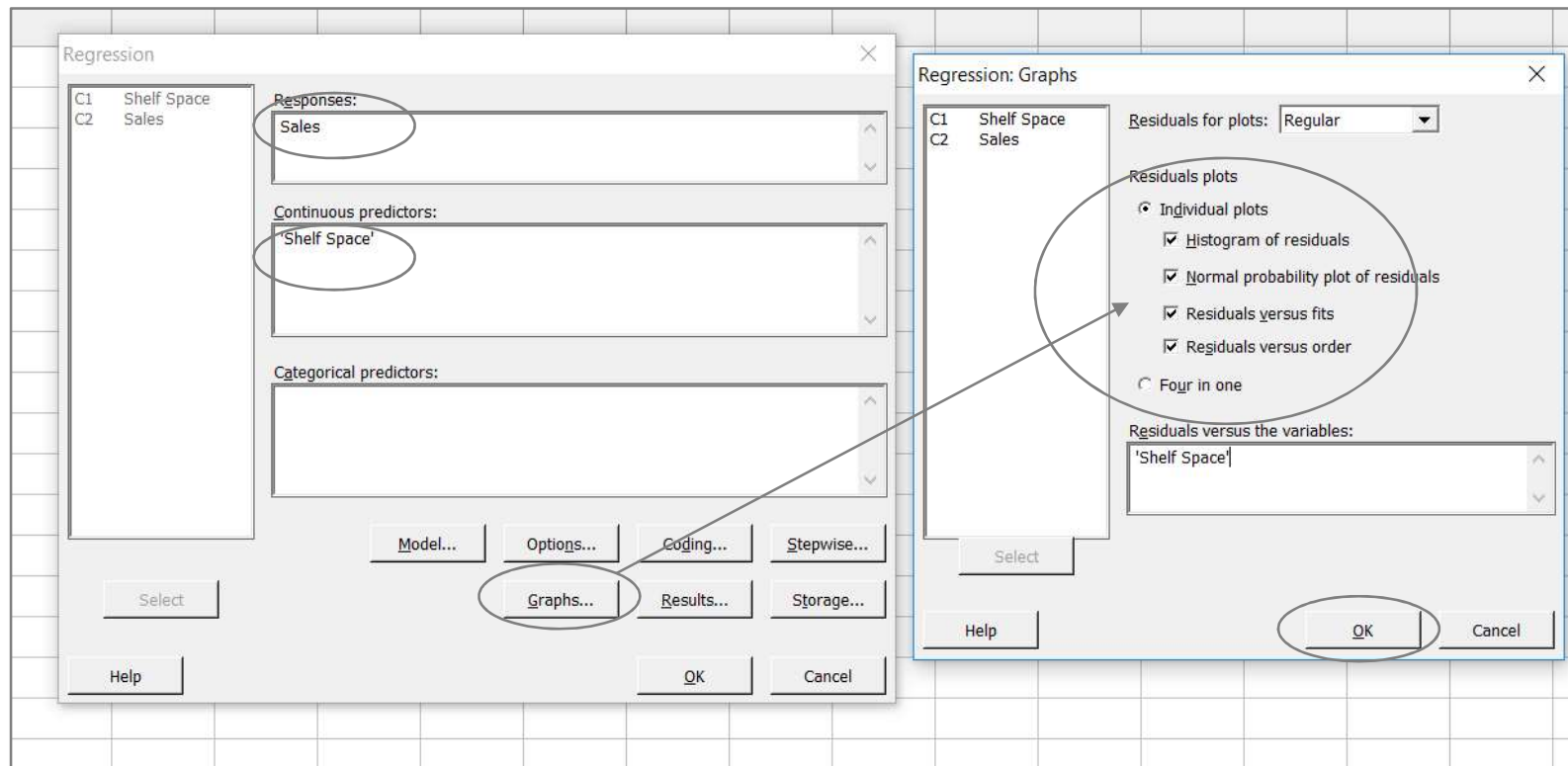
Tool Bar Menu > Stat > Regression > Regression

- To use the results of regression, assumptions about residuals must be satisfied
- What are the assumptions about residuals?
  - Residuals are normally distributed with mean of zero
  - Residuals show no pattern (random)
  - Residuals have constant variance (homogeneous variance or no heteroscedasticity)
  - Residuals are independent of the values of regressor (x) variables
  - Residuals are independent of each other

# Assumptions for Regression

Residuals are error in the fit of regression line

- Difference between the observed value of response variable and fitted value

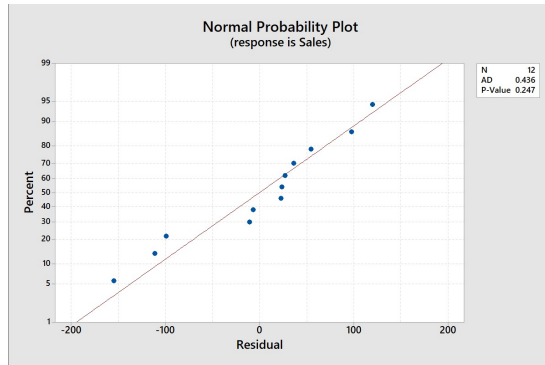


# Fit Regression Model

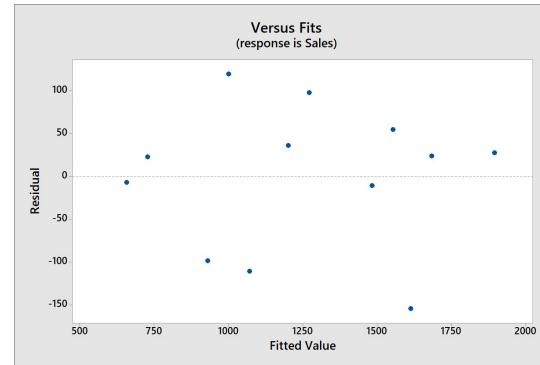
## Graphs - Four-in-One Residual Plot

- The four-in-one residual plot displays four different residual plots together in one graph window. This layout can be useful for comparing the plots to determine whether your model meets the assumptions of the analysis. The residual plots in the graph include:
- Histogram - indicates whether the data are skewed or outliers exist in the data
- Normal probability plot - indicates whether the data are normally distributed, other variables are influencing the response, or outliers exist in the data
- Residuals versus fitted values - indicates whether the variance is constant, a nonlinear relationship exists, or outliers exist in the data
- Residuals versus order of the data - indicates whether there are systematic effects in the data due to time or data collection order

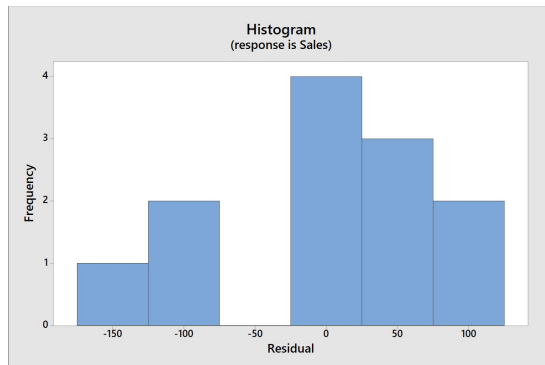
# Assumptions for Regression



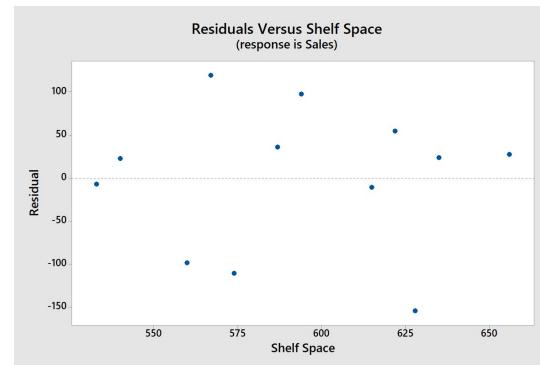
Residuals are normally distributed around mean of zero



Residuals do not exhibit heteroscedasticity (they have homogenous variance)  
Residuals are randomly distributed



Histogram of residuals resemble a normal distribution with mean of zero

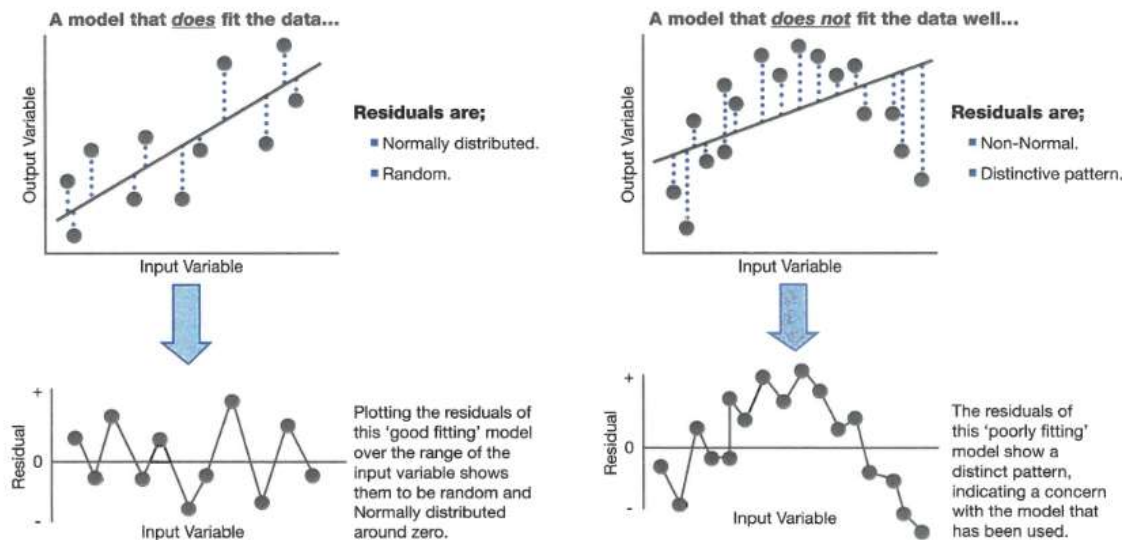


Residuals are independent of 'Shelf Space' variable

No assumptions were violated Regression results are valid

# Checking the Model – Analysis of Residuals

- Confirming that your regression model is a reasonable fit can be done visually with simple, regression, because you are able to see the fitted line plot. However this is not possible with Multiple regression since it is more than two dimensional. Instead, the residual errors between the model and the data points can be analysed in order to decide if the model is a good fit for the data. This is possible because the behavior of residuals for a good fitting model is well established as described below.



# Anexas Consultancy Services

## Multiple Regression

# Module Objectives

By the end of this module participant will be able to:

- Determine, for a given response variable, the key process input variables from a set of multiple input variables
- Perform multiple linear regression for a given set of response variable using several input variables
- Perform model diagnostics and validate assumptions
- Use regression model to predict the value of a response variable for given values of predictor variables



# Why Learn Multiple Regression?

- Explore the existence of relationship between a dependent variable and several independent variables
- Screen multiple input variables and determine which variables have the biggest impact on the response variable
- Describe the nature of relationship with an equation and use it for prediction

# What is Multiple Regression?

- Procedure of establishing relationship between a continuous type response variable and two or more independent variables
- Multiple regression equation can be used to predict a response based on values of predictor variables
- Multiple regression equation takes the form

$$Y = f (x_1, x_2, x_3, \dots)$$

# Types of Multiple Regression

## Multiple Linear Regression

Multiple regressor (x) variables such as  $x_1, x_2, x_3$  and model linear with respect to coefficients

Example1:  $y = a_0 + a_1 x_1 + a_2 x_2 + a_3 x_3 + \text{error}$

Example2:  $y = a_0 + a_1 x_1 + a_2 x_2 + a_3 x_2^2 + \text{error}$

## Multiple Non-Linear Regression

Multiple regressor (x) variables such as  $x_1, x_2, x_3$  and model non-linear with respect to coefficients

Example:  $y = (a_0 + a_1 x_1) / a_2 x_2 + a_3 x_3 + \text{error}$

This module focuses on multiple linear regression applying general least squares method

# Multicollinearity

- A condition in which two or more independent variables (x variables) are correlated (pairwise and more complex linear relationships)
- When used in multiple regression model, they contribute to redundant information
- For example, fuel economy of a truck = f (truck load, engine horse power)
- But truck load may be correlated with engine horse power
- Truck load and horse power provide some overlapping information leading to potential problems

# Problems Due to Multicollinearity

- Multicollinearity can cause severe problems
  - calculations of coefficients and standard errors are affected (unstable, inflated variances)
  - difficulty in assessing any particular variable's effect
  - opposite signs (from what is expected) in the estimated parameters
  - if two input variables  $x_1$  and  $x_2$  are highly correlated, then p-value for both might be high

# Detecting Multicollinearity

- High values of pairwise correlation (generally  $> 0.8$ ) provide warnings of potential multicollinearity problems
- If the above two variables are strongly correlated, one of them should be removed from regression model

# Variance Inflation Factor

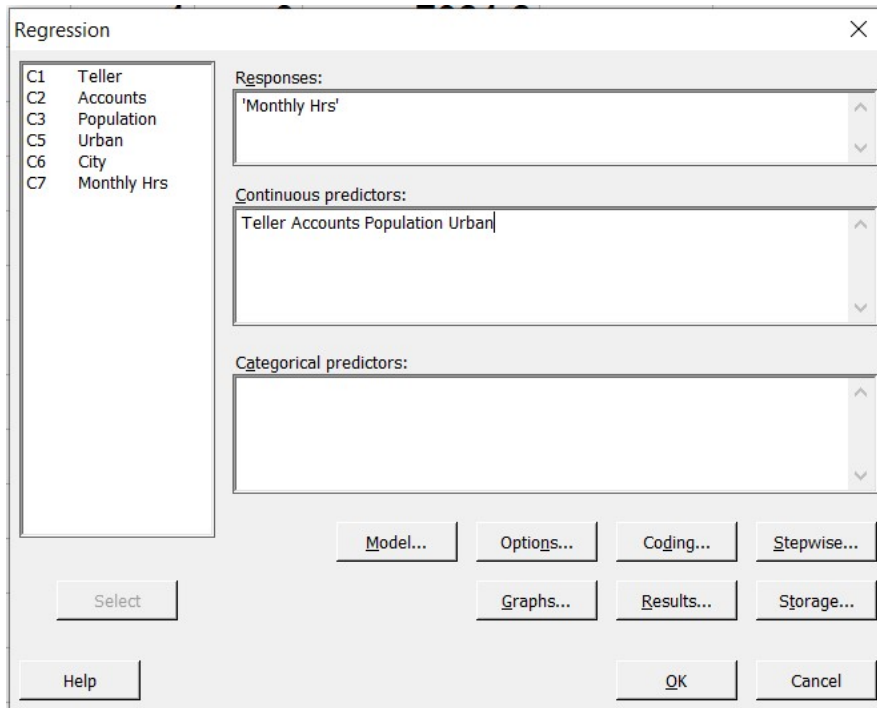
- A metric, called variance inflation factor (VIF) calculates the degree of multicollinearity

$$\text{VIF} = \frac{1}{1 - R_i^2}$$

- $R_i^2$  is the  $R^2$  value obtained when  $X_i$  is regressed against other  $X$
- A large VIF implies that at least one variable is redundant
- $\text{VIF} > 10$ : high degree of multicollinearity - cause for serious concern ( $R_i^2 > .9$ )
- $\text{VIF} > 5$ : moderate degree of multicollinearity ( $0.8 < R_i^2 < 0.9$ )
- Guideline: Ensure that  $\text{VIF} < 5$  when possible

# Calculating VIF

Minitab displays VIF values in the session window through *Stat > Regression > Regression > Fit Regression Model* menu



## Regression Analysis: Monthly Hrs versus Teller, Accounts, Population, Urban

### Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	4	34308448	8577112	5345.10	0.000
Teller	1	2719647	2719647	1694.83	0.000
Accounts	1	41	41	0.03	0.876
Population	1	8948965	8948965	5576.83	0.000
Urban	1	123813	123813	77.16	0.000
Error	12	19256	1605		
Total	16	34327704			

### Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
40.0583	99.94%	99.93%	99.90%

### Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	1094	148	7.41	0.000	
Teller	0.9510	0.0231	41.17	0.000	10.59
Accounts	0.0091	0.0571	0.16	0.876	8.31
Population	0.056381	0.000755	74.68	0.000	1.04
Urban	238.5	27.2	8.78	0.000	1.95

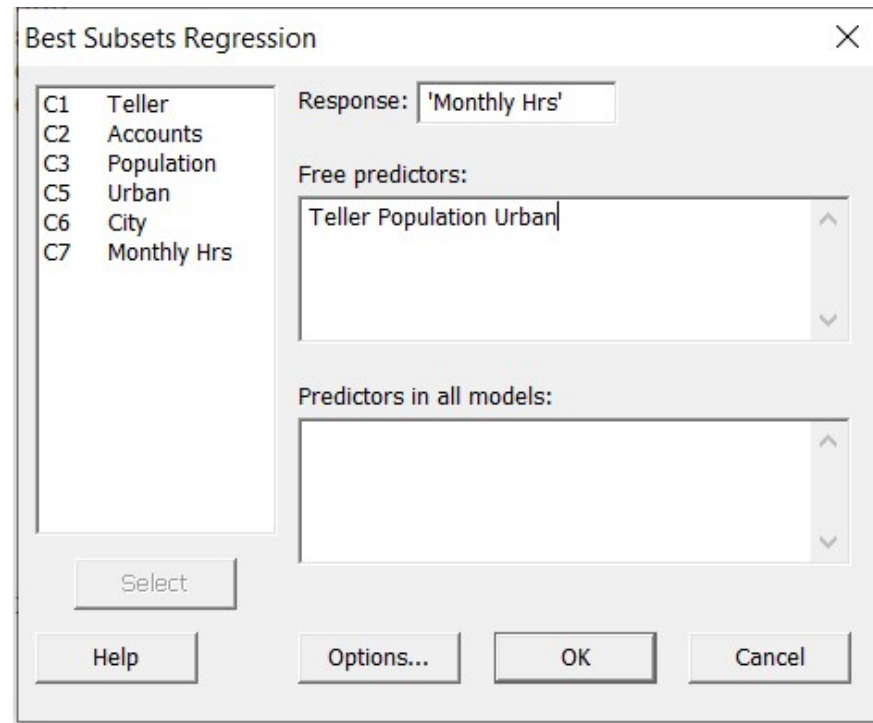


# Predictor Variable Selection

- What combination of predictor variables is best for the regression model?
- Three options in Minitab:
  - Stepwise: procedure to add and remove variables to the regression model to produce a useful subset of predictors
  - Best Subsets: procedure to give best fitting regression model that can be constructed with one variable, two variable, three variable, etc. models
  - Regression: once the best model is selected, use Regression to get more detailed diagnostics

# Best Subsets

Tool Bar Menu > Stat > Regression > Regression > Best Subsets



Use ‘Best Subsets’ technique to select a group of likely models for further analysis.

# Best Subsets Statistics

- Select the smallest subset that fulfills certain statistical criteria
- Minitab displays  $R^2$ ,  $R^2$  (adjusted), C-p, and s statistics
  - $R^2$  (large  $R^2$  is desired; use to compare models with the same number of terms)
  - adjusted  $R^2$  (large is desired; use to compare models with different number of terms)
  - s (standard deviation of error terms; small is desired)
  - Mallows' C-p statistic (small is desired; Guideline: want C-p  $\leq$  number of terms in model)

# Putting It All Together

Multiple regression objective: Establish a model with high prediction ability and minimum multicollinearity

## Multiple regression steps:

1. Remove variables contributing to multicollinearity from the predictors
2. Use remaining variables and apply Best Subsets to evaluate best predictor candidates for the model
3. Choose the best candidate and complete regression analysis
4. Perform model diagnostics to identify outliers and unusual observations
5. Analyze residuals for violation of assumptions
6. Assess predictive capability using new observations

## Business Process Example: Bank Labor Hours

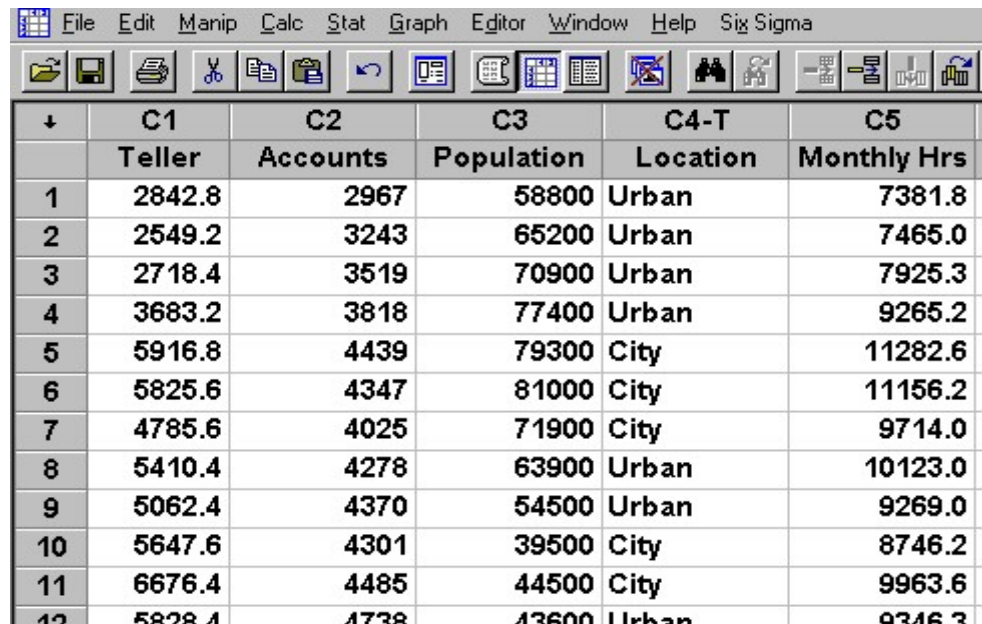
A banking institution wants to produce an empirical equation that will estimate personnel needs its branches. The following data was collected from its existing branches at various locations. The response variable (Y) for the study was monthly labor hours. The input variables were average number of daily teller transactions (x1), average count of total number of accounts (x2), location of branch (x3), population within 20 Km radius (x4). The data is recorded in the file ***Banking.mtw***.

# Business Process Example: Bank Labor Hours

- Establish a multiple regression model to predict monthly labor hours using the predictor variables
- Perform model diagnostics to detect any outliers or unusual observations
- Validate any assumptions used for creating the model

# Example: Bank Labor Hours

- **Banking.mtw**
  - Output variable: Monthly Hrs
  - Input variables: Teller, Accounts, Population, Location
  - Practical questions about the data and process?



The screenshot shows a Minitab software window with the following menu: File, Edit, Manip, Calc, Stat, Graph, Editor, Window, Help, Six Sigma. The toolbar includes icons for file operations, editing, and statistical analysis. The data table below is displayed in a grid format with columns labeled C1 through C5 and rows numbered 1 through 12.

↓	C1	C2	C3	C4-T	C5
	Teller	Accounts	Population	Location	Monthly Hrs
1	2842.8	2967	58800	Urban	7381.8
2	2549.2	3243	65200	Urban	7465.0
3	2718.4	3519	70900	Urban	7925.3
4	3683.2	3818	77400	Urban	9265.2
5	5916.8	4439	79300	City	11282.6
6	5825.6	4347	81000	City	11156.2
7	4785.6	4025	71900	City	9714.0
8	5410.4	4278	63900	Urban	10123.0
9	5062.4	4370	54500	Urban	9269.0
10	5647.6	4301	39500	City	8746.2
11	6676.4	4485	44500	City	9963.6
12	5828.4	4738	43600	Urban	9316.3

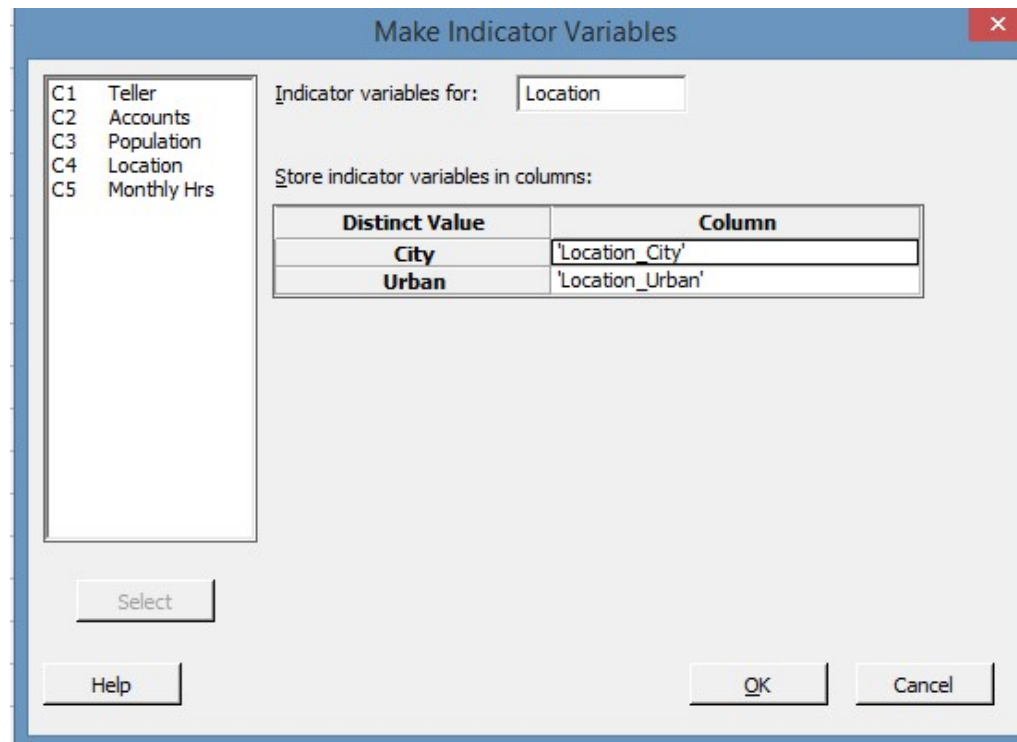
# Example: Bank Labor Hours

- ***Dealing with indicator variables (dummy variables)***
  - To use independent variables which are categorical (e.g, location, gender) in regression, first create “indicator” variables (dummy variables)
  - Indicators are simply 1’s and 0’s which are used like binary code.
  - Each level of a categorical variable is assigned a column.
  - If a row of data is associated with that level of the variable, the value in that column for that row of data will be 1. If not associated with that level, the value will be 0.



# Example: Bank Labor Hours

1. Create an indicator variable for “Location”
  - Toolbar>Calc> Make indicator Variables



# Example: Bank Labor Hours

- Result should follow the pattern below
- Order in which the columns are named is important. For alphanumeric data, the columns are created in the order specified as the value order (alphabetic, order of appearance, user defined).

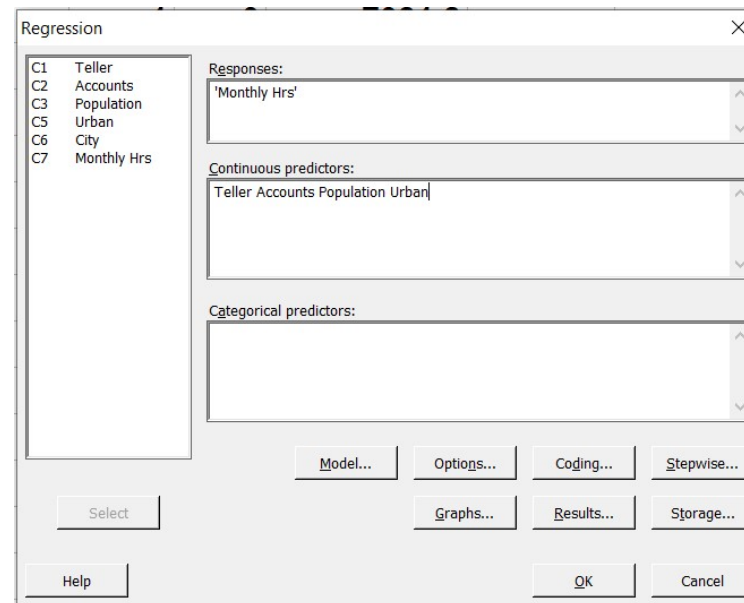
Location	Urban	City	M
Urban	1	0	
Urban	1	0	
Urban	1	0	
Urban	1	0	
City	0	1	
City	0	1	
City	0	1	
Urban	1	0	
Urban	1	0	
City	0	1	
City	0	1	
Urban	1	0	

## Example: Bank Labor Hours

- ***Create an indicator variable for “Location”***
- When categorical variables are used in the regression model, one column of indicator values is NEVER included.
- In the example, use either “Urban” or “City” since it would be redundant to use both. (If “Urban”=0, then the observation must be “City”)

# Example: Bank Labor Hours

- **Multiple regression steps:**
- Remove variables contributing to multicollinearity from the predictors
- Identify if multicollinearity is a problem by using variance inflation factors (VIF) measurements
- 1. *Stat> Regression> Regression> Fit Regression Model*
- 2. Fill in the dialog as shown below:



# Example: Bank Labor Hours

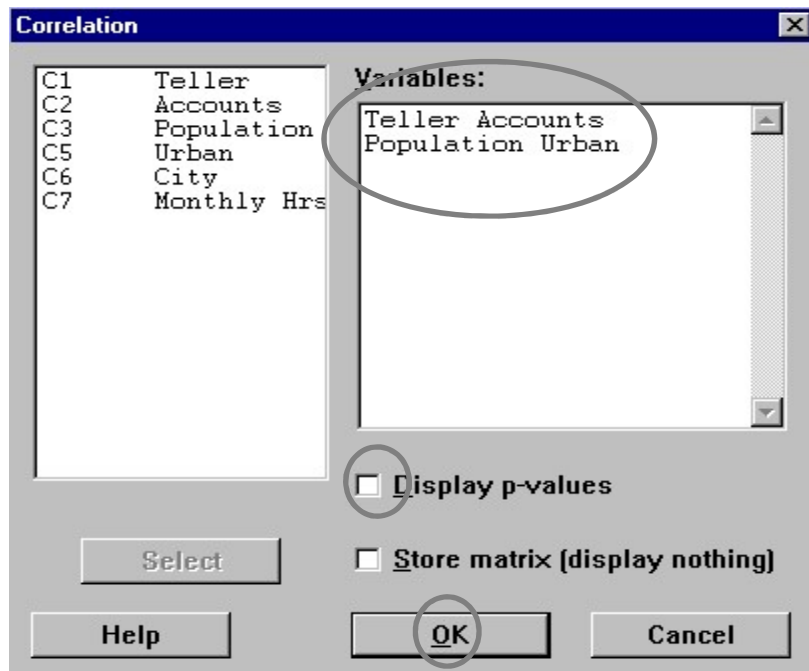
Serious multicollinearity problem!  
 Want VIF < 5

The regression equation is  
 Monthly Hrs = 1094 + 0.951 Teller + 0.0091 Accounts + 0.0564  
 Population + 238 Urban

Predictor	Coef	SE Coef	T	P	VIF
Constant	1094.3	147.8	7.41	0.000	
Teller	0.95103	0.02310	41.17	0.000	10.6
Accounts	0.00913	0.05706	0.16	0.876	8.3
Populati	0.0563806	0.0007550	74.68	0.000	1.0
Urban	238.49	27.15	8.78	0.000	1.9

# Example: Bank Labor Hours

- b. Identify pairwise correlations between x variables
  1. *Stat> Basic Statistics> Correlation..*
  2. Fill in the dialog as shown below:
  - 3: Press Ok



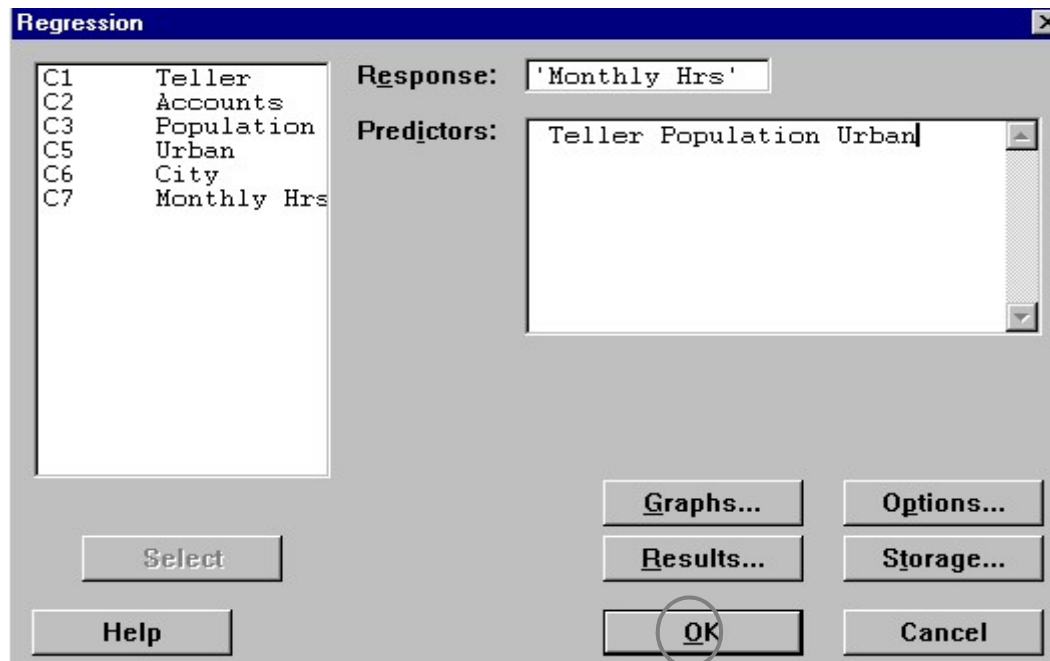
# Example: Bank Labor Hours

Next step: remove Accounts and check multicollinearity

	Teller	Accounts	Populati
Accounts	0.918		
Populati	-0.024	-0.082	
Urban	-0.573	-0.372	-0.126

# Example: Bank Labor Hours

Remove “Accounts” and repeat regression and check for VIF values





# Example: Bank Labor Hours

All VIF <5

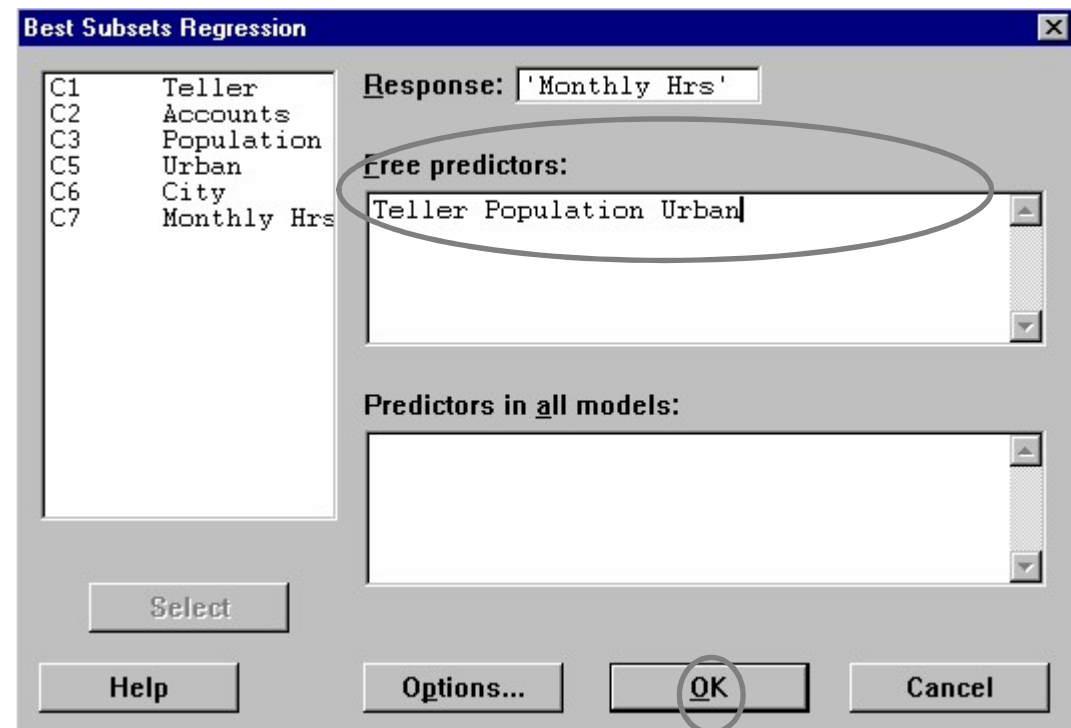
Predictor	Coef	SE Coef	T	P	VIF
Constant	1114.38	74.88	14.88	0.000	
Teller	0.954454	0.008388	113.78	0.000	1.5
Populati	0.0563707	0.0007237	77.89	0.000	1.0
Urban	240.49	23.18	10.38	0.000	1.5

# Example: Bank Labor Hours

- **Multiple regression steps:**

2. Use remaining variables and apply Best Subsets to evaluate best predictor candidates for the model

1. Stat> Regression> Best Subsets..
2. Fill in the dialog as shown below:
- 3: Press Ok



# Example: Bank Labor Hours

Next step: remove LENGTH and check VIF values

Response is Monthly Hrs

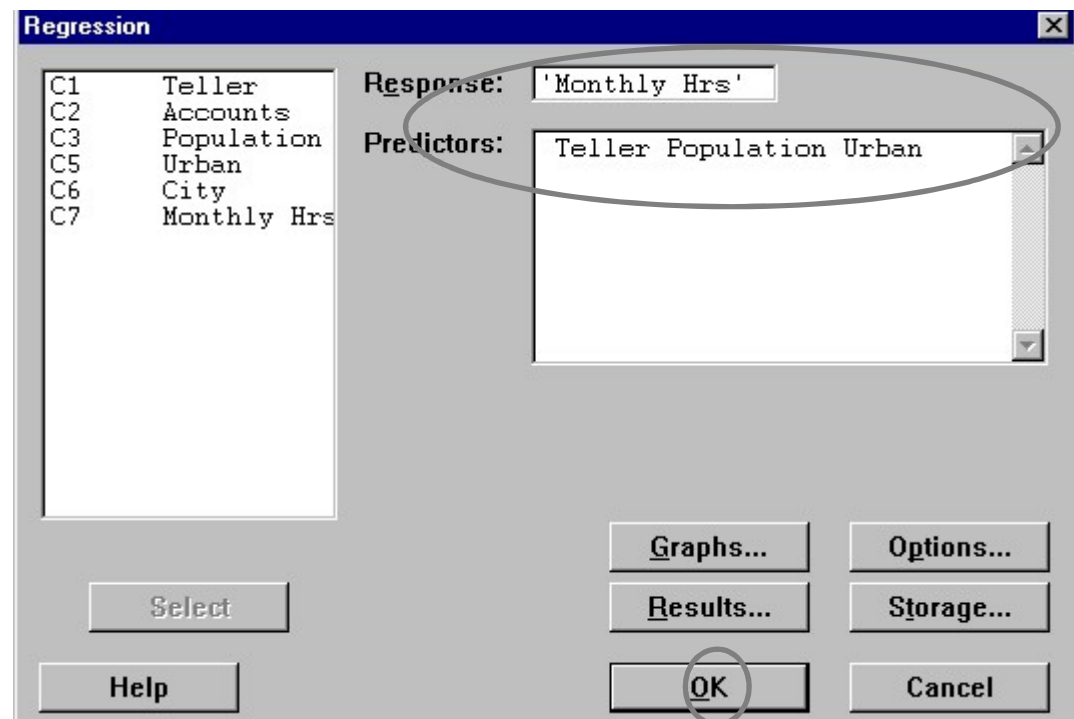
Vars	R-Sq	R-Sq (adj)	C-p	S	P o T p e u U l l r l a b e t a r i n
1	73.7	71.9	6075.6	776.22	X
1	25.7	20.8	2E+04	1303.6	X
2	99.5	99.4	109.7	113.11	X X
2	73.7	70.0	6069.0	802.90	X X
3	99.9	99.9	4.0	38.528	X X X

# Example: Bank Labor Hours

Multiple regression steps:

3. Choose the best candidate and complete regression analysis

1. Stat> Regression> Regression
2. Fill in the dialog as shown below:
3. Press Ok



# Example: Bank Labor Hours

The regression equation is  
 Monthly Hrs = 1114 + 0.954 Teller + 0.0564 Population  
 + 240 Urban

Predictor	Coef	SE Coef	T	P
Constant	1114.38	74.88	14.88	0.000
Teller	0.954454	0.008388	113.78	0.000
Populati	0.0563707	0.0007237	77.89	0.000
Urban	240.49	23.18	10.38	0.000

S = 38.53

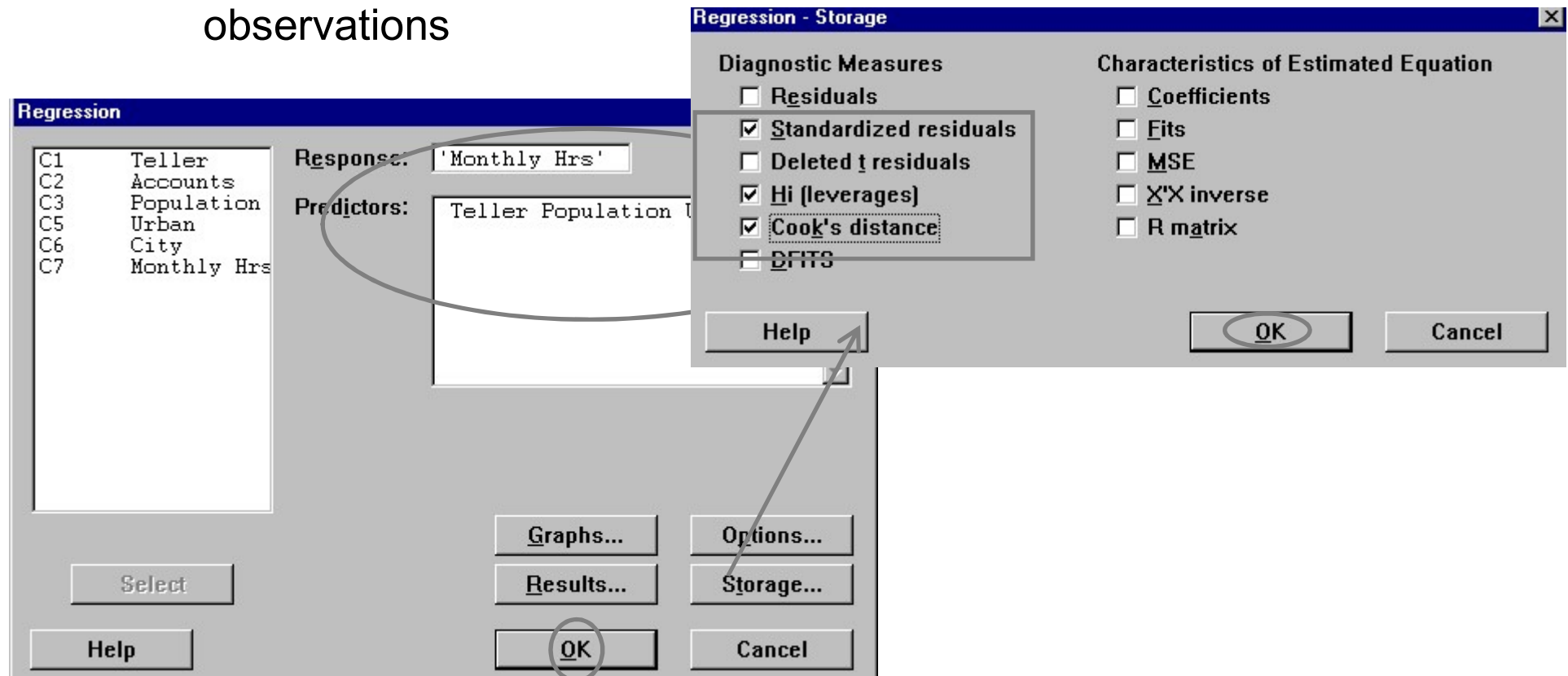
R-Sq = 99.9%

R-Sq(adj) = 99.9%

# Example: Bank Labor Hours

Multiple regression steps:

4. Perform model diagnostics to identify outliers and unusual observations



# Example: Bank Labor Hours

## Outliers/ Unusual Observations

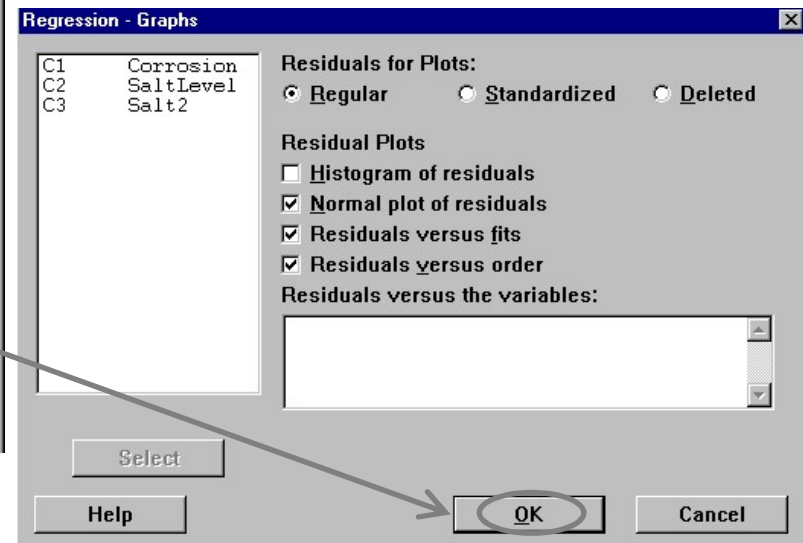
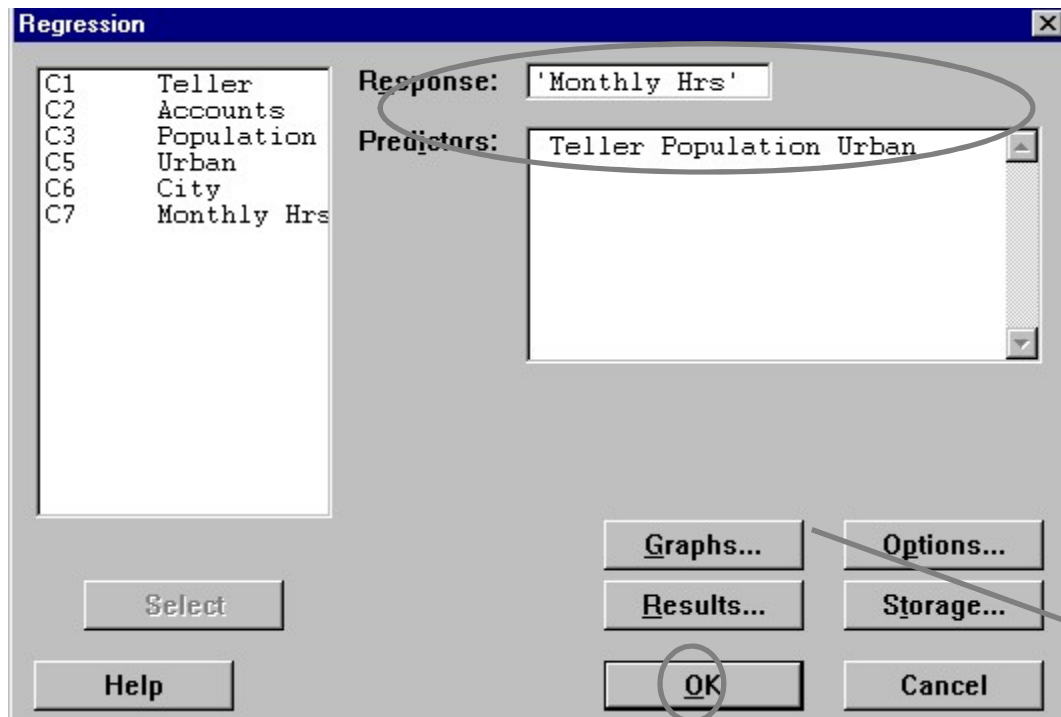
C7	C8	C9	C10
Monthly Hrs	SRES1	HI1	COOK1
7381.8	-0.03088	0.243202	0.000077
7465.0	0.05043	0.275047	0.000241
7925.3	-0.62889	0.256412	0.034096
9265.2	0.92022	0.197013	0.051941
11282.6	1.45481	0.181257	0.117139
11156.2	-2.44597	0.197162	0.367313
9714.0	-0.60984	0.197691	0.022910
10123.0	0.05897	0.158210	0.000163
9269.0	0.28358	0.152478	0.003617
8746.2	0.49623	0.398760	0.040829
9963.6	-0.98303	0.304931	0.105985
9346.3	-0.91801	0.315137	0.096947
9734.0	0.65238	0.172624	0.022199
10390.2	1.78362	0.127810	0.116546
10773.6	-0.36476	0.139764	0.005404
12673.2	0.62914	0.293429	0.041094
11776.1	-0.57219	0.389073	0.052128

**Potential trouble spots:**  
**Standardized residual > 2**  
**Leverage >  $2p/n = 0.4706$**   
**Cook's distance > 1**

**What actions should be taken with the outliers and influential observations, if any ?**

# Example: Bank Labor Hours

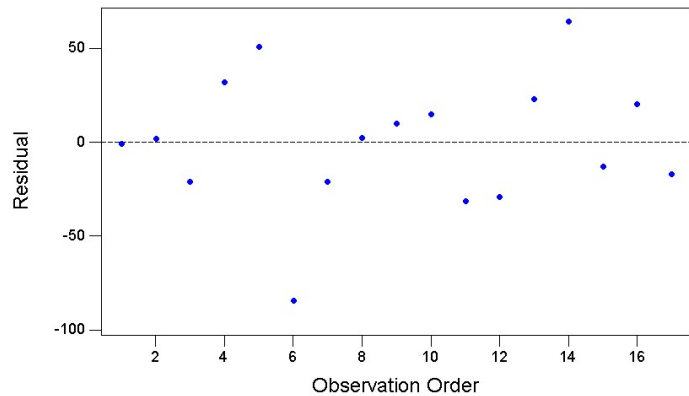
## 5. Analyze residuals for violation of assumptions



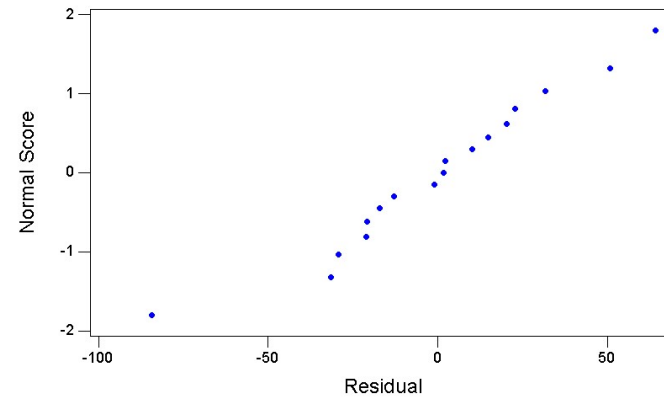


# Example: Bank Labor Hours Residual Plot Analysis

Residuals Versus the Order of the Data  
(response is Monthly)

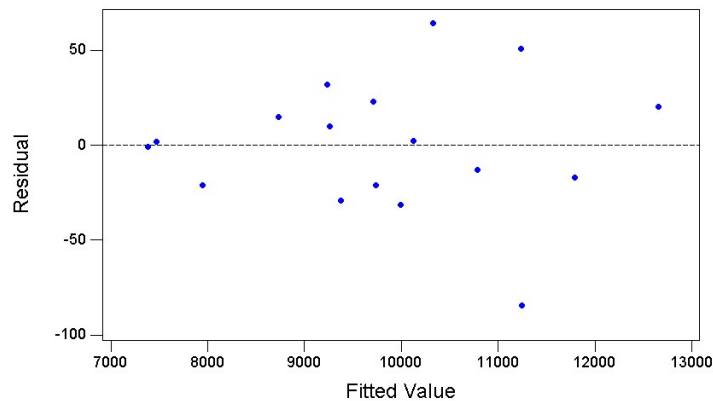


Normal Probability Plot of the Residuals  
(response is Monthly)

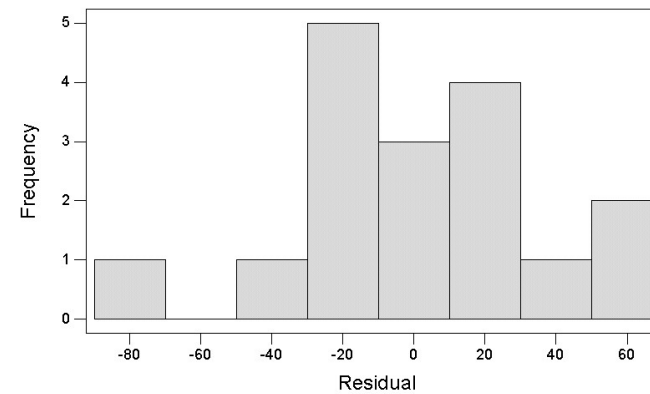


**No evidence of violation  
of assumptions**

Residuals Versus the Fitted Values  
(response is Monthly)



Histogram of the Residuals  
(response is Monthly)



# Industrial Process Example: Alumina Deposition

A chemical engineer wants to model the process of placing Alumina / Ceria on ceramic honeycomb substrate as an acid water slurry. The Key Process Output Variable (KPOV) of interest is the 'wet weight pickup' measured in grams per cubic inch of substrate volume of slurry on each ceramic substrate. Thirteen input variables affecting the response variable (wet weight pickup) were also identified.

Before performing design of experiments, engineer wants to narrow down the number of input variables affecting the response variable.

# Industrial Process Example: Alumina Deposition

Data collected as part of a passive study conducted to identify the leverage variables affecting the 'wet weight pickup' is given in (*Multiwet.mtw*). Perform a multiple regression to model the relationship between input and output variables.

# Example: Alumina Deposition

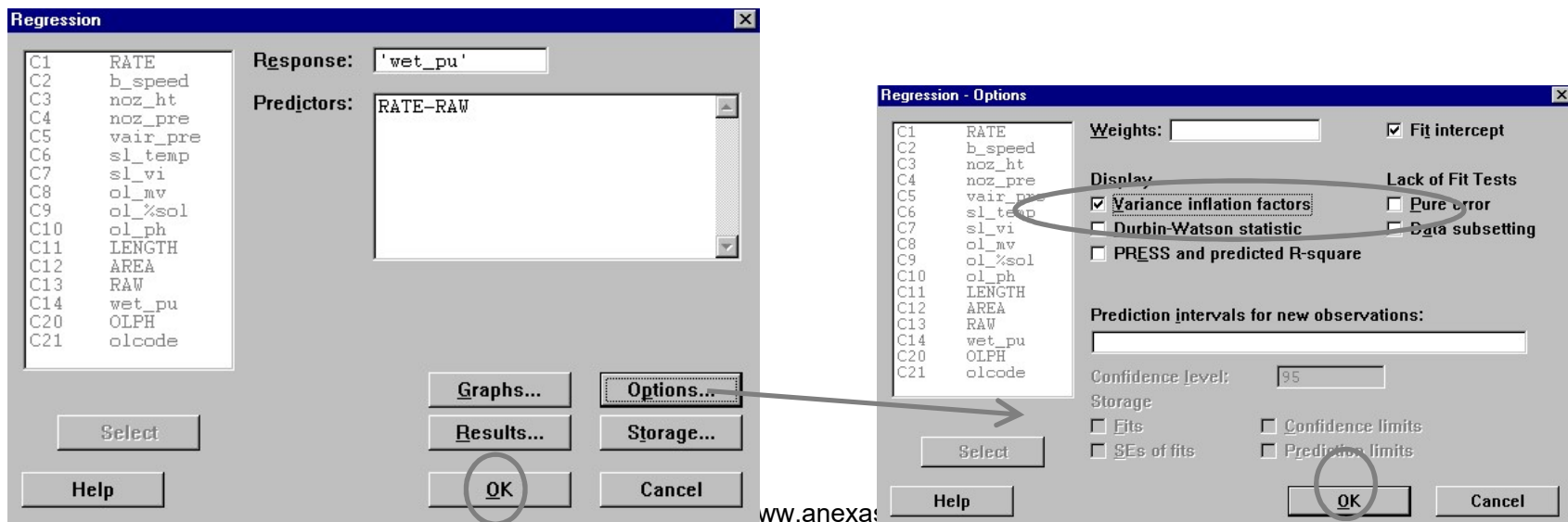
## ***Multiwet.mtw***

- Output variable: wet\_pu
- Input variables: RATE, b\_speed, noz\_ht, noz\_pre, vair\_pre, sl\_temp, sl\_vi, ol\_mv, ol\_%sol, ol\_ph, LENGTH, AREA, RAW
- Practical questions about the data and process?

# Example: Alumina Deposition

## Multiple regression steps:

1. Remove variables contributing to multicollinearity from the predictors
  - a. Identify if multicollinearity is a problem by using variance inflation factors (VIF) measurements
    1. *Stat > Regression > Regression*
    2. Fill in the dialog as shown below:



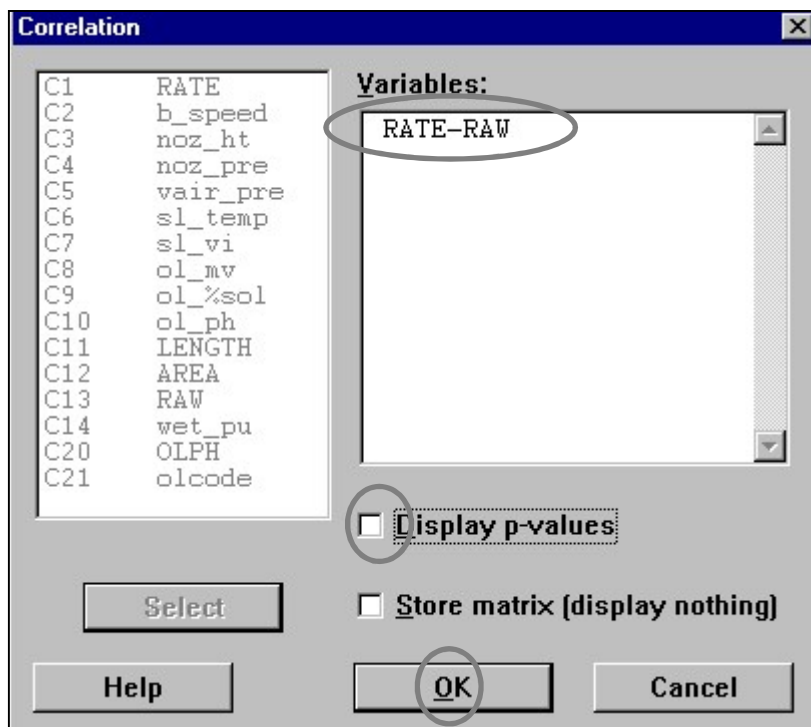
# Example: Alumina Deposition

Serious multicollinearity problem! Want VIF <5

Predictor	Coef	SE Coef	T	P	VIF
Constant	-0.503	3.733	-0.13	0.893	
RATE	0.03827	0.02531	1.51	0.139	13.7
b_speed	0.01821	0.01121	1.62	0.113	5.3
noz_ht	-0.6738	0.8084	-0.83	0.410	134.7
noz_pre	0.5272	0.1106	4.77	0.000	11.8
vair_pre	0.006180	0.007003	0.88	0.383	8.3
sl_temp	-0.02794	0.01442	-1.94	0.060	9.0
sl_vi	-0.001636	0.004425	-0.37	0.714	2.3
ol_mv	0.02743	0.02616	1.05	0.301	2.2
ol_%sol	0.03942	0.01781	2.21	0.033	2.1
ol_ph	-0.2010	0.1225	-1.64	0.109	5.2
LENGTH	-0.4734	0.2634	-1.80	0.080	109.7
AREA	-0.11130	0.03847	-2.89	0.006	64.5
RAW	0.4473	0.3337	1.34	0.188	5.0

# Example: Alumina Deposition

- b. Identify pairwise correlations between x variables
  1. *Stat> Basic Statistics> Correlation..*
  2. Fill in the dialog as shown below:
  3. Press Ok



# Example: Alumina Deposition

Next step: remove noz\_ht and check multicollinearity

	RATE	b_speed	noz_ht	noz_pre	vair_pre	sl_temp	sl_vi	ol_mv
b_speed	0.164							
noz_ht	0.661	0.462						
noz_pre	-0.451	0.044	0.138					
vair_pre	0.202	0.332	-0.042	-0.469				
sl_temp	-0.445	-0.197	-0.781	-0.118	0.335			
sl_vi	-0.122	-0.185	-0.131	-0.110	-0.031	0.250		
ol_mv	-0.007	0.512	0.328	0.262	0.289	-0.193	-0.206	
ol_%sol	-0.275	-0.529	-0.372	0.081	-0.251	0.261	0.147	-0.369
ol_ph	0.534	0.391	0.723	0.064	0.335	-0.540	-0.290	0.436
LENGTH	-0.516	-0.490	-0.959	-0.161	-0.030	0.755	0.019	-0.397
AREA	0.240	0.573	0.857	0.503	-0.073	-0.704	-0.204	0.542
RAW	-0.485	-0.526	-0.508	-0.086	-0.226	0.263	0.386	-0.213

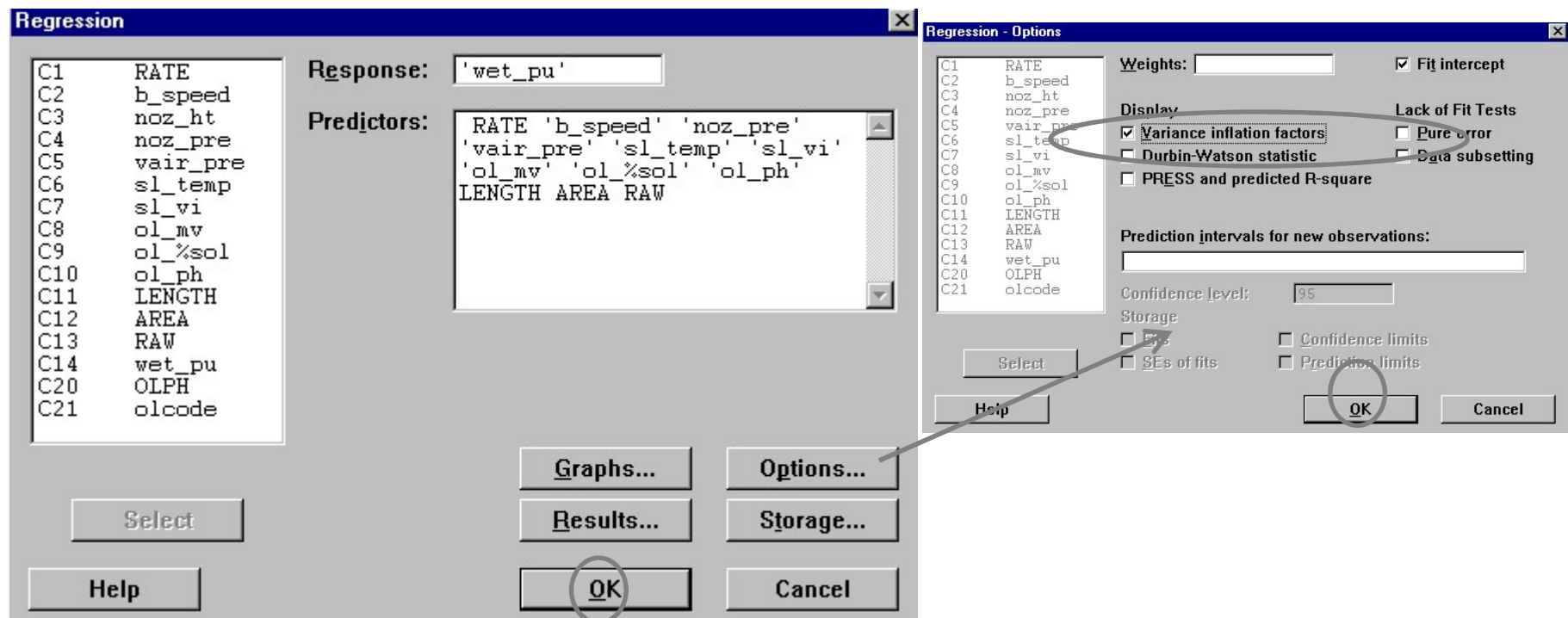
  

	ol_%sol	ol_ph	LENGTH	AREA
ol_ph	-0.462			
LENGTH	0.421	-0.703		
AREA	-0.382	0.686	-0.891	
RAW	0.277	-0.579	0.360	-0.448



# Example: Alumina Deposition

- c. Remove 'noz\_ht' and assess VIF values
- 1. *Stat> Regression> Regression*
- 2. Fill in the dialog as shown below:



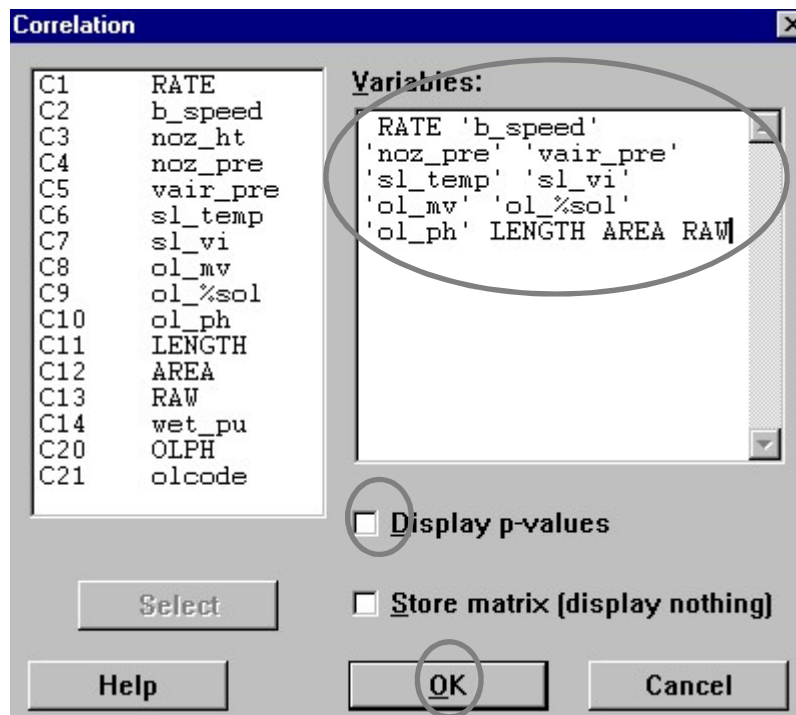
# Example: Alumina Deposition

Several factors still have high VIF! Want VIF <5

Predictor	Coef	SE Coef	T	P	VIF
Constant	-1.920	3.310	-0.58	0.565	
RATE	0.02193	0.01595	1.37	0.177	5.5
b_speed	0.01763	0.01114	1.58	0.122	5.3
noz_pre	0.5305	0.1100	4.82	0.000	11.8
vair_pre	0.009866	0.005409	1.82	0.076	5.0
sl_temp	-0.03070	0.01398	-2.20	0.034	8.5
sl_vi	-0.000349	0.004130	-0.08	0.933	2.0
ol_mv	0.02591	0.02599	1.00	0.325	2.2
ol_%sol	0.03325	0.01613	2.06	0.046	1.7
ol_ph	-0.2139	0.1210	-1.77	0.085	5.1
LENGTH	-0.2901	0.1445	-2.01	0.052	33.3
AREA	-0.11591	0.03792	-3.06	0.004	63.1
RAW	0.5904	0.2850	2.07	0.045	3.7

# Example: Alumina Deposition

- d. Remove 'noz\_ht' and assess VIF values
  1. *Stat> Basic Statistics> Correlation..*
  2. Fill in the dialog as shown below:
  3. Press Ok



# Example: Alumina Deposition

Next step: remove AREA and check multicollinearity

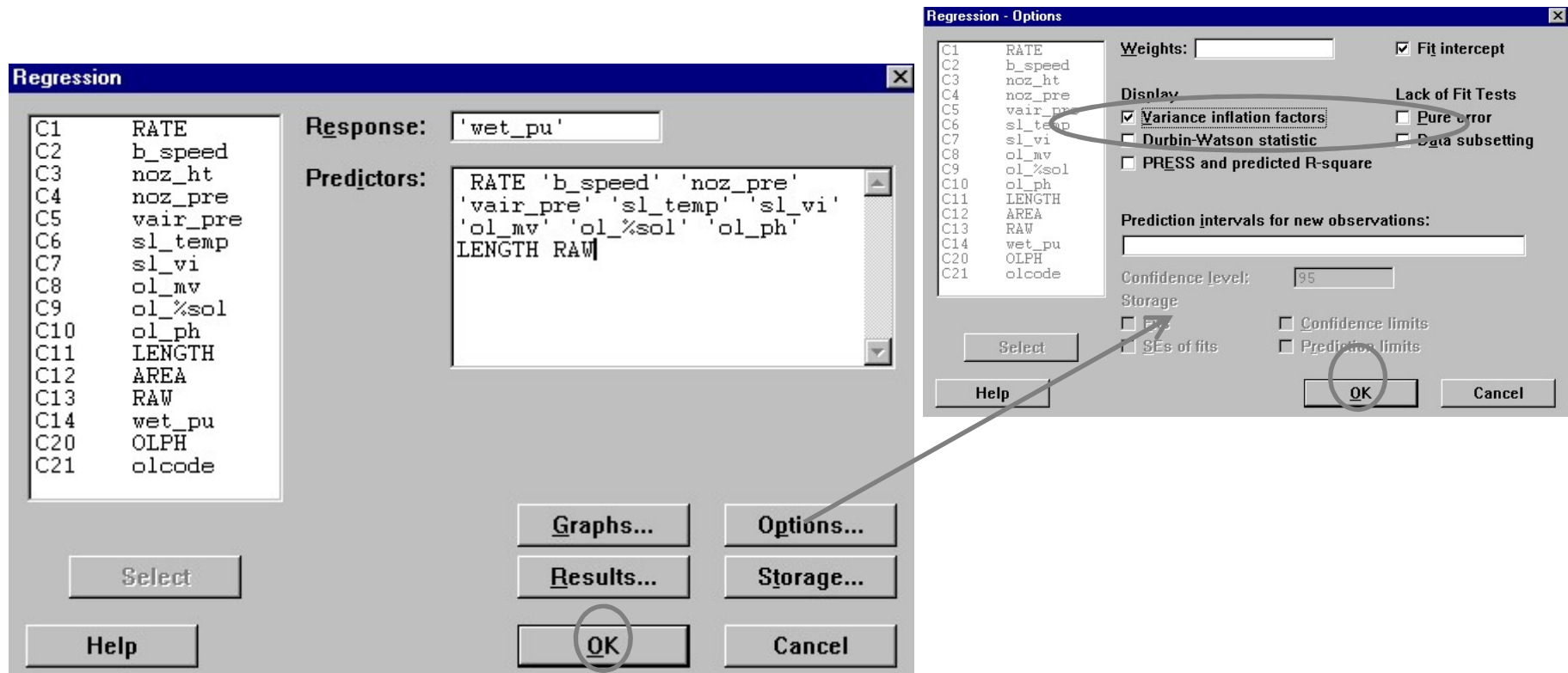
	RATE	b_speed	noz_pre	vair_pre	sl_temp	sl_vi	ol_mv	ol_%sol
b_speed	0.164							
noz_pre	-0.451	0.044						
vair_pre	0.202	0.332	-0.469					
sl_temp	-0.445	-0.197	-0.118	0.335				
sl_vi	-0.122	-0.185	-0.110	-0.031	0.250			
ol_mv	-0.007	0.512	0.262	0.289	-0.193	-0.206		
ol_%sol	-0.275	-0.529	0.081	-0.251	0.261	0.147	-0.369	
ol_ph	0.534	0.391	0.064	0.335	-0.540	-0.290	0.436	-0.462
LENGTH	-0.516	-0.490	-0.161	-0.030	0.755	0.019	-0.397	0.421
AREA	0.240	0.573	0.503	-0.073	-0.704	-0.204	0.542	-0.382
RAW	-0.485	-0.526	-0.086	-0.226	0.263	0.386	-0.213	0.277

	ol_ph	LENGTH	AREA
LENGTH	-0.703		
AREA	0.686	-0.891	
RAW	-0.579	0.360	-0.448

# Example: Alumina Deposition

- e. Remove 'AREA' and assess VIF values
  1. *Stat > Regression > Regression*
  2. Fill in the dialog as shown below:



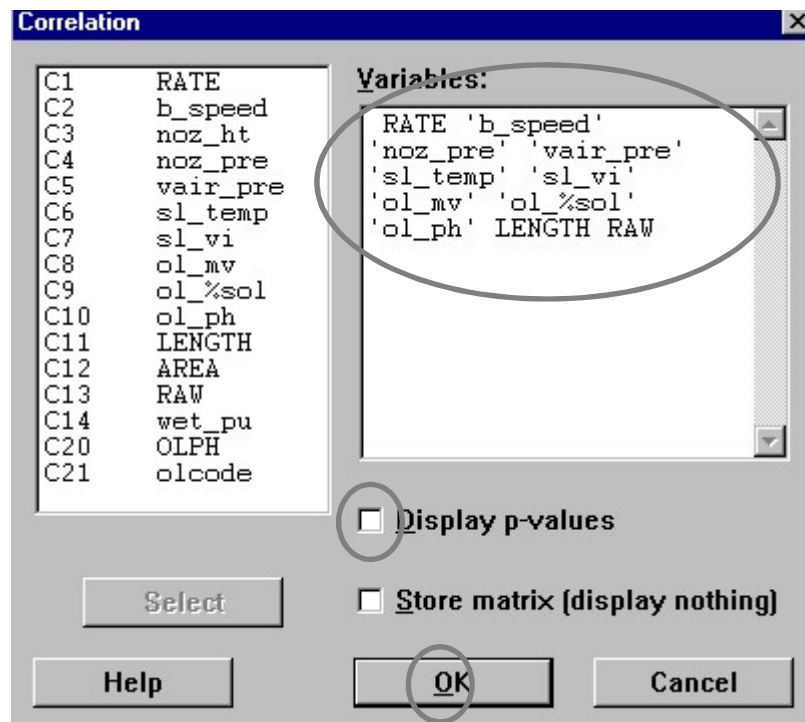
# Example: Alumina Deposition

Several factors still have high VIF! Want VIF <5

Predictor	Coef	SE Coef	T	P	VIF
Constant	-4.649	3.512	-1.32	0.193	
RATE	0.03761	0.01664	2.26	0.029	4.9
b_speed	-0.00136	0.01019	-0.13	0.894	3.7
noz_pre	0.27940	0.08070	3.46	0.001	5.2
vair_pre	0.007805	0.005913	1.32	0.195	4.9
sl_temp	-0.01220	0.01389	-0.88	0.385	6.9
sl_vi	0.003396	0.004345	0.78	0.439	1.8
ol_mv	0.00540	0.02767	0.20	0.846	2.0
ol_%sol	0.02337	0.01741	1.34	0.187	1.7
ol_ph	-0.3187	0.1279	-2.49	0.017	4.7
LENGTH	0.08079	0.08651	0.93	0.356	9.8
RAW	0.7490	0.3087	2.43	0.020	3.5

# Example: Alumina Deposition

- f. Remove 'AREA' and assess VIF values
  1. *Stat> Basic Statistics> Correlation..*
  2. Fill in the dialog as shown below:
  3. Press Ok



# Example: Alumina Deposition

Next step: remove LENGTH and check VIF values

	RATE	b_speed	noz_pre	vair_pre	sl_temp	sl_vi	ol_mv	ol_%sol
b_speed	0.164							
noz_pre	-0.451	0.044						
vair_pre	0.202	0.332	-0.469					
sl_temp	-0.445	-0.197	-0.118	0.335				
sl_vi	-0.122	-0.185	-0.110	-0.031	0.250			
ol_mv	-0.007	0.512	0.262	0.289	-0.193	-0.206		
ol_%sol	-0.275	-0.529	0.081	-0.251	0.261	0.147	-0.369	
ol_ph	0.534	0.391	0.064	0.335	-0.540	-0.290	0.436	-0.462
LENGTH	-0.516	-0.490	-0.161	-0.030	0.755	0.019	-0.397	0.421
RAW	-0.485	-0.526	-0.086	-0.226	0.263	0.386	-0.213	0.277

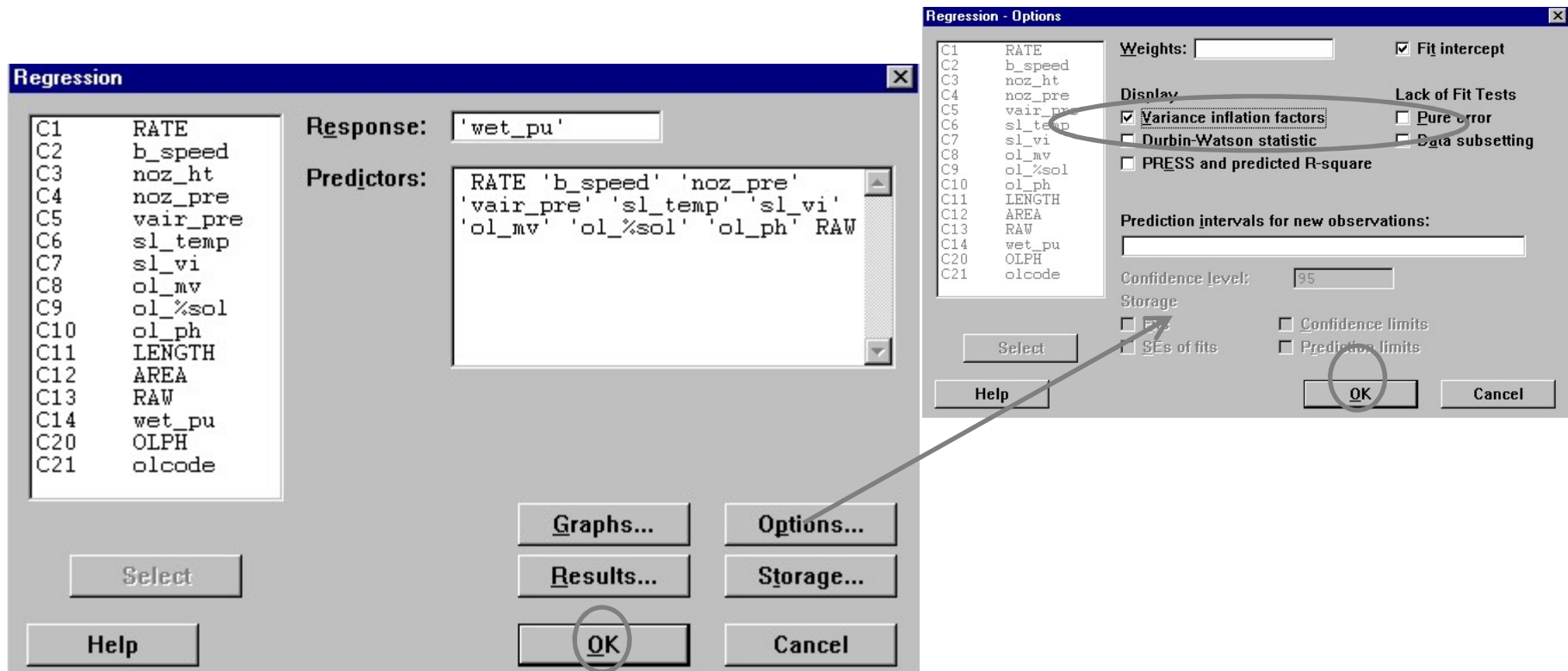
  

	ol_ph	LENGTH
LENGTH	-0.703	
RAW	-0.579	0.360



# Example: Alumina Deposition

- g. Remove 'LENGTH' and assess VIF values
- 1. *Stat> Regression> Regression*
- 2. Fill in the dialog as shown below:



# Example: Alumina Deposition

All VIF <5

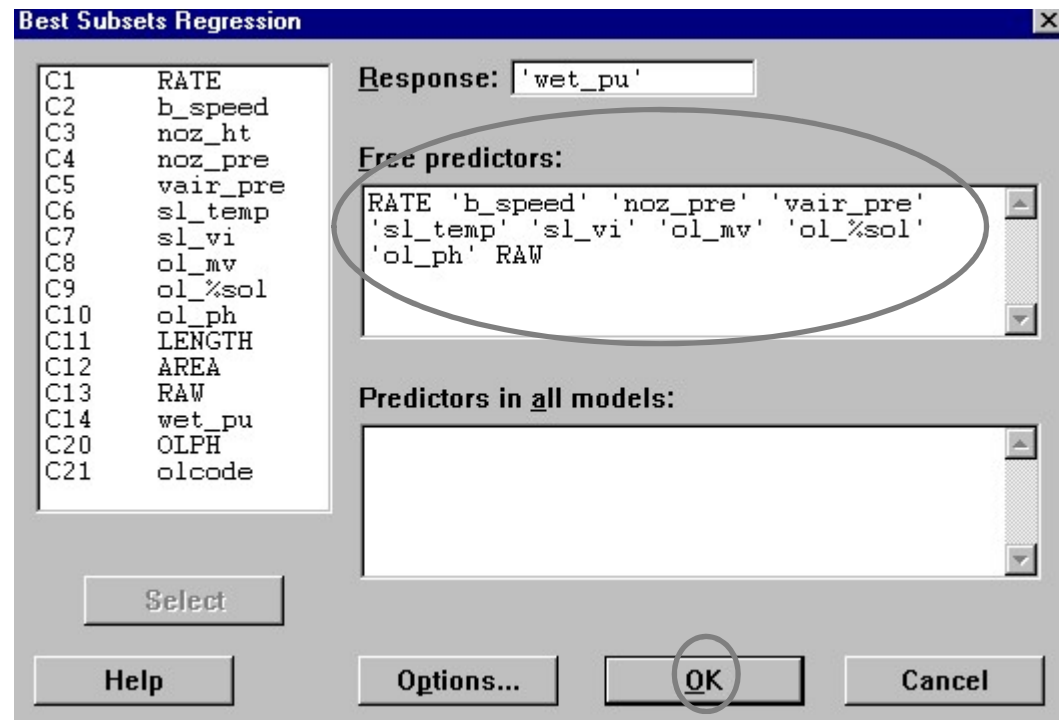
Predictor	Coef	SE Coef	T	P	VIF
Constant	-3.111	3.097	-1.00	0.321	
RATE	0.02845	0.01343	2.12	0.040	3.2
b_speed	-0.006774	0.008372	-0.81	0.423	2.5
noz_pre	0.23409	0.06439	3.64	0.001	3.3
vair_pre	0.005700	0.005457	1.04	0.303	4.2
sl_temp	-0.002936	0.009704	-0.30	0.764	3.4
sl_vi	0.001197	0.003646	0.33	0.744	1.3
ol_mv	0.00778	0.02751	0.28	0.779	2.0
ol_%sol	0.02297	0.01738	1.32	0.194	1.7
ol_ph	-0.3436	0.1248	-2.75	0.009	4.5
RAW	0.6230	0.2772	2.25	0.030	2.9

# Example: Alumina Deposition

## Multiple regression steps:

- Use remaining variables and apply Best Subsets to evaluate best predictor candidates for the model

- Stat > Regression > Best Subsets..*
- Fill in the dialog as shown below:
- Press Ok



# Example: Alumina Deposition

Response is wet\_pu

Next step: remove LENGTH  
and check VIF values

```

v
b n a s      o
_ o i l      l
s z r _ s o _ o
R p _ _ t l l % l
A e p p e _ _ s _ R
T e r r m v m o p A
E d e e p i v l h W
    
```

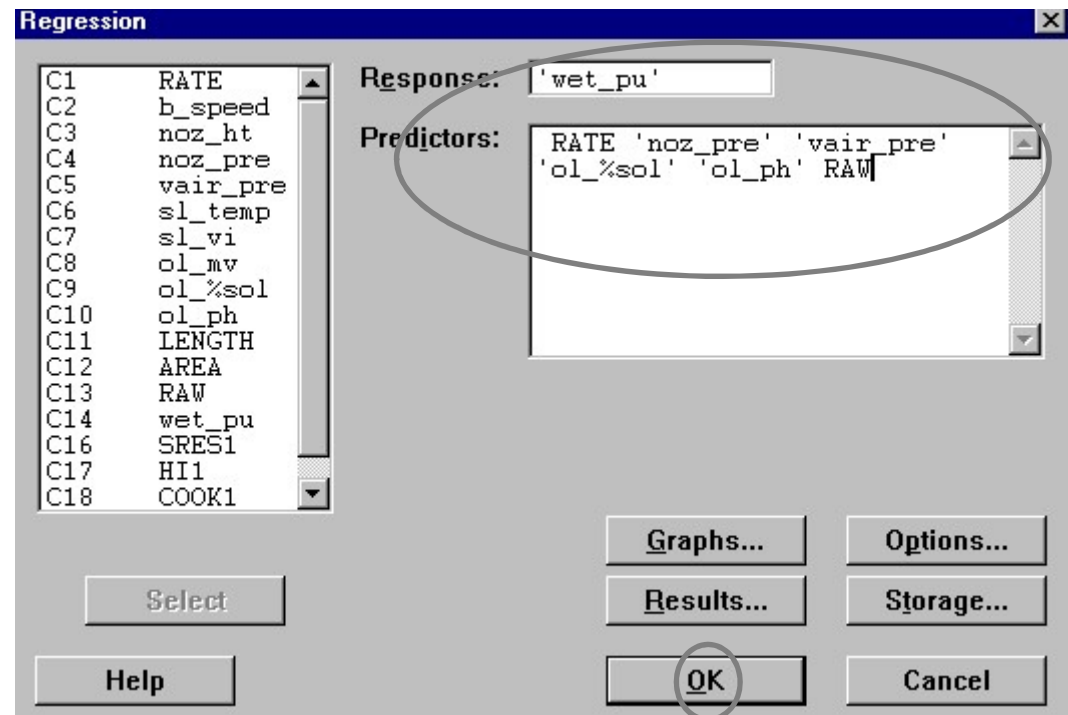
Vars	R-Sq	R-Sq(adj)	C-p	S	E	d	e	e	p	i	v	l	h	W	
1	31.8	30.4	25.0	0.16911										X	
1	27.1	25.6	29.9	0.17484										X	
2	45.6	43.3	12.4	0.15255		X							X		
2	41.6	39.2	16.6	0.15808		X							X		
3	52.4	49.4	7.2	0.14423		X							X	X	
3	52.1	49.0	7.6	0.14475		X	X						X		
4	56.8	53.1	4.5	0.13885	X	X							X	X	
4	56.0	52.1	5.4	0.14022		X							X	X	X
5	60.0	55.5	3.2	0.13512	X	X							X	X	X
5	58.2	53.6	5.1	0.13808	X	X	X						X	X	
6	61.4	56.1	3.7	0.13422	X	X	X						X	X	X
6	60.5	55.1	4.7	0.13579	X	X	X	X					X	X	X
7	61.8	55.6	5.3	0.13502	X	X	X	X					X	X	X
7	61.4	55.2	5.7	0.13570	X	X	X	X	X				X	X	X
8	61.9	54.7	7.2	0.13647	X	X	X	X	X	X			X	X	X
8	61.9	54.7	7.2	0.13647	X	X	X	X			X		X	X	X
9	62.0	53.7	9.1	0.13797	X	X	X	X	X	X	X		X	X	X
9	62.0	53.6	9.1	0.13799	X	X	X	X	X	X	X	X	X	X	X

# Example: Alumina Deposition

## Multiple regression steps:

3. Choose the best candidate and complete regression analysis

1. *Stat> Regression> Best Subsets..*
2. Fill in the dialog as shown below:
- 3: Press Ok



## Example: Alumina Deposition

The regression equation is

$$\text{wet\_pu} = -4.75 + 0.0309 \text{ RATE} + 0.235 \text{ noz\_pre} + 0.00452 \text{ vair\_pre} + 0.0286 \text{ ol\_sol} - 0.313 \text{ ol\_ph} + 0.758 \text{ RAW}$$

Predictor	Coef	SE Coef	T	P
Constant	-4.754	2.184	-2.18	0.035
RATE	0.03093	0.01245	2.48	0.017
noz_pre	0.23479	0.05661	4.15	0.000
vair_pre	0.004525	0.003571	1.27	0.212
ol_%sol	0.02865	0.01470	1.95	0.058
ol_ph	-0.31336	0.09312	-3.37	0.002
RAW	0.7576	0.2142	3.54	0.001

S = 0.1342

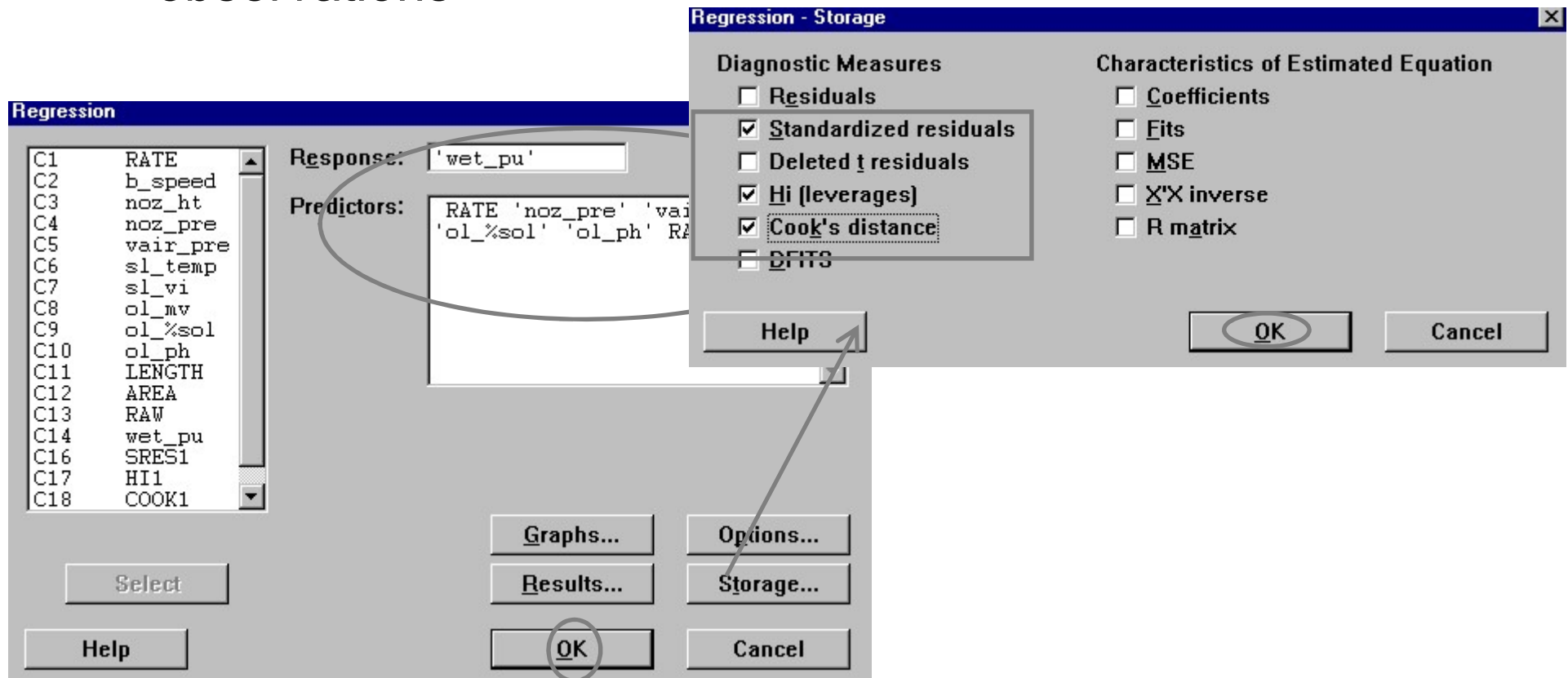
R-Sq = 61.4%

R-Sq(adj) = 56.1%

# Example: Alumina Deposition

Multiple regression steps:

4. Perform model diagnostics to identify outliers and unusual observations



# Example: Alumina Deposition

## Outliers/ Unusual Observations

	SRES1	HI1	COOK1
6	2.73754	0.100785	0.119993
7	0.64728	0.117710	0.007985
8	-0.75073	0.140894	0.013204
9	2.60919	0.082681	0.087660
10	0.08823	0.121181	0.000153
11	-1.95640	0.184598	0.123786
12	0.06623	0.050418	0.000033
13	-0.54417	0.049358	0.002196
14	-2.60239	0.048948	0.049794
15	-1.43050	0.044407	0.013585
16	-0.19782	0.084059	0.000513
17	0.52585	0.163286	0.007709
18	-0.30950	0.072739	0.001073
19	-0.70209	0.090436	0.007001
20	0.58939	0.108930	0.006067
21	-0.47165	0.054785	0.001842
22	0.36363	0.058157	0.001166
23	-0.13536	0.139829	0.000426
24	-0.26053	0.093215	0.000997
25	1.83274	0.418808	0.345778
26	-0.76024	0.423783	0.060724
27	0.08687	0.240779	0.000349

Potential trouble spots:  
 Standardized residual > 2  
 Leverage >  $2p/n = 0.3529$   
 Cook's distance > 1

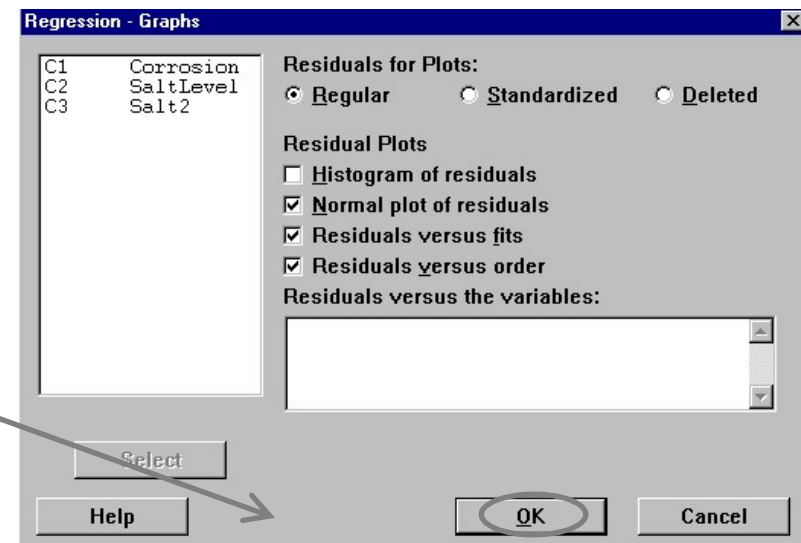
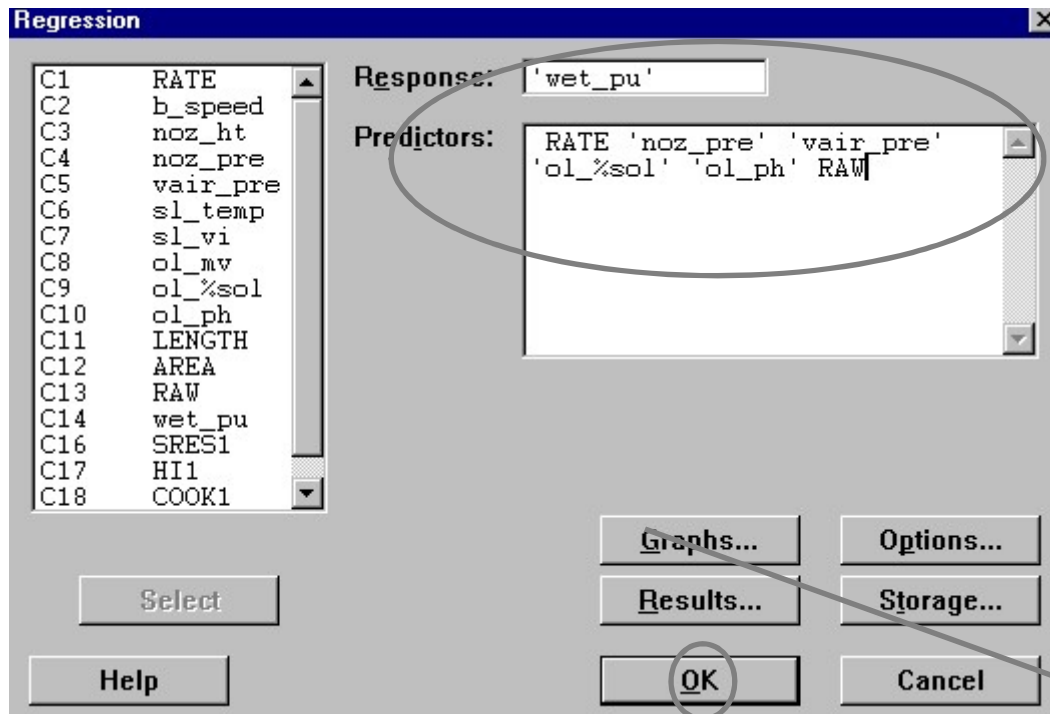
What actions should be taken with the outliers and influential observations?



# Example: Alumina Deposition

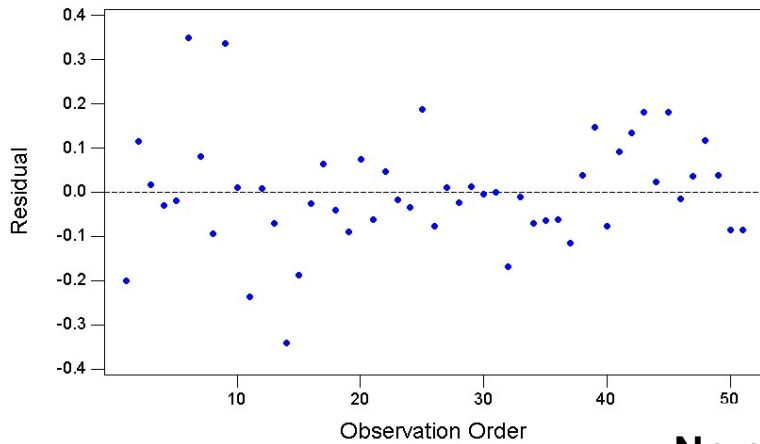
Multiple regression steps:

5. Analyze residuals for violation of assumptions

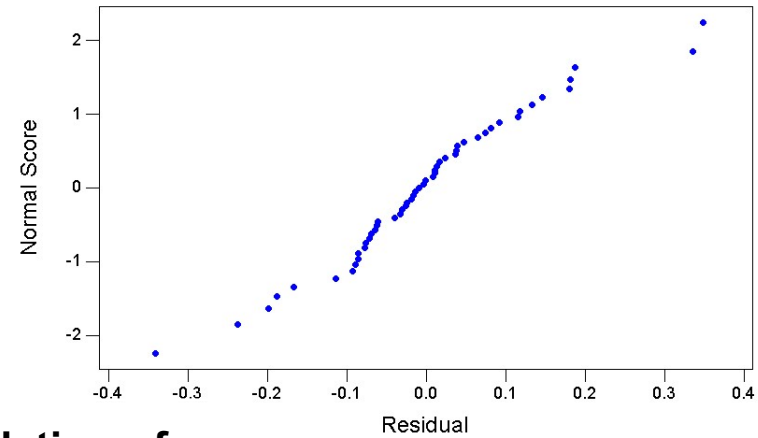


# Example: Alumina Deposition - Residual Plot Analysis

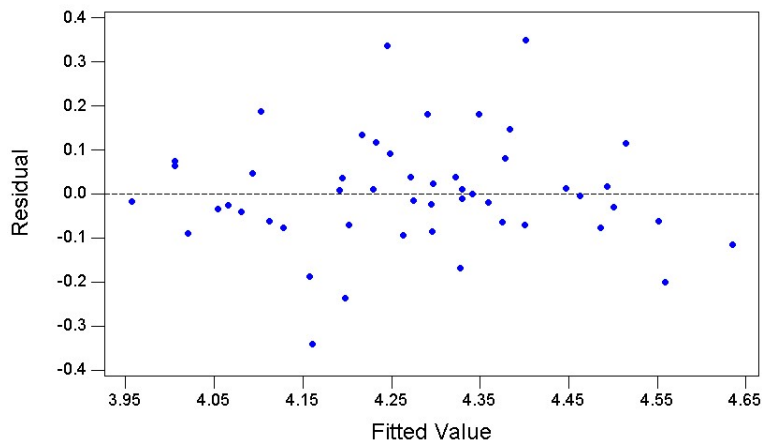
Residuals Versus the Order of the Data  
(response is wet\_pu)



Normal Probability Plot of the Residuals  
(response is wet\_pu)

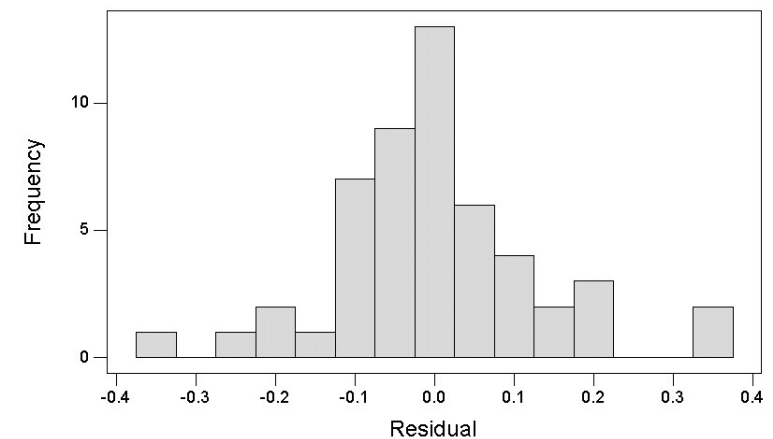


Residuals Versus the Fitted Values  
(response is wet\_pu)



**No evidence of violation of assumptions**

Histogram of the Residuals  
(response is wet\_pu)



# Business Process Case Study: Compensation Debate

You are a Six Sigma Black Belt team working for Sigma Soft, a successful software company with locations in New York, Seattle and Phoenix.

Recently, complaints were filed by three software specialists at Sigma Soft's Seattle office alleging they are victims of discriminatory salary practices. Specifically, the three employees charge that salaries at Sigma Soft are based on social networking abilities and gender bias.

Sigma Soft prides itself on having a fair and equitable corporate citizenship. Understandably, the company is taking the accusations very seriously.

The Executive VP of human resources has asked your Six Sigma Black Belt team to investigate the issues surrounding the employees' complaints and determine their validity.

During a preliminary investigation you discovered the controversy was triggered in part by a recent magazine article on software specialists. Among other things, the article alleged that women software specialists with similar education and experience as male counterparts received 20% less financial compensation.

# Business Process Case Study: Compensation Debate

Following statistics were also published in the magazine article regarding disparity in compensation for software specialists at different geographical locations:

<b>COMPENSATION STATISTICS FOR SOFTWARE SPECIALIST</b>			
<b>LOCATION</b>	<b>New York</b>	<b>Seattle</b>	
Average Compensation, \$	\$140,000	\$105,000	
Average IS experience	9.2 years	9.2 years	

# Business Process Case Study: Compensation Debate

The personnel profiles on the three employees lodging complaints against Sigma Soft are as follows.

<b>COMPENSATION DATA FOR SOFTWARE SPECIALISTS</b>			
	<b><u>Employee1</u></b>	<b><u>Employee2</u></b>	<b><u>Employee3</u></b>
<b>Compensation, \$</b>	72,000	90,000	94,100
<b>IS experience</b>	7 years	10 years	11 years
<b>Education</b>	MS	BS	BS
<b>Other experience</b>	3 years	3 years	1 year
<b>Sex</b>	Female	Female	Female
<b>Age</b>	35	33	30

# Business Process Case Study: Compensation Debate

In addition your team has obtained compensation information (**salary.mtw**) for comparable positions in three software companies, including Sigma Soft and two competitors.

Team breakout exercise:

- How is your team going to approach the problem? Create a list of practical questions that will aid in your investigation.
- Create a flipchart for your thought processes.

Data in **salary.mtw**

# Business Process Case Study: Compensation Debate

- Utilize the compensation data to analyze the issues and determine the validity of the employees' complaints as well as the magazine's claims.
  - Is the magazine information regarding average compensation and gender bias accurate?
  - Is there a difference in salaries between men and women at Sigma Soft?
  - Is there a difference in salaries between women at Sigma Soft and women in the other software companies?
- Establish a multiple regression model to predict a software specialist's salary using input variables.
  - Perform model diagnostics to detect any outliers or unusual observations.
  - Validate any assumptions used in creating the multiple regression model
- Utilize the regression model to predict the salaries of the three employees making accusations against Sigma Soft.
- Are the employees' complaints valid?
- Are salaries at Sigma Soft gender biased?

# Anexas Consultancy Services

## Logistic Regression Analysis

Discrete Ys



# Days Between Maintenance and Chances of Good Start-Ups

- Suppose you work on a chemical blending process for a small-volume, highly regulated product
- It is a one-shift operation and you must shut down and clean the blender at the end of each day
- There is no preventive maintenance schedule in place: maintenance is called after certain mechanical problems are observed and procedures are done at the end of the shift
- You are interested in establishing a preventive maintenance schedule and would like to learn how many days you can go between maintenance procedures and still have a chance of a good start-up each day (machine starts on first attempt)

# Days Between Maintenance and Chances of Good Start-Ups, cont.

- You check the log for the last 89 maintenance procedures
  - What data would you collect to answer this question?
  - How would you display them?
  - Can you predict an attribute outcome (yes/no) with a continuous variable?

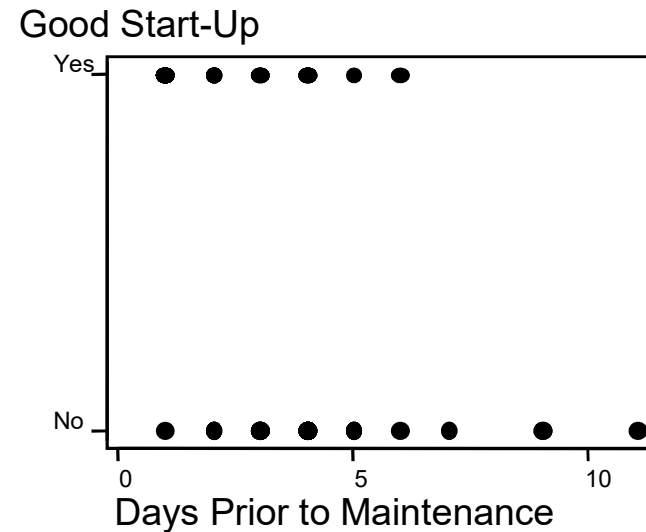
# Days Between Maintenance and Chances of Good Start-Ups, cont.

<u>Maintenance Procedure</u>	<u>Days Prior to Maintenance</u>	<u>Good Start-Up First Run Next Day?</u>
1	2	Yes
2	1	Yes
3	7	No
4	2	Yes
5	5	No
.	.	.
86	2	Yes
87	4	No
88	2	Yes
89	3	Yes

# Possible Ways to Display Discrete Y-Data

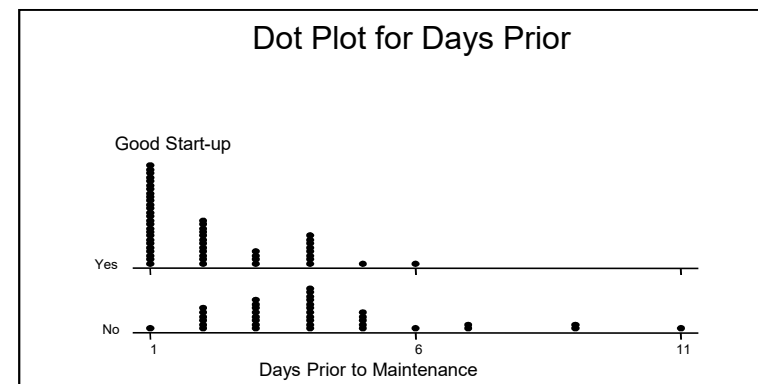
## 1. Scatter plot of raw data

- Inappropriate data for scatter plot—Y is discrete-attribute, not continuous
- Thus, the plot is not useful; cannot understand relationship or make predictions based on this pattern



## 2. Stratified dot plot

- Useful
- Can start to see patterns that reveal information about the relationship

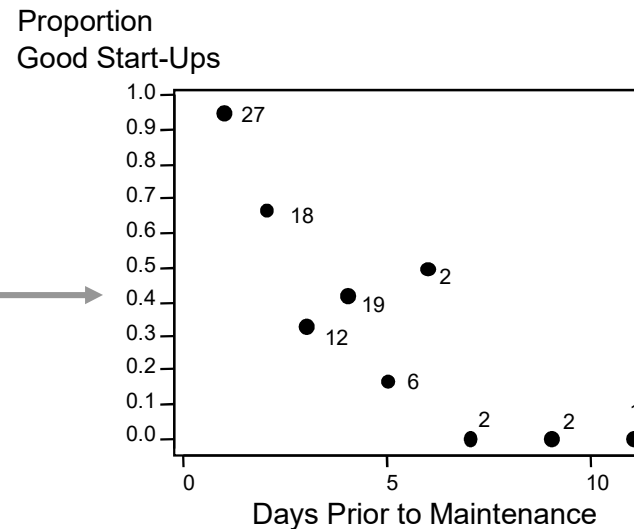


# Possible Ways to Display Discrete Y-Data, cont.

## 3. Scatter plot of summarized data

- Very useful
- Can pick out patterns to help us answer our question

Each point is labeled with the number of maintenance procedures at those conditions



# Use Proportions for Discrete Data

- Logistic regression predicts the **proportion** or **probability** of a particular Y attribute
- In other words, you would like to know the percent of times the chemical blender works right the first run of the day after a maintenance procedure

Raw Data

Maintenance Procedure	(X) Days Prior to Maint	(Y) Good Start-Up
1	2	Yes
2	1	Yes
3	7	No
.	.	.
87	4	No
88	2	Yes
89	3	Yes

X = continuous data

Y = discrete attribute data

A particular Y-attribute

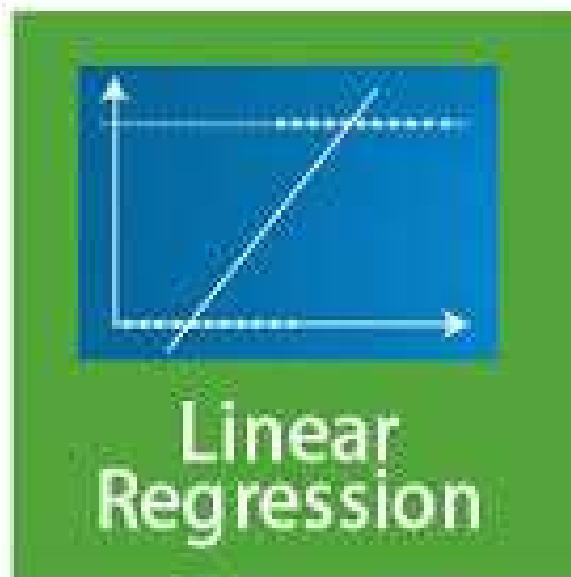


# Appropriate Data for Y in Logistic Regression

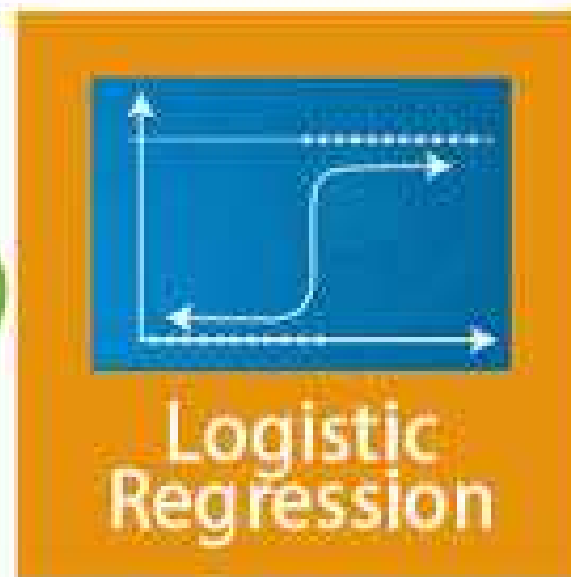
## Type of data for Y

- Discrete, attribute
- Two levels only
  - “Success”
  - “Failure”
- Choose one and call it an **event**
  - People are usually more interested in successes, but you can just as easily predict failures
  - Minitab will want to know which level interests you and will call it the “event” (of interest)

# Linear Regression Vs Logistic Regression



VS

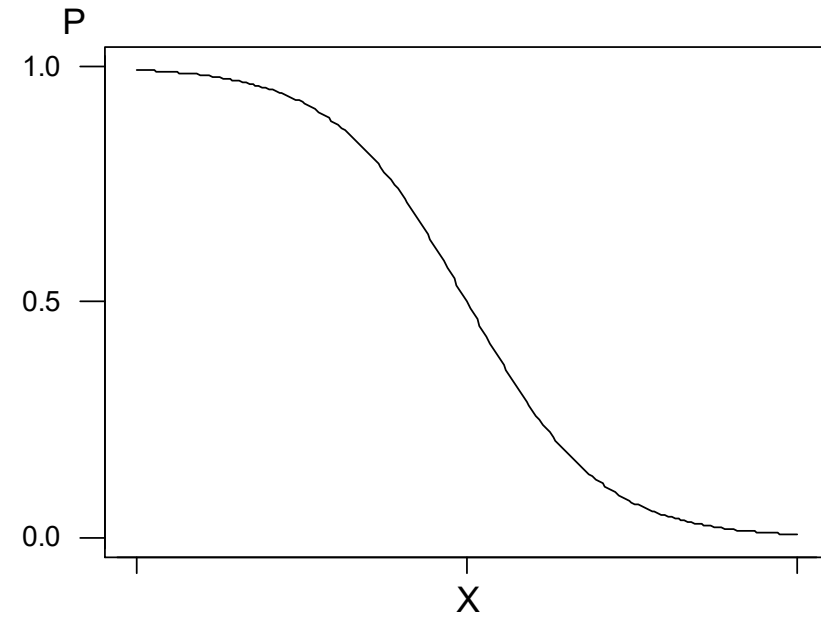
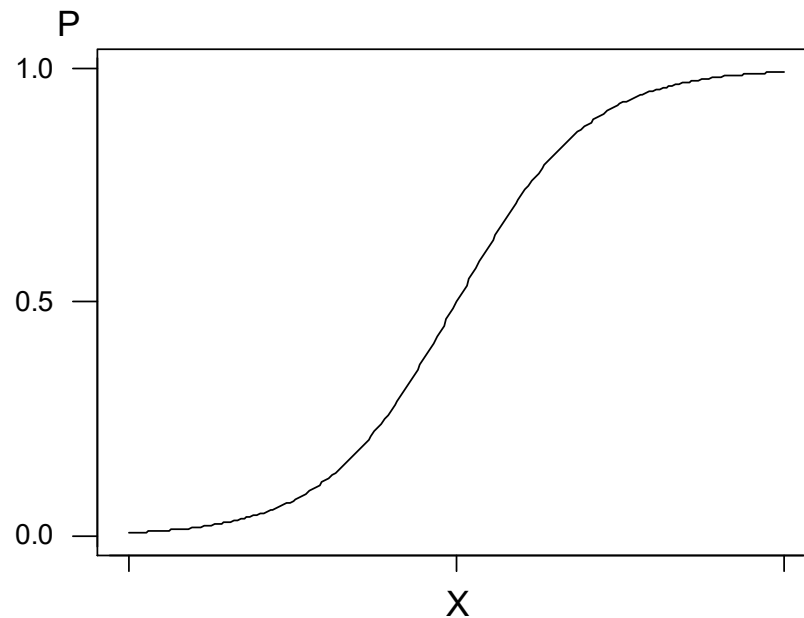




# Logistic Regression Fits a Curve to the Data

The relationship between an event probability ( $p$ ) and a predictor ( $X$ ) is usually curved, not straight

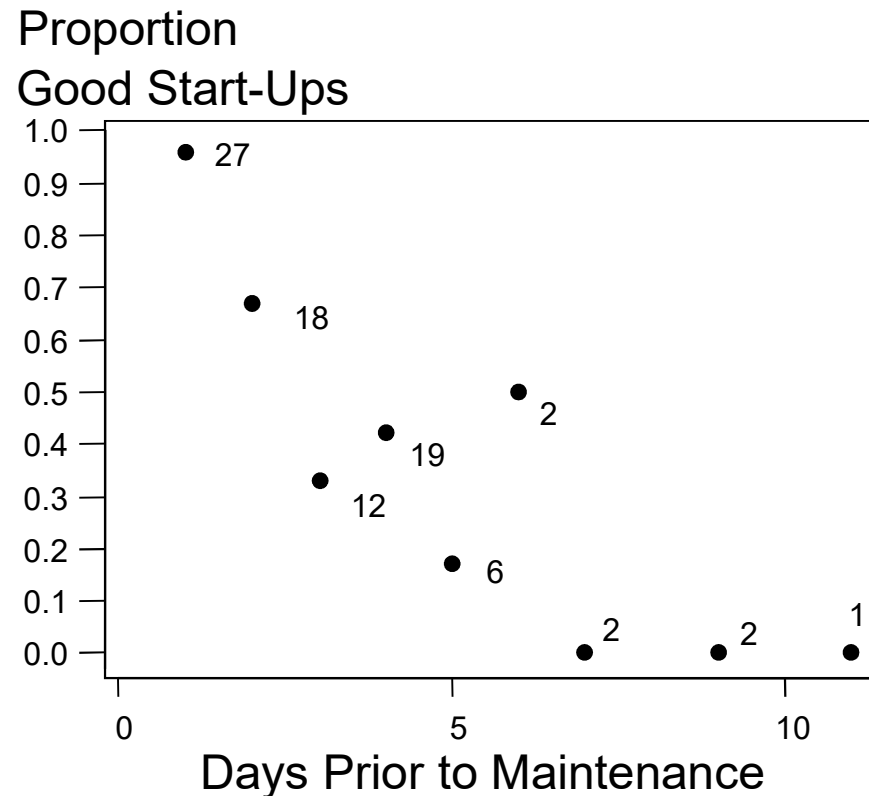
- It looks like a tilted “S,” backward “S,” or some portion of an “S”
- It has asymptotes at 0 and 1



# Logistic Regression Fits a Curve to the Data, cont.

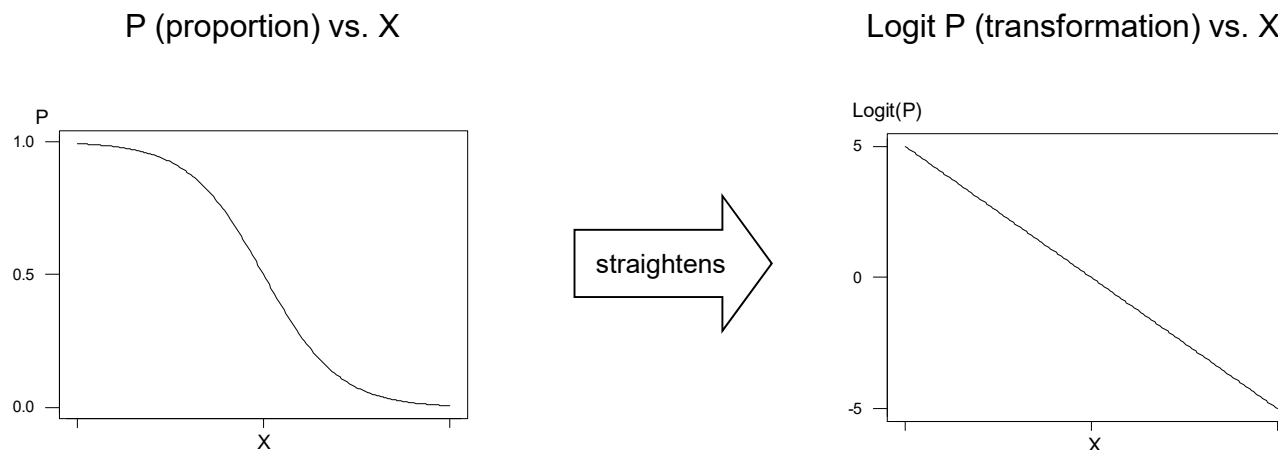
## Maintenance example

- The proportion of good start-ups decreases as the days prior to maintenance increase, but not at a constant rate
- Draw a curve that might fit the points on this plot



# How Does Logistic Regression Work?

- The Y-variable is actually  $P$ , the proportion of Y-attributes at each  $X$ -value
- The logit transformation straightens the S-shaped relationship between  $P$  and  $X$



- Logistic regression gets its name from this transformation

# Doing Logistic Regression

- We will use Minitab to:
  - Obtain  $b_0$  and  $b_1$
  - Back-transform the fits to obtain predicted P's (event probabilities or EPROs) for each value of X
- Logistic regression is an advanced topic
  - Much of Minitab's output is beyond the scope of this course
  - The interpretation of  $b_1$  is not simple
  - We will use plots of the back-transformed fits vs. X to interpret the relationship

# Anexas Consultancy Services

## Regression Analysis

Logistic Regression with Discrete Y's

# Minitab Follow-Along: Logistic Regression

- **Data: *PrevMain.mtw***

1. Do a logistic regression analysis on the “Good Start-ups” predicted from “Days Prior to Maint.”:

Stat > Regression > Binary Logistic Regression

The screenshot shows the Minitab Binary Logistic Regression dialog box with the following settings and annotations:

- Response:** C3 GoodStartups
- Frequency (optional):** C4 %GoodStartups
- Response in response/frequency format:**  (Unselected)
- Response in event/trial format:**  (Selected)
  - Number of events:** GoodStartups
  - Number of trials:** Procedures
- Model:** DaysPrioroMaint
- Factors (optional):** (Empty)
- Buttons:** Select, Graphs..., Options..., Storage... (circled), Cancel, Help

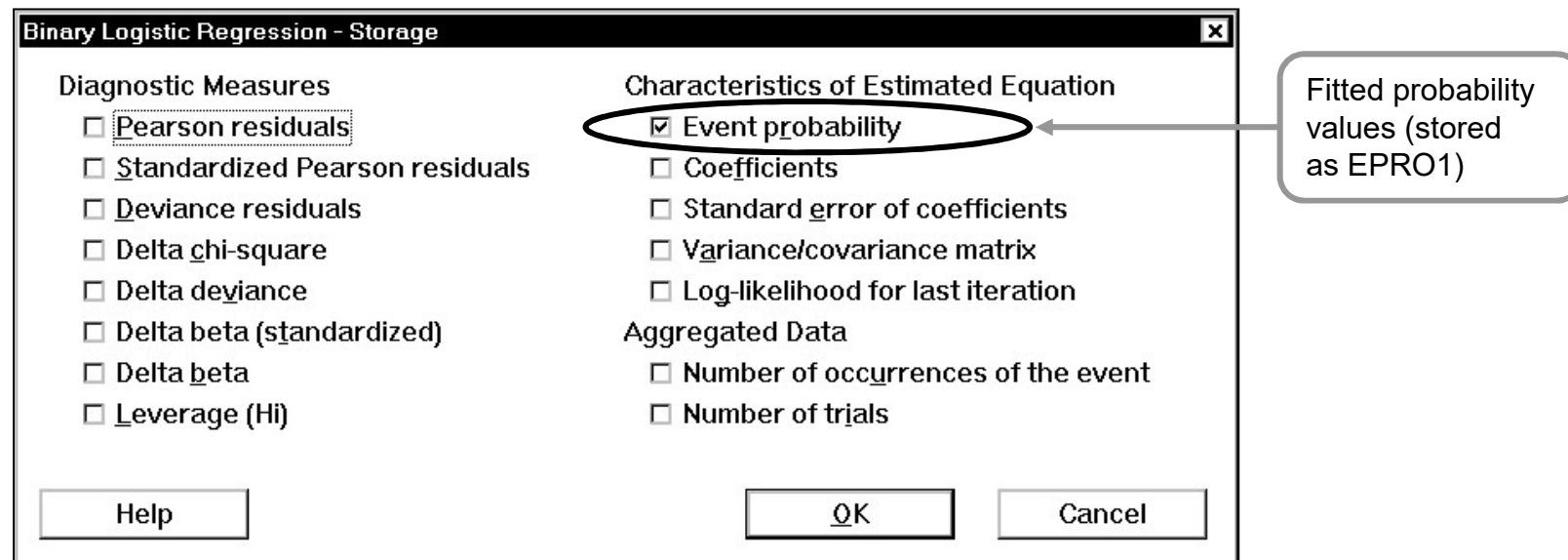
Annotations:

- Choose the appropriate way data are stored in worksheet:** Points to the response and frequency list.
- For “raw” data:** Points to the response in response/frequency format radio button.
- For summary data, number good out of total:** Points to the event/trial format radio button and its associated fields.
- List all terms (X’s) in the model:** Points to the Model field.
- List discrete X’s (again), if any; for our case leave it empty:** Points to the Factors (optional) field.

# Minitab Follow-Along: Logistic Regression, cont.

2. Store the fitted Y's (known as event probabilities) to make a fitted line plot later:

Stat > Regression > Binary Logistic Regression > Storage



# Minitab Output: Logistic Regression

## Binary Logistic Regression: GoodStartups, Procedures versus DaysPriortoM

Link Function: Logit

Response Information

GoodStartups	Event	52
	Non-event	37
Procedures	Total	89

Logistic Regression Table

Predictor	Coef	SE Coef	Z	P	Odds Ratio	95% CI Lower	95% CI Upper
Constant	2.82120	0.617574	4.57	0.000			
DaysPriortoMaint	-0.858419	0.196212	-4.37	0.000	0.42	0.29	0.62

Logistic regression equation:

$$\ln \frac{p}{(1-p)} = 2.82 - 0.858(\text{Days Prior})$$

LogLikelihood = 44.643

Test that all slopes are zero: G = 31.555, DF = 1, PValue = 0.000

GoodnessofFit Tests

Method	ChiSquare	DF	P
Pearson	9.772	7	0.202
Deviance	8.497	7	0.291
HosmerLemeshow	5.769	3	0.123

Table of Observed and Expected Frequencies:

(See HosmerLemeshow Test for the Pearson ChiSquare Statistic)

Value	Group					Total
	1	2	3	4	5	
Success						
Obs	2	8	4	12	26	52
Exp	1.4	6.7	6.7	13.5	23.7	
Failure						
Obs	11	11	8	6	1	37
Exp	11.6	12.3	5.3	4.5	3.3	
Total	13	19	12	18	27	89

Measures of Association:

(Between the Response Variable and Predicted Probabilities)

Pairs	Number	Percent	Summary Measures	
Concordant	1483	77.1%	Somers' D	0.66
Discordant	217	11.3%	GoodmanKruskal Gamma	0.74
Ties	224	11.6%	Kendall's Taua	0.32
Total	1924	100.0%		

Beyond scope of course

Both slope and intercept are significant

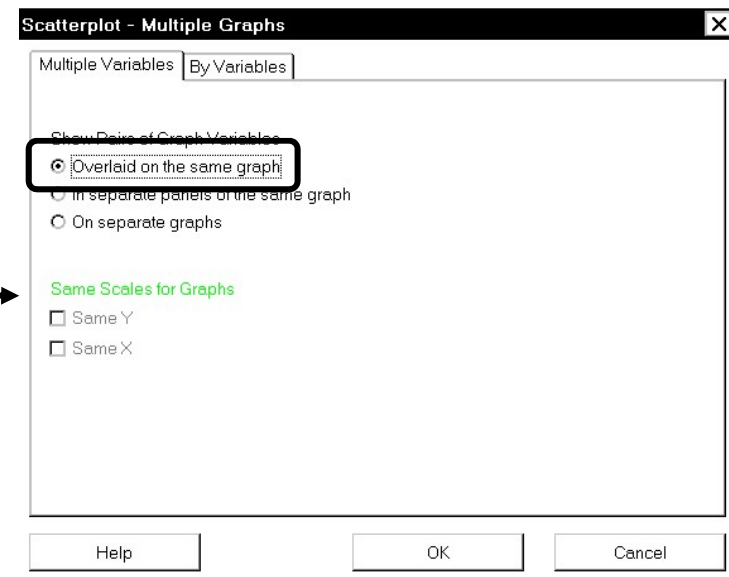
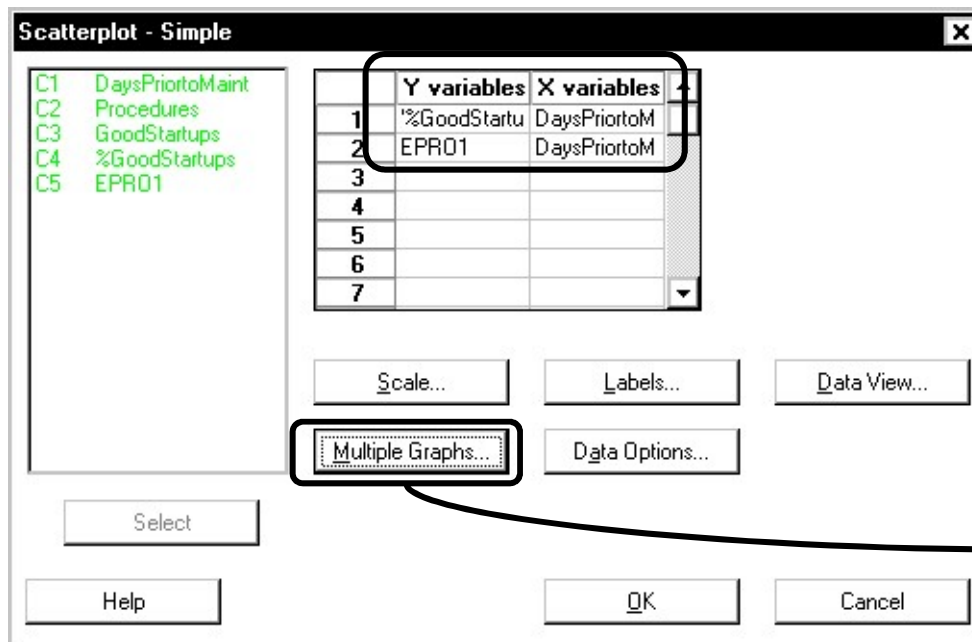
Testing for a lack of fit (curve through points). Since P > 0.05, conclude no lack of fit in any of three tests.



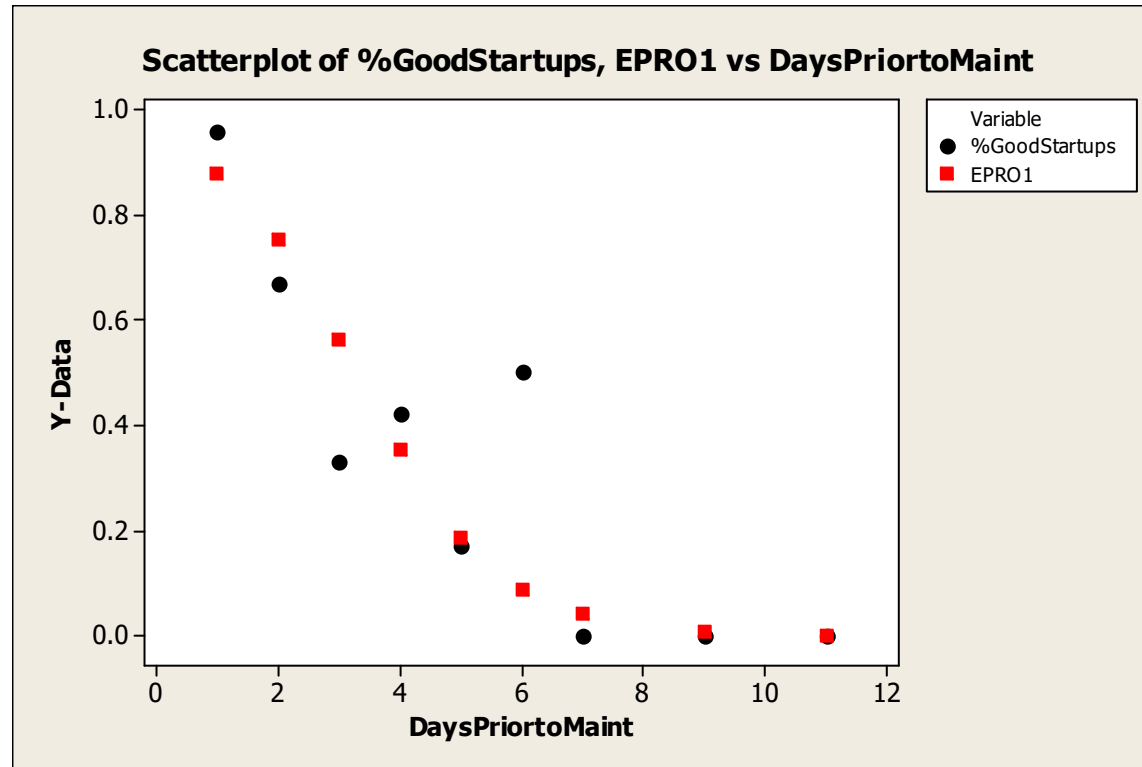
# Minitab Follow-Along: Logistic Regression, cont.

3. Make a fitted curve plot for the data:

Graph > Scatter plot > Simple



# Minitab Follow-Along: Logistic Regression, cont.



**Conclusion:** If you schedule maintenance every two days, you'll have a 75% chance of a good start-up the next day.

# Anexas Consultancy Services

## Regression Analysis

Logistic Regression with Discrete Y's

# Multiple Logistic Regression

- You can try to improve the prediction of event probabilities by using several predictors (X's), not just one
  - The predictors (X's) can be either continuous or discrete
  - The multiple logistic regression equation is

$$\log_e \left( \frac{p}{1-p} \right) = b_0 + b_1 X_1 + b_2 X_2 + \dots + b_k X_k$$

- If any of the X's are discrete (in groups), you must designate one group to be the reference group
  - The coefficient for a discrete X group compares its intercept with the reference group (see discussion in previous section)
- Multiple logistic regression is useful for identifying key drivers (X's), particularly those that affect Y (event probabilities) in combination with each other

# Practice: Applications of Logistic Regression

- **Objective:** Practice identifying predictor variables for logistic regression situations
- **Instructions:**
  - Circle the outcome you would define as a “success” for each discrete Y
  - List some possible predictors
- **Time:** 5 minutes

Discrete Y's	Predictors (X's)
Shipment is on time or late	
Customer buys product or not	
Deal was won or lost	
Customer retained this year or not	
Product works or not	
Machine breaks down or not	

# Answers: Applications of Logistic Regression

Discrete Y's	Predictors (X's)
Shipment is <b>on time</b> or late	<ul style="list-style-type: none"> <li>• Size of order</li> <li>• Number of other orders in queue</li> </ul>
Customer <b>buys</b> product or not	<ul style="list-style-type: none"> <li>• Product price</li> <li>• Presence of CTQ</li> </ul>
Deal was <b>won</b> or lost	<ul style="list-style-type: none"> <li>• Cost</li> <li>• Experience level of company representative</li> </ul>
Customer <b>retained</b> this year or not	<ul style="list-style-type: none"> <li>• Length of time with company</li> <li>• Amount price increased or decreased</li> </ul>
Product <b>works</b> or not	<ul style="list-style-type: none"> <li>• Number of defects</li> <li>• Quality of materials</li> </ul>
Machine breaks down or <b>not</b>	<ul style="list-style-type: none"> <li>• Type of maintenance procedure used</li> <li>• Experience level of maintenance operator</li> </ul>

## Exercise: Do a Multiple Logistic Regression (Part 1)

- **Objective:** Practice doing a multiple logistic regression in Minitab and interpreting the results
- **Data:** *Retain.mtw*
- **Background:**
  - Suppose you work for a company that manufactures custom and standard surgical devices for doctors; competition in this market is fierce
  - You are interested in finding the key factors that help predict whether your company will retain a customer next year or not
  - Last year's data on 63 customers are given below
  - The event of interest is “customers kept”
  - Q-Score range is 1 – 10, 1 being the best score and 10 is worst
- **Time:** 25 minutes

# Exercise: Do a Multiple Logistic Regression (Part 1), cont.

## ■ **Instructions:**

- Work in pairs
- Open the appropriate Minitab window and enter the answers to questions 1 through 4 before proceeding

Stat > Regression > Binary Logistic Regression

1. Which variable is Y?
2. Are these raw data or a summary table of counts?
  - Choose the appropriate data format in Minitab
3. How many predictors (X's) are there?
  - List each in Minitab's "Model" box
4. How many of the predictors (X's) are discrete?
  - List only the discrete X's in Minitab's optional "Factors" box

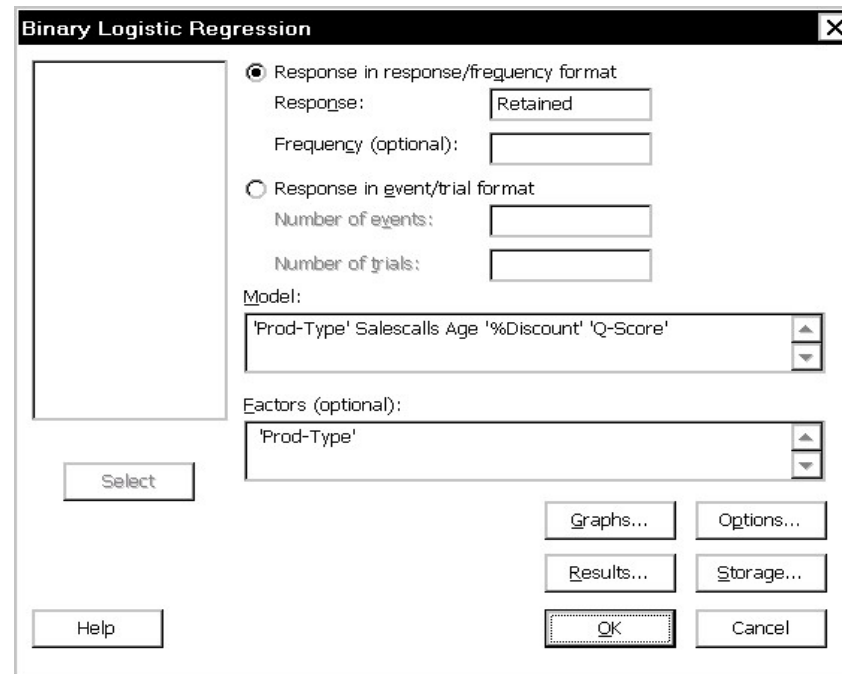


# Answers: Do a Multiple Logistic Regression (Part 1)

## Results for Questions 1 through 4

- 1. Y variable = **Retained**
- 2. Raw data or summary? **Raw data**
- 3. How many predictors are there? **Five**
- 4. How many predictors are discrete?  
**One (Product type)**

This is what your Minitab window should look like so far

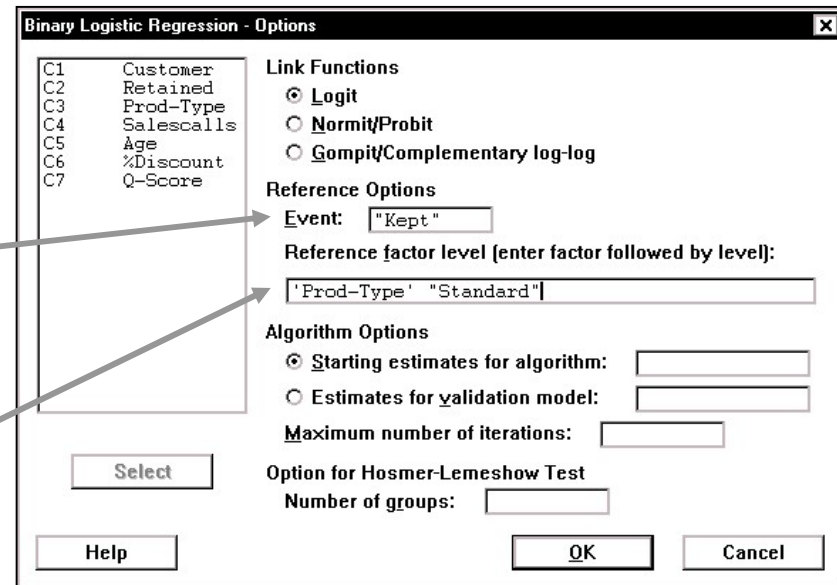


# Exercise: Do a Multiple Logistic Regression (Part 2)

- Select the Options button in Minitab and designate:
  - a. The event of interest
    - Select the Options button; type in the event of interest in double quotes in the box labeled “A” below
  - b. The reference group (or the discrete X)
    - Type the factor name followed by the level of X that you want to be the reference group in the box labeled “B” below
    - Type the factor name in single quotes and the level in double quotes

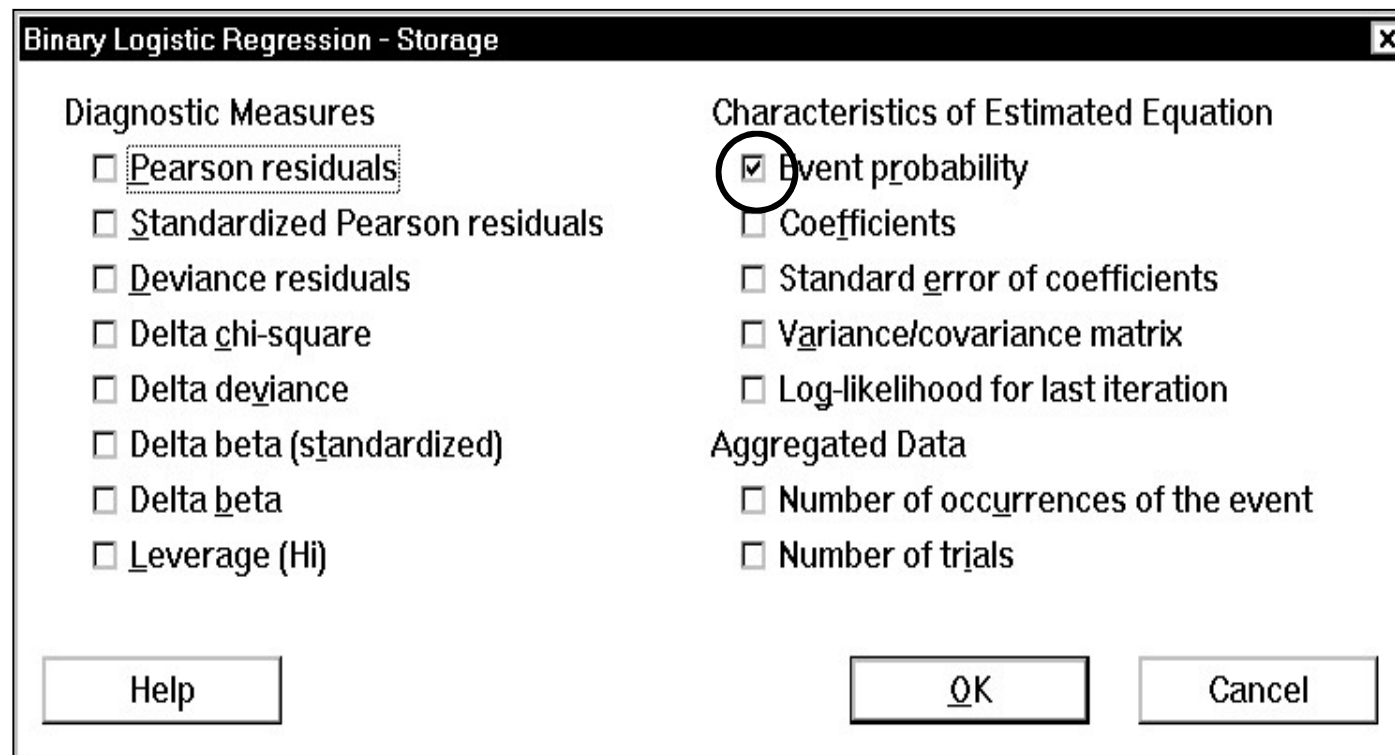
a. If this box is left empty, the default event becomes the one in **last** alphabetical order; in this case the default would be “lost.”

b. Use double quotes for discrete X reference group if it is text. If this box is left empty, the default reference group becomes the **first** one in alphabetical order; in this case the default is “custom.”



# Exercise: Do a Multiple Logistic Regression (Part 2), cont.

- Select the Storage button in Minitab and check “Event probability” to store it so you can make plots with it
  - This is the same as the fits in regular regression



## Exercise: Do a Multiple Logistic Regression (Part 2), cont.

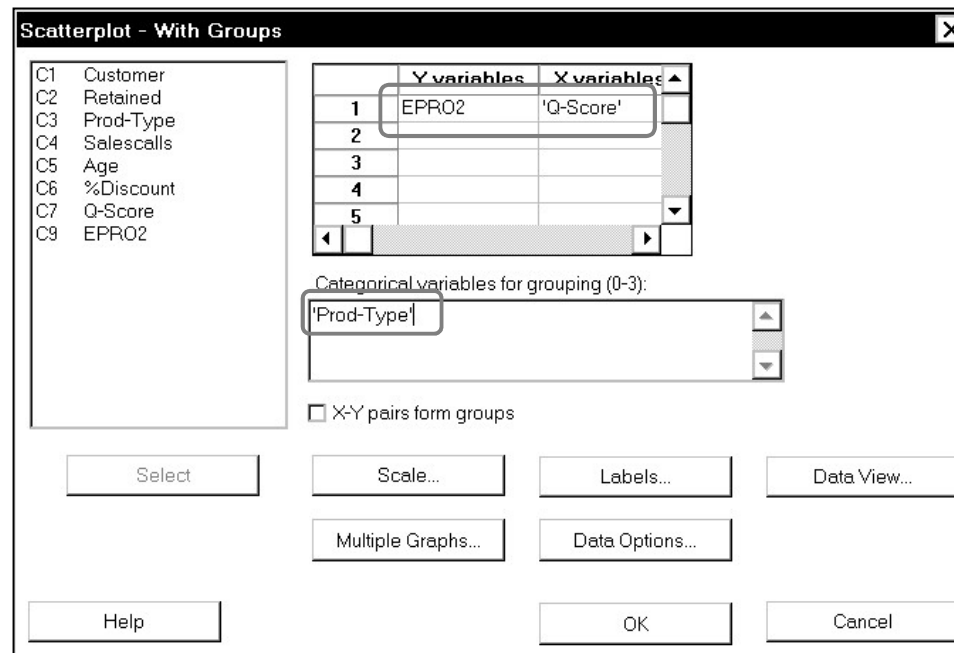
5. Which predictors (X's) help explain whether a customer will be kept or lost?
6. Remove the non-significant predictors (X's) and redo the multiple logistic regression
  - a. Be prepared to report an interpretation of the output to the whole group
  - b. Which type of product has the greatest probability of retaining customers?

# Exercise: Do a Multiple Logistic Regression (Part 2), cont.

7. a. Create a scatter plot of predicted event probabilities vs. the significant X-values; stratify by the discrete X groups

Graph > Scatter plot

b. Interpret the plot



# Answers: Do a Multiple Logistic Regression (Part 2)

## Minitab output for full model

**Binary Logistic Regression: Retained versus ProdType, Salescalls,...**  
 Link Function: Logit

Response Information

Variable	Value	Count	(Event)
Retained	Kept	47	(Event)
	Lost	14	
	Total	61	

\* NOTE \* 61 cases were used  
 \* NOTE \* 2 cases contained missing values

Logistic Regression Table

Predictor	Coef	SE Coef	Z	P	95% CI		
					Odds Ratio	Lower	Upper
Constant	1.959	3.056	0.64	0.522			
ProdType Custom	2.218	1.152	1.92	0.054	9.18	0.96	87.89
Salescal	0.1157	0.1133	1.02	0.307	1.12	0.90	1.40
Age	0.3737	0.2873	1.30	0.193	1.45	0.83	2.55
%Discoun	0.1416	0.1927	0.73	0.463	1.15	0.79	1.68
QScore	0.7476	0.2508	2.98	0.003	0.47	0.29	0.77

LogLikelihood = 19.518  
 Test that all slopes are zero: G = 26.683, DF = 5, PValue = 0.000

GoodnessofFit Tests

Method	ChiSquare	DF	P
Pearson	46.617	50	0.610
Deviance	39.036	50	0.869
HosmerLemeshow	2.978	8	0.936

[Some output deleted to save space (and it is all beyond the scope of this course).]

**Product type is discrete. Recall that "Standard" is the reference product. In this regression, this is the slope adjustment for "Custom," relative to "Standard."**

**Event of interest is "customer kept."**

**Beyond scope of class**

**Two significant predictors (X's)**

**No significant lack of fit since P > 0.05**

# Answers: Do a Multiple Logistic Regression (Part 2), cont.

## Question 5: Answer (cont.)

- Product type and Q-Score are significant predictors of whether or not we will retain customers next year
- We will remove the non-significant predictors (X's) and redo the regression

# Answers: Do a Multiple Logistic Regression (Part 2), cont.

## Question 6: Output for reduced model

The positive coefficient means that we have a higher chance of retaining customers who purchase custom products than we have of retaining customers who purchase standard products.

The negative coefficient correlates to the probability of keeping the customer. As the Q-Score increases, the chance of keeping the customer decreases. (Low Q-Scores are good.)

**Binary Logistic Regression: Retained versus ProdType, QScore**

Link Function: Logit

Response Information

Variable	Value	Count	
Retained	Kept	48	(Event)
	Lost	15	
	Total	63	

Logistic Regression Table

Predictor	Coef	SE Coef	Z	P
Constant	5.810	1.767	3.29	0.001
<b>ProdTyp</b>				
Custom	1.7091	0.8840	1.93	0.053
QScore	0.8672	0.2398	3.62	0.000

Odds Ratio	95% CI	
	Lower	Upper
5.52	0.98	31.24
0.42	0.26	0.67

Beyond scope of class

LogLikelihood = 22.403

Test that all slopes are zero: G = 24.352, DF = 2, PValue = 0.000

GoodnessofFit Tests

Method	ChiSquare	DF	P
Pearson	21.240	13	0.068
Deviance	16.272	13	0.235
HosmerLemeshow	2.206	4	0.698

[Some output removed.]

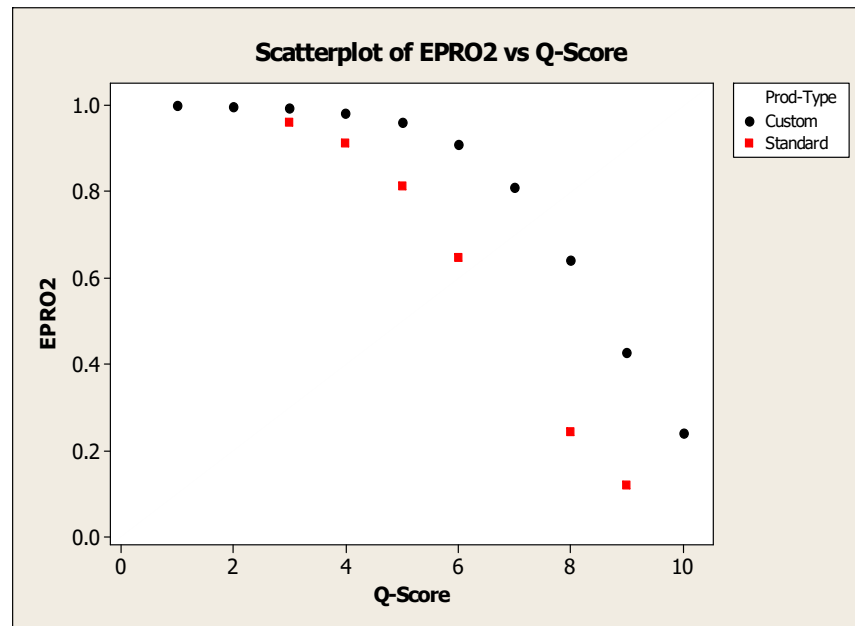


# Answers: Do a Multiple Logistic Regression (Part 2), cont.

## Question 7: Scatter plot

### Conclusions from scatter plot

- Your chance of keeping customers who purchase standard products decreases sharply with a quality score (QScore) of six or higher
- Overall, you have a better chance of keeping customers who purchase custom products



# Applications to Your Business

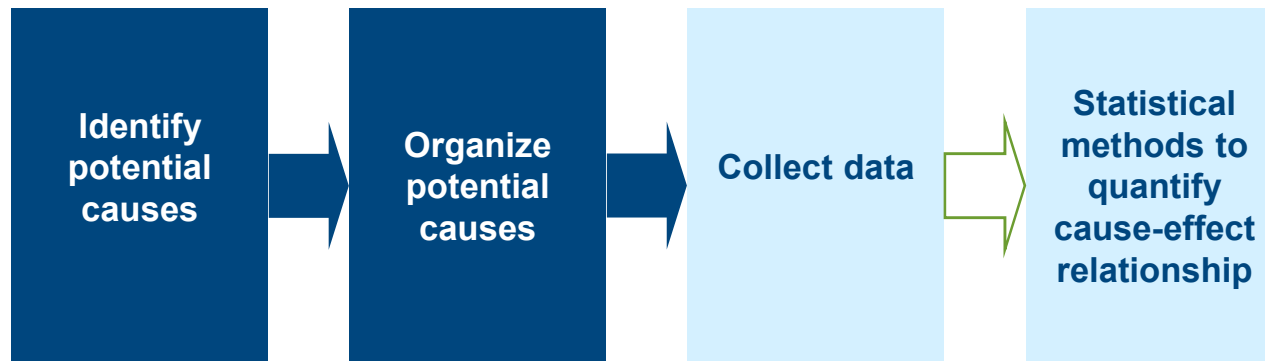
- **Objective:** Practice identifying applications of logistic regression in your business
- **Instructions:**
  - Work alone or, if there are other participants whose work is related to yours, in pairs or small groups
  - Think of a discrete Y-variable—an output or result that has a particular attribute that interests you
  - List some possible predictors of the attribute
  - Would logistic regression be useful to you? Why or why not?
  - Volunteers will be asked to share with the whole group
- **Time:** 5 minutes

# Anexas Consultancy Services

## Process Analysis (Lean)

Identifying Causes with Process Maps

# You Are Here

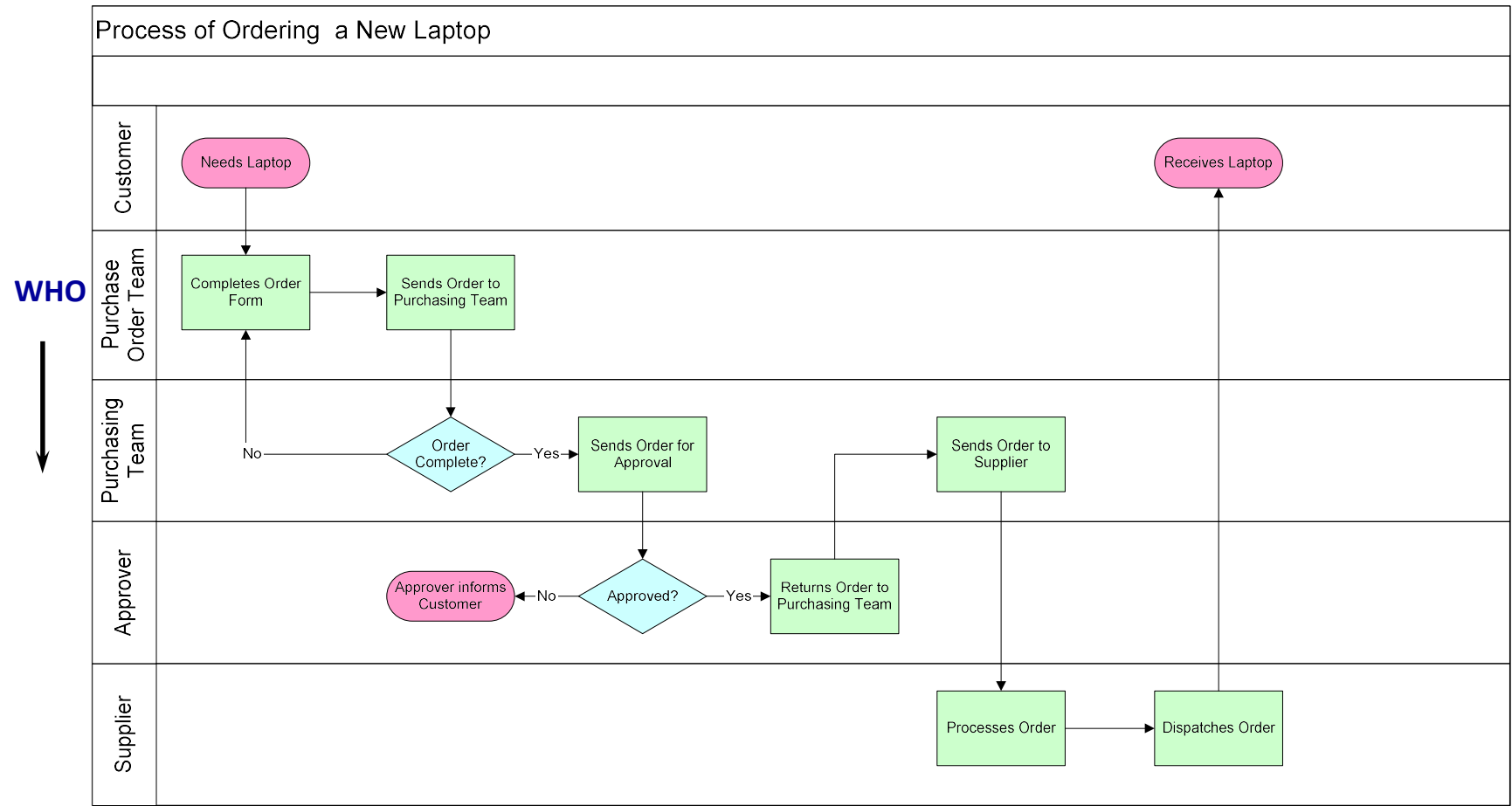


# Anexas Consultancy Services

Identify the Value-Added Path

“The Happy Path”

# Process Mapping



**WHO**

↓

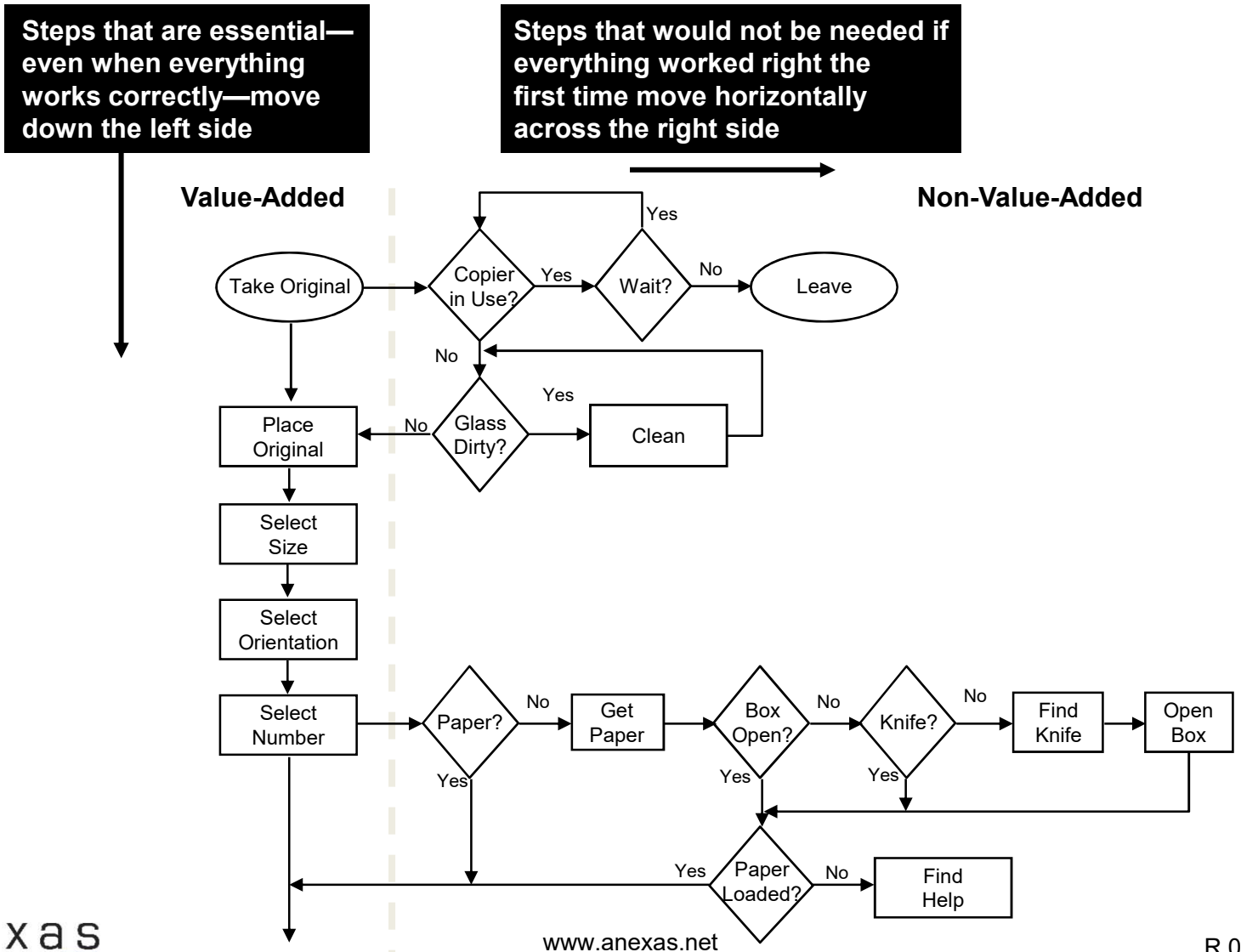
**PROCESS FLOW** →

# Value-Added and Non-Value-Added Steps

- Value-added steps:
  - Customers are willing to pay for it
  - Physically change the product
  - Are done right the first time
- Non-value-added steps:
  - Not essential to produce output
  - Include:
    - Defects, errors, omissions
    - Preparation/setup, control/inspection
    - Overproduction, processing, inventory
    - Transporting, motion, waiting, delays



# Opportunity Process Map Features



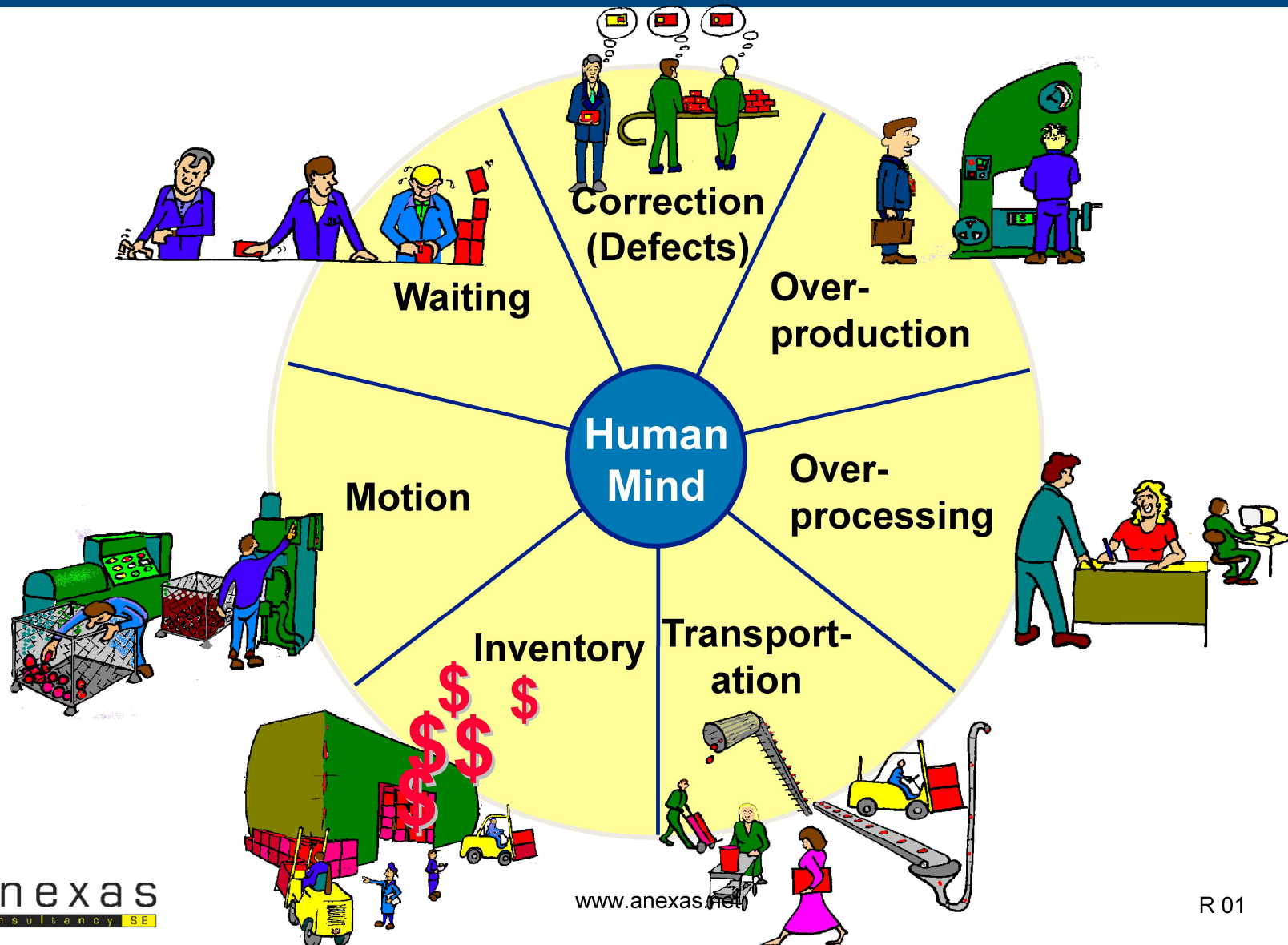


# Anexas Consultancy Services

## Identifying Flow Issues

Waste

# Major Categories of Waste: TIMWOOD



# Anexas Consultancy Services

## Identifying Flow Issues

Cycle Time

# Value Analysis Matrix

- You can track specific types of non-value-added time with a value analysis matrix
- This helps clarify not only the types of waste present in the process, but also the percentage of the overall process that each non-value-added step adds

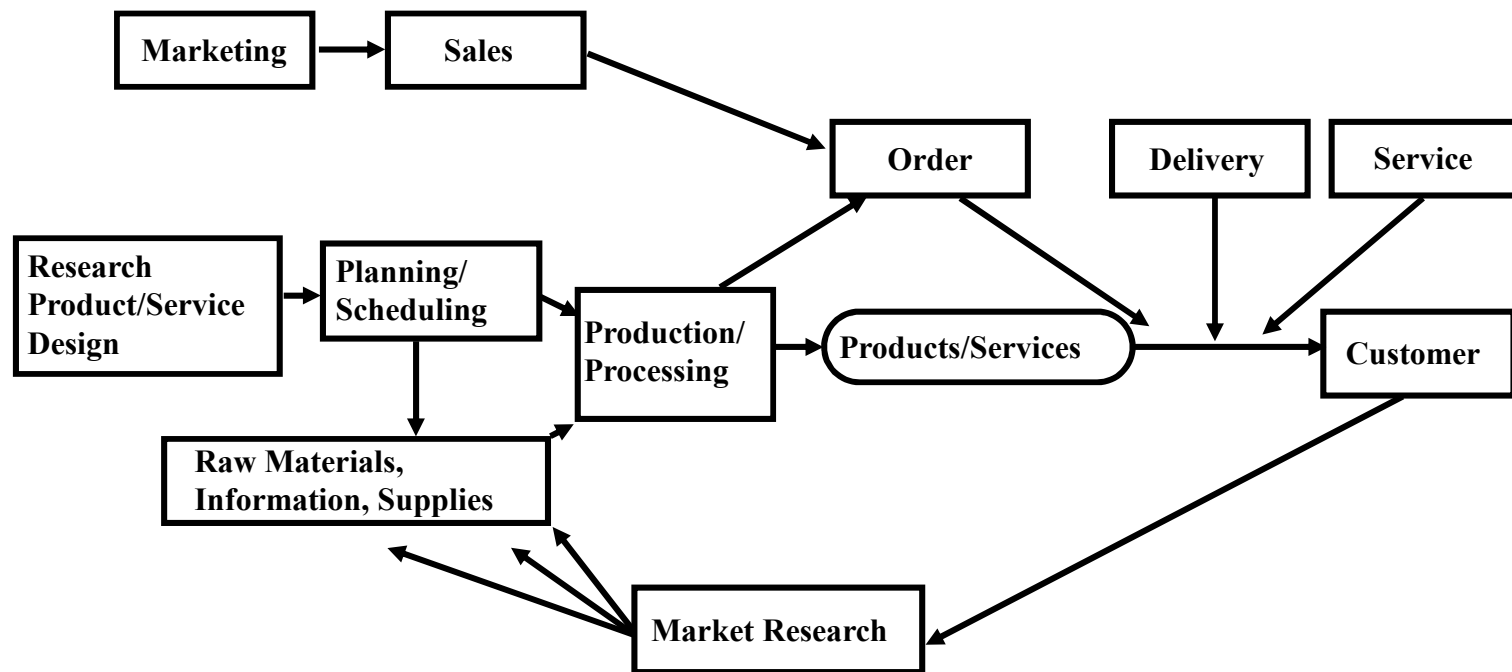
Process step	1	2	3	4	5	6	7	8	9	10	Total	%Total
Time (hours)	12	10	1	10	20	6	10	1	10	20	100	100%
Value-added			✓					✓			2	2%
Non-value-added												
Fixing errors									✓		10	10%
Prep/set-up												
Control/inspection						✓					6	6%
Delay	✓				✓					✓	52	52%
Transporting/motion		✓		✓			✓				30	30%
<b>Total</b>											100	100%

# Anexas Consultancy Services

## The Value Stream

# Work as a System

Making improvements that last requires us to see work as a system—the result of a series of interactive functions, operations, methods, and processes

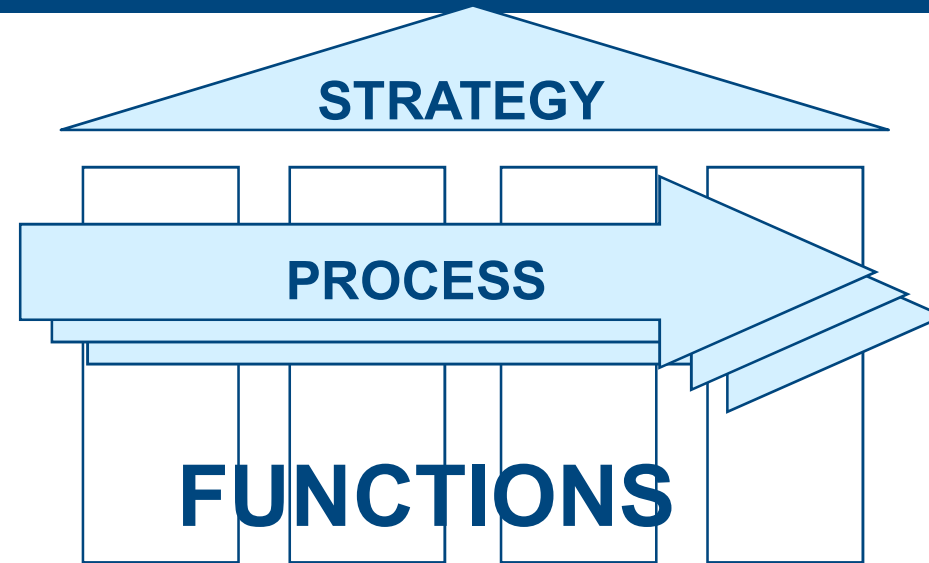


**Visualize the workflow from end to end!**

# Quality, Systems, and Processes

- Quality is judged by customers based on the output of a process or the system
- Focusing only on the individual worker will not lead to greatly improved quality that lasts
- To improve quality, the processes and system must be improved
- Simply focusing on or defining a process or the system is **not** improvement—we have to make changes and use data to show the change is improvement

# The Real Organization

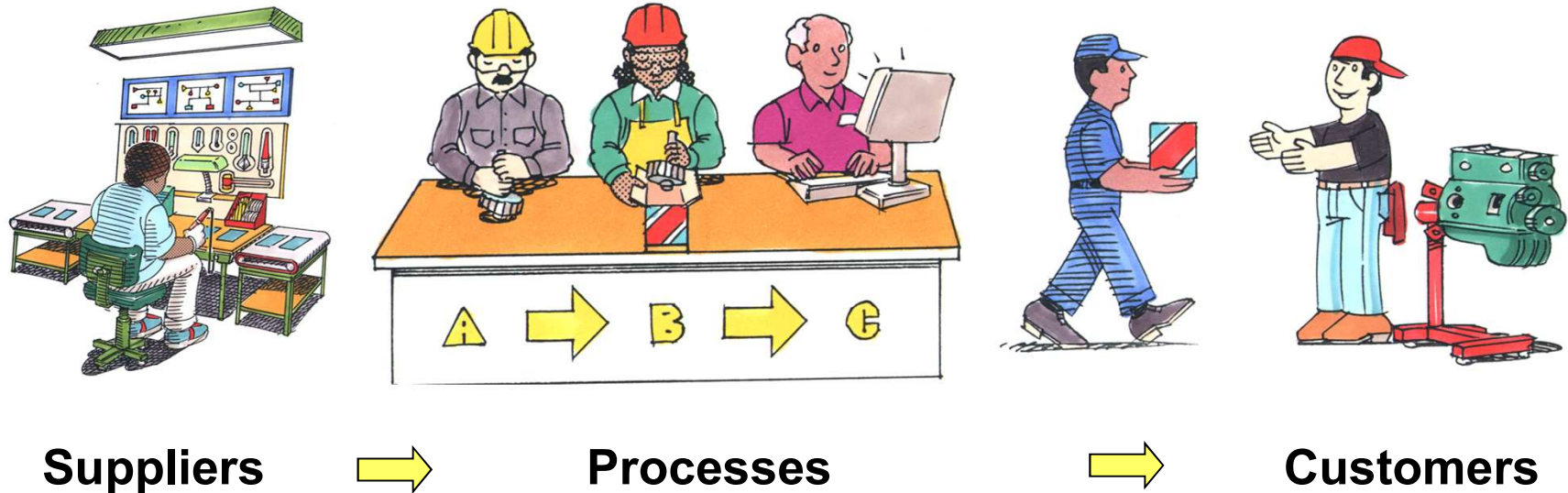


- Products and services for customers are not produced by functions, but by processes that tend to cross functional lines
- Some of these processes tend to be the way customers identify with the company
- Functions are (or should be) the residence and building place for competencies
- Process capabilities and competencies are the key leverage points to achieve strategy (desired state of the organization)



# What Is the Value Stream?

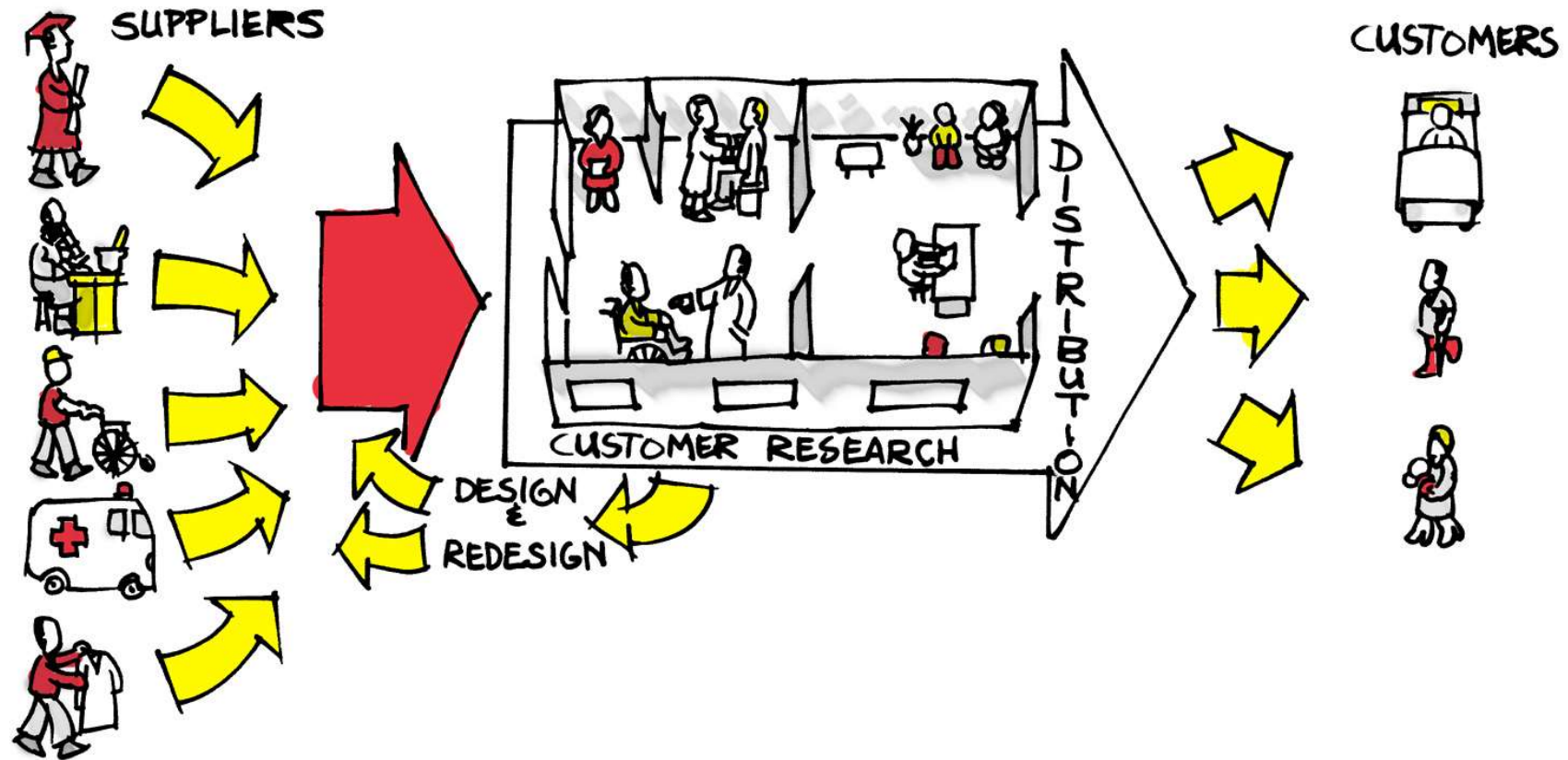
The value stream is the entire set of activities required to bring a product or service into the hands of the customer<sup>1</sup>



<sup>1</sup> From *Lean Thinking*, Womack and Jones, 1996.

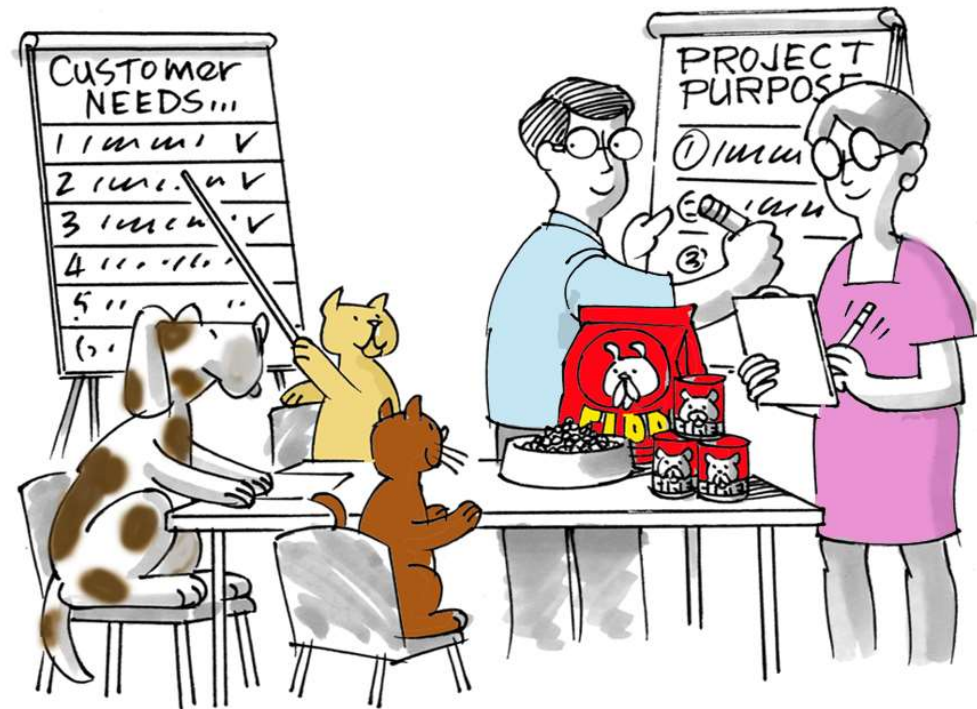
# What Is the Value Stream?, cont.

Value streams can be defined for any industry



# Value Stream and Customer Requirements

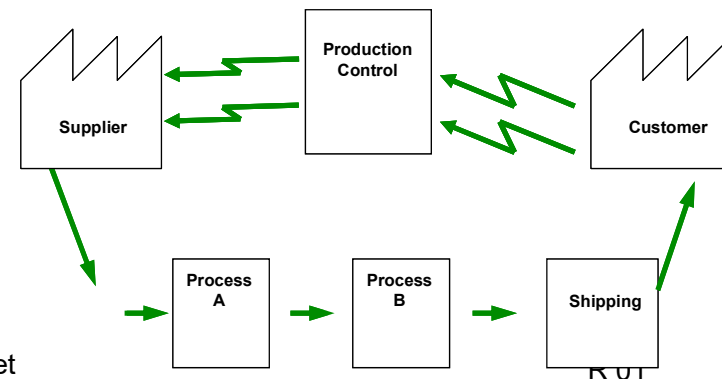
- The value stream exists to provide products and services that meet the customer requirements
- Understanding customer requirements is critical to the value stream mapping process



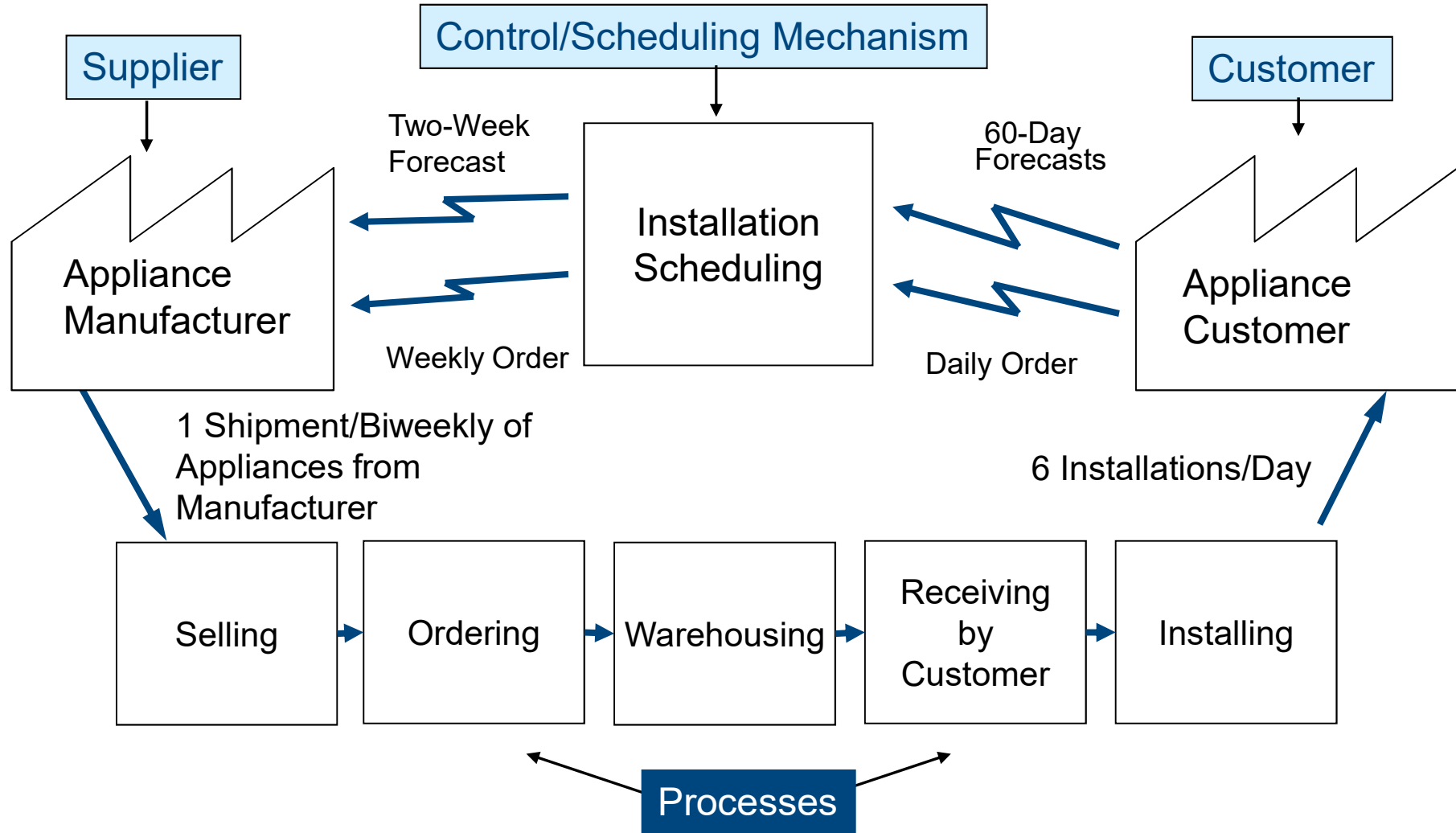
# What Is a Value Stream Map (VSM)?

A workflow visualization tool for representing how customer needs are met

- By showing the flow of materials and information, waste is easier to see and opportunities to improve are easier to identify
- A picture of the entire value stream:
  - Includes material and information flows
  - Can be applied to any industry, product, or service
  - Is a paper-and-pencil technique



# Example: High-Level Value Stream—Appliance Sales and Installation



# Selecting a Value Stream

- Maintain a customer focus
- Select a product/service family
- Stay narrowly focused; create the flow for one product family or one service at a time

## **Financial Services Company**

### **Service Families**

Credit cards

Auto loans

Direct banking

Commercial loans

## **Truck and Railcar Loading Manufacturer**

### **Product Families**

Loading racks and platforms

Loading arms

Fall prevention systems

# Origin of Value Stream Mapping

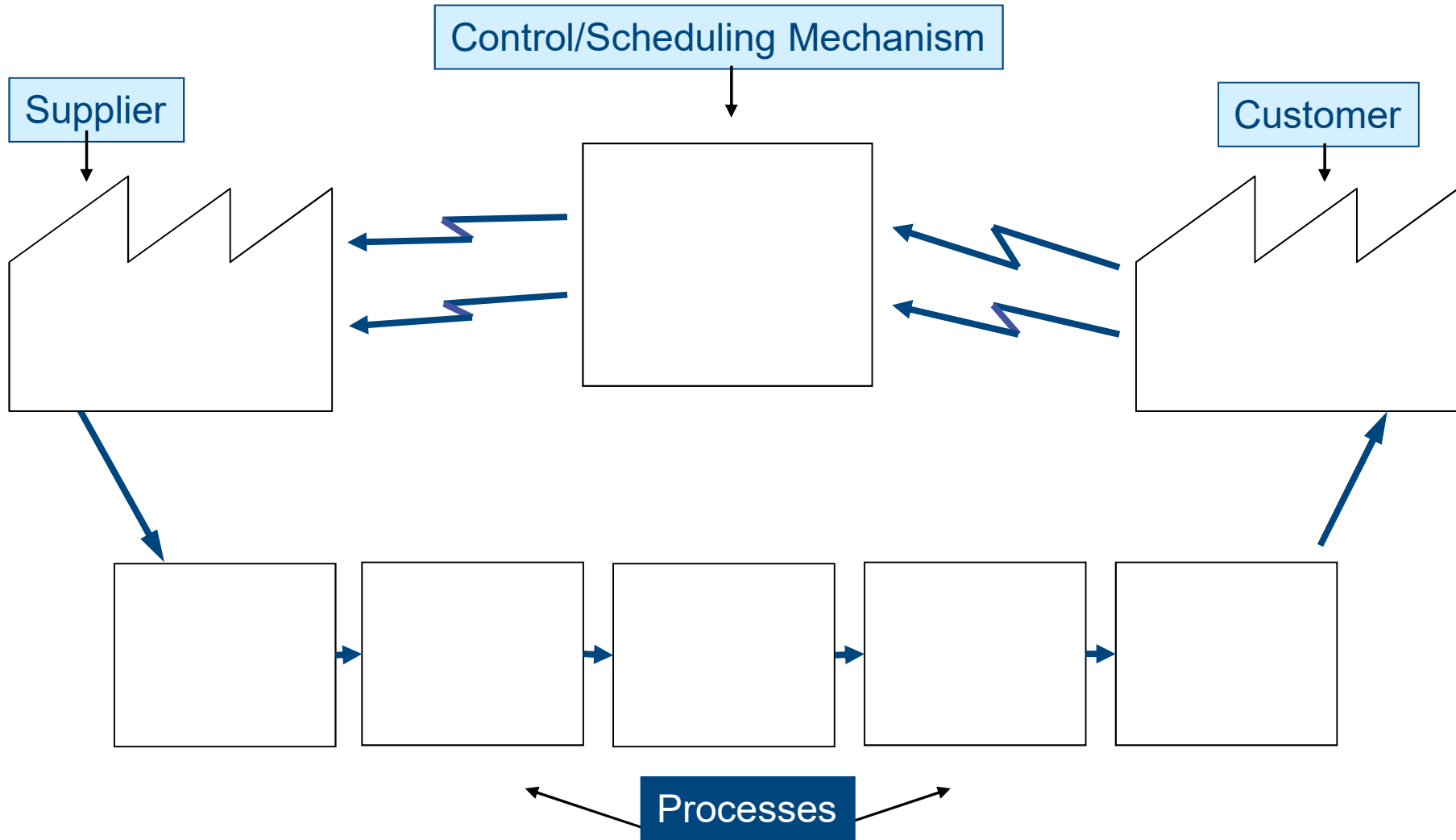
- Originated as part of Lean production concepts
- Originated in Japan as part of the Toyota Production System
- Originally called “material and information flow”
- Popularized in U.S. with the development of Toyota Supplier Support Centers (1990s–present)
- Method was documented and published in the U.S. in 1999 under the term “value stream mapping”

# Exercise: Identifying a High-Level Value Stream

- **Objective:** Identify a value stream for your workplace
- **Instructions:**
  - Work individually
  - Select a product or service family in your work area, department, or enterprise
  - Use the template to document the value stream
  - Note that for this exercise, we are interested in identifying just the value stream itself, not all of the details we will add later
  - Be prepared to share with the larger group
- **Time:** 15 minutes

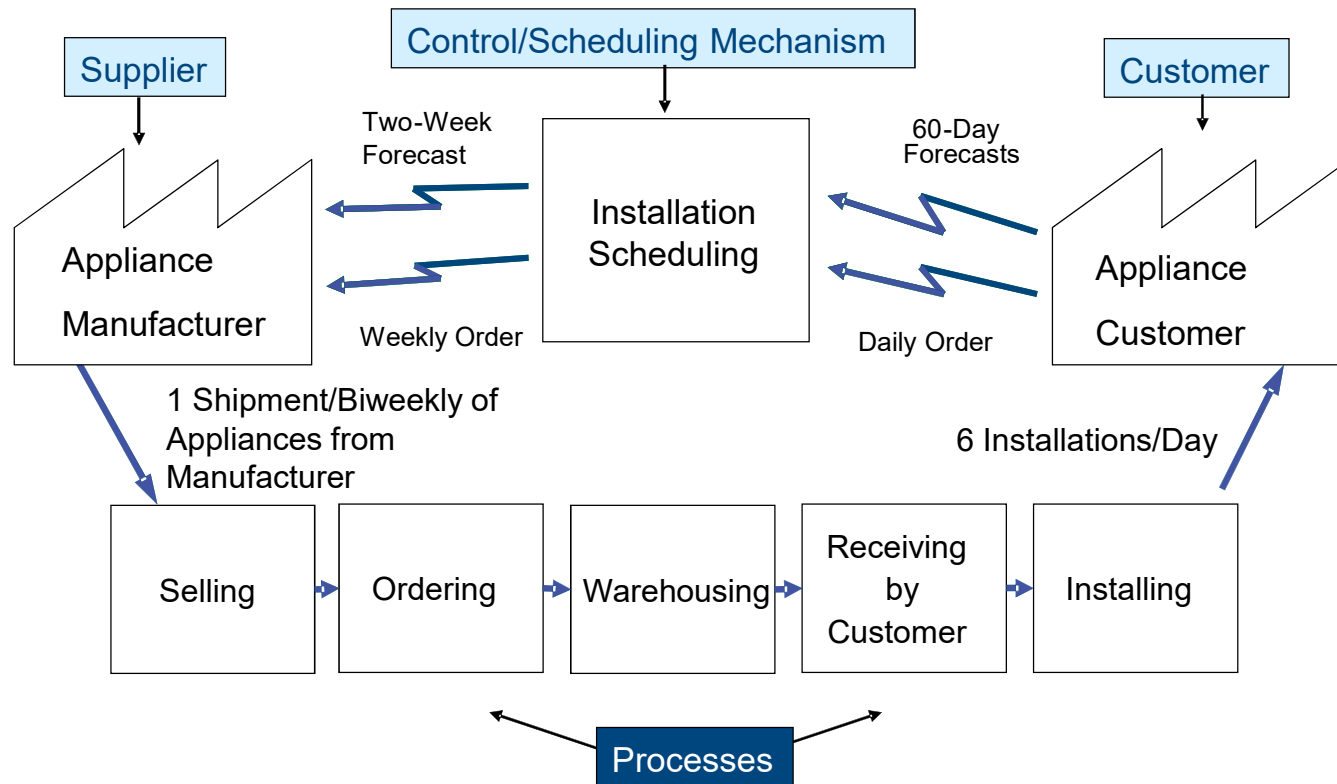


# Exercise: Identifying a High-Level Value Stream, cont.



# High-Level Value Stream

Can you identify waste and improvement opportunities from the high level?



**More detail is needed in order to be useful.**

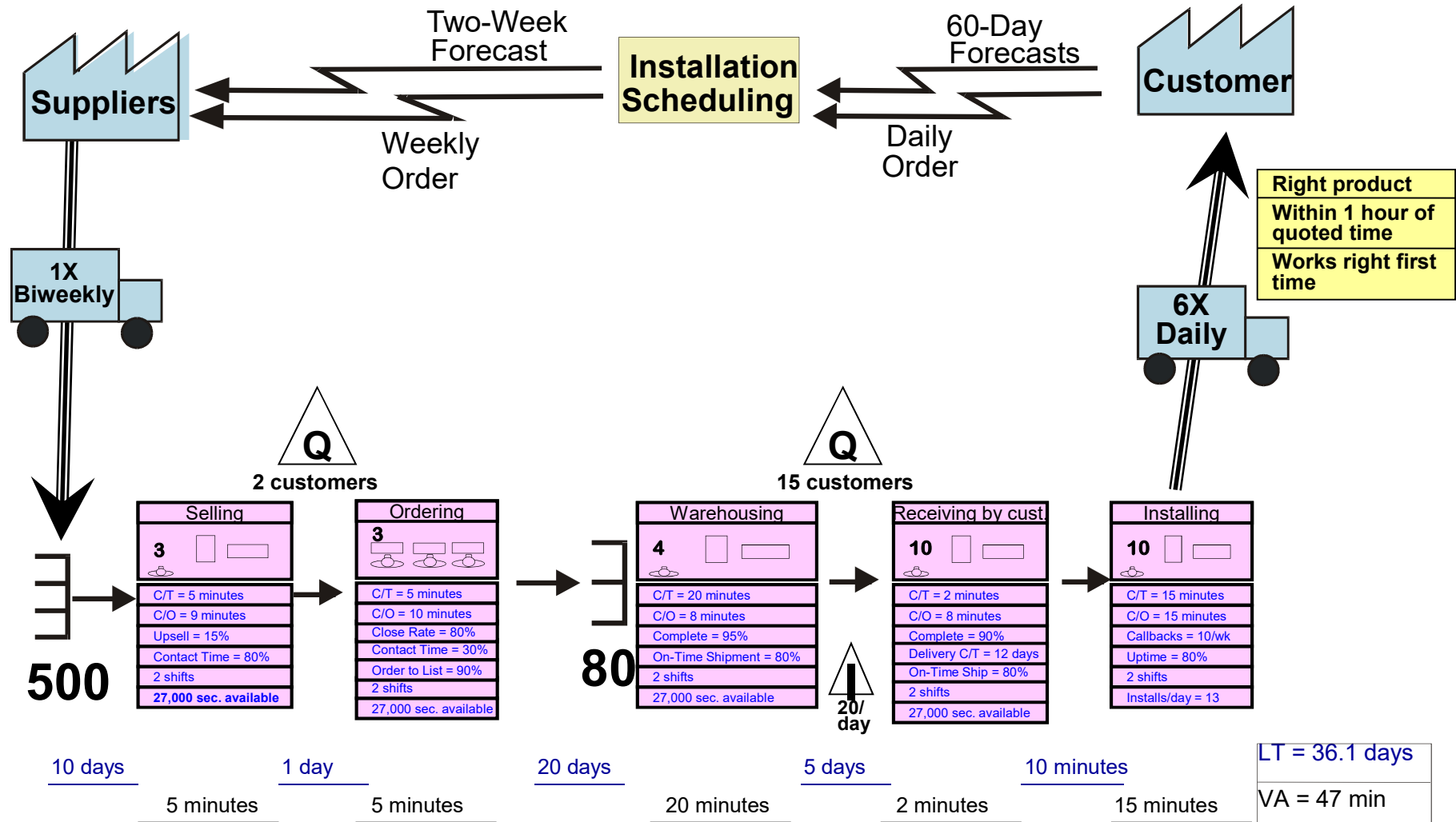
# Anexas Consultancy Services

Comprehensive Value  
Stream Maps

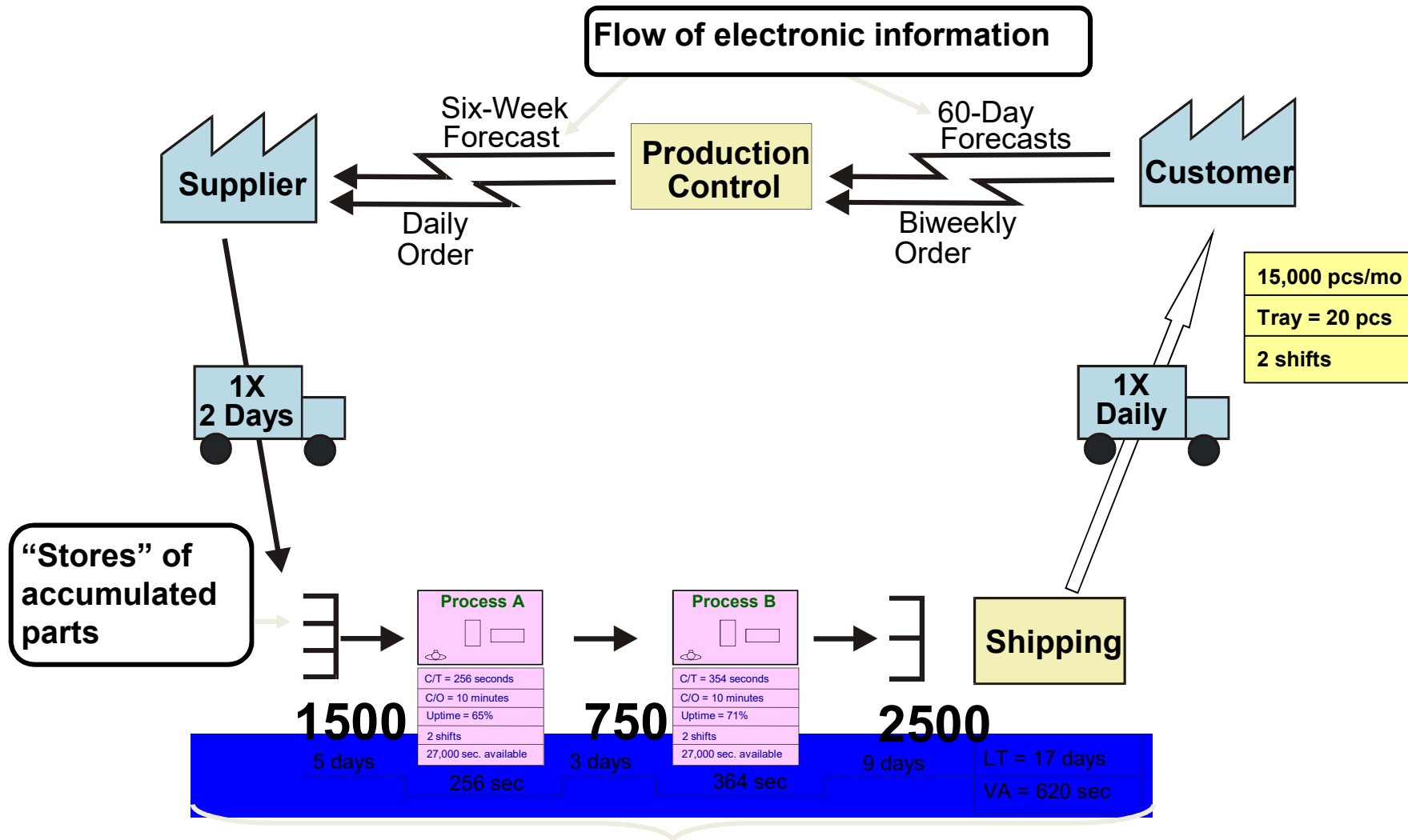
# Elements of Comprehensive Value Stream Maps

- Supplier capabilities
- Process requirements and timing
- Process performance data
- Customer requirements
- Required production control/service scheduling information
- Indication of how the product/service flow is pulled and leveled through the processes
- Stated objectives of total efficiency

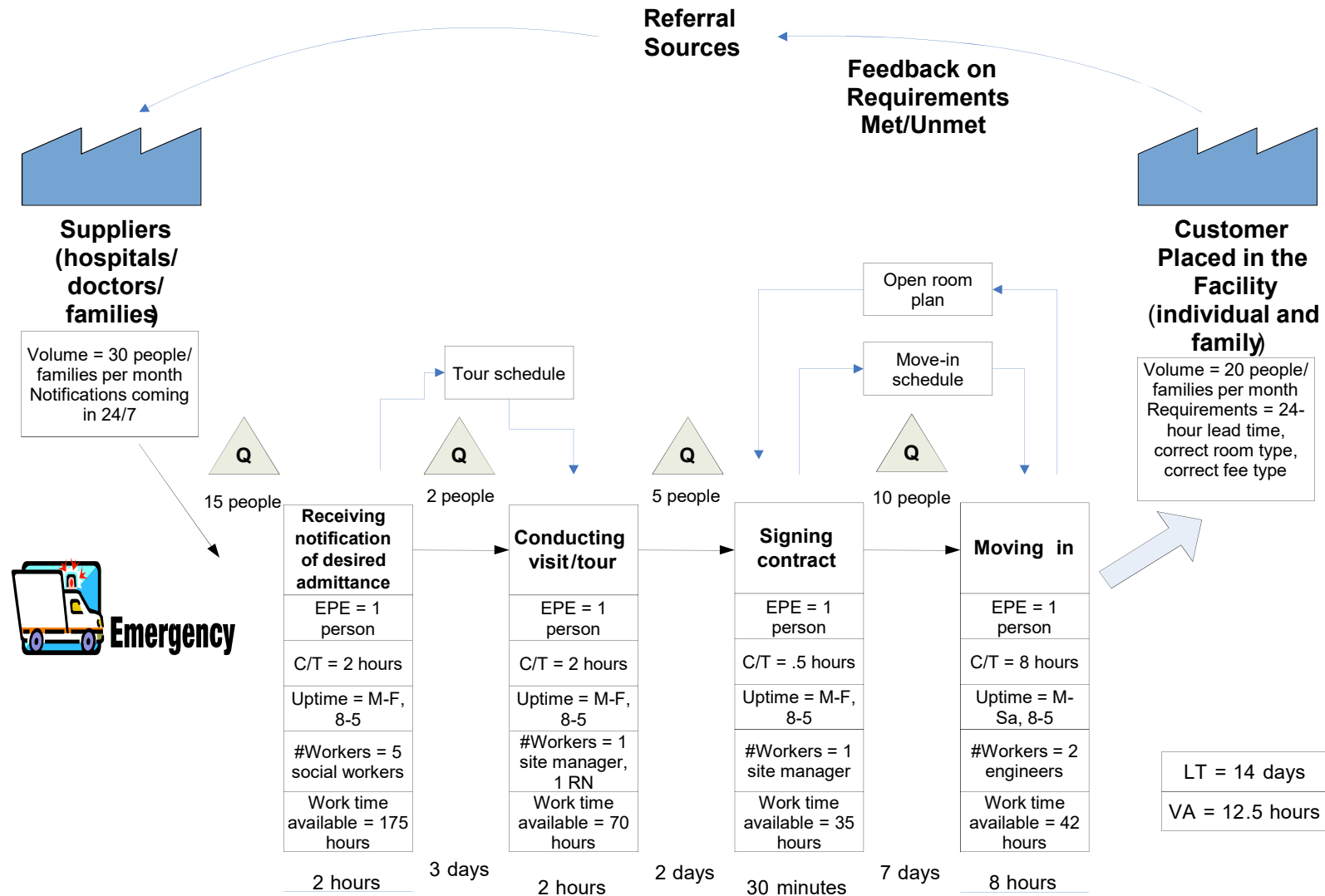
# Example: Comprehensive VSM—Appliance Sales and Installation



# Example: Manufacturing



# Example: Healthcare



# What's Different About Value Stream Mapping?

- Broader range of information gathered and displayed
- Broader scope (from door to door)
- Typically a higher level view than most process maps
- Future oriented
- Used as a strategic business planning tool to manage a change process



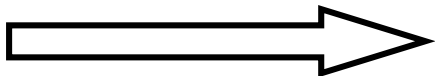
# Basic Symbols for Value Stream Maps

- Value stream maps use symbols as abbreviations
- Basic symbols can be used for any value stream, whether manufacturing, administrative, transactional, etc.
- Some symbols may be generic for a particular type of industry
- Some organizations have symbols that are specific to that organization

# Basic Symbols for Value Stream Maps, cont.



Use arrows on all lines to show clear direction of flow



Use a wide arrow to show finished goods or services to the customer



Indicates electronic communication

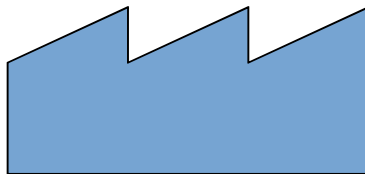


Indicates process (i.e., assembling, underwriting, etc.); each box represents one process



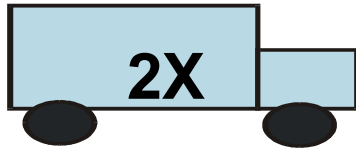
Indicates “stores” of shelves; accumulation location for parts/materials, documents, or emails directly following a process or operation with predetermined quantity requirements

**1350**



Icon may be used to represent customer or supplier

# Basic Symbols for Value Stream Maps, cont.



Transportation, shipping, or delivery icon: information inside box designates frequency of delivery



Signaling cards (kanban): signaling pull of work from process downstream

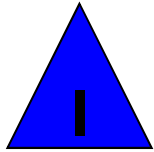
Process B	
C/T = 56 seconds	
C/O = 10 minutes	
Uptime = 80%	
2 shifts	
27,000 sec. available	

Data box: contains performance data for the particular process



Indicates the timeline: the total time the product or service flows through system—lead time

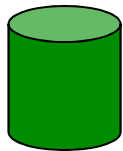
# Basic Symbols for Value Stream Maps, cont.



Denotes inventory waiting for the next process



Denotes work waiting in queue for the next process



Denotes electronic storage



Denotes opportunity for improvement



Represents a person

# Benefits of Value Stream Mapping

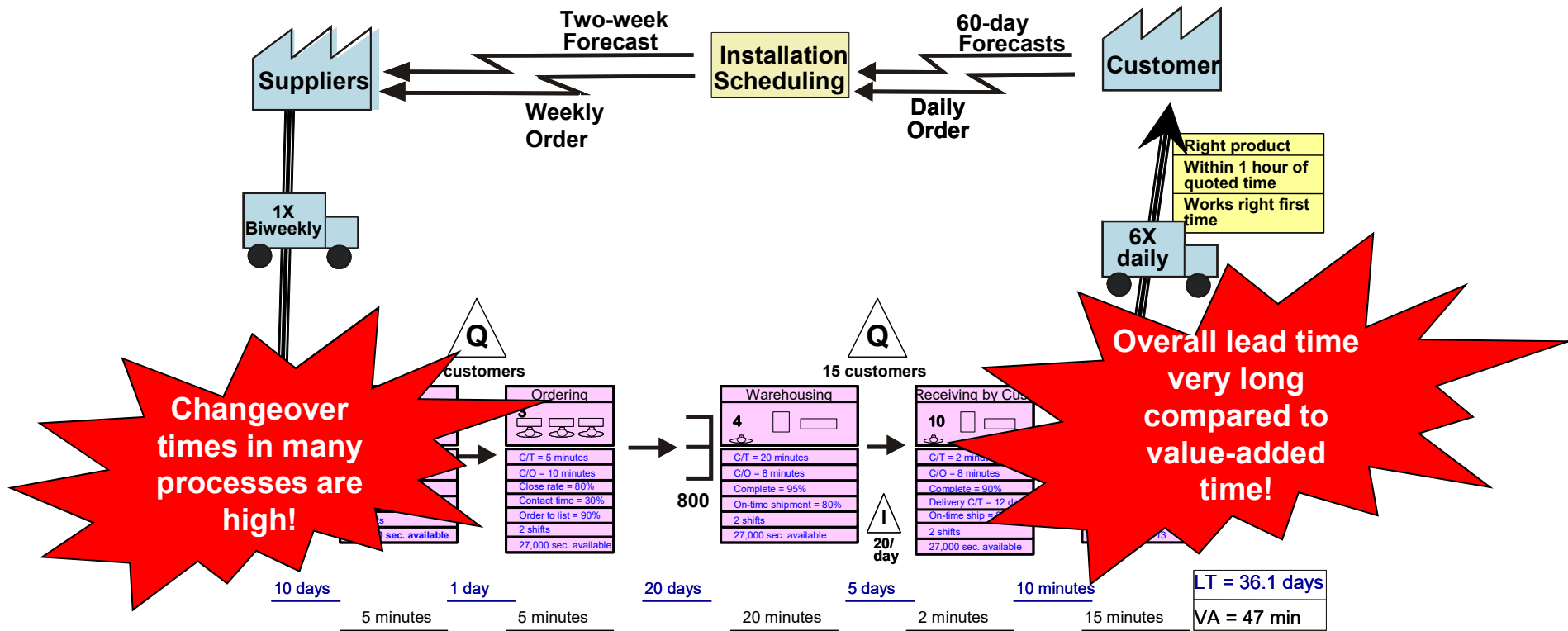
- Forces improvement on the whole, not just on optimizing individual processes
- Provides a common language for talking about processes
- Creates common understanding of how value is produced
- Helps employees see how their function or process fits into the overall value stream
- Visually identifies where waste occurs
- Helps to focus improvement efforts
- Shows linkage between product/service flow and information flow
- Shows performance data

# Anexas Consultancy Services

See the Waste in the  
Value Stream

# What to Look for

- When you are analyzing your current state map, you are looking for opportunities to reduce or eliminate the waste
- Many issues become apparent just by creating a visual



## Exercise: See the Waste

- **Objective:** Practice identifying waste using a current state value stream map
- **Instructions:**
  - Use either:
    - A value stream map from your own organization
    - The example provided by the instructor
  - Identify waste and opportunities for improvement
    - What, if anything, can you see with a value stream map you could not see using the types of waste?
  - Be prepared to report out
- **Time:** 15 minutes



# Summary

- The value stream is the entire set of activities required to bring a product or service into the hands of the customer<sup>1</sup>
- Value streams are typically created for a single product or service family
- A current state map makes the flow of work visible and helps you see the waste

<sup>1</sup>From *Lean Thinking*, Womack and Jones, 1996.

## Summary, cont.

- Value stream maps may look very different depending on the value stream and the industry
- Future state value stream maps are used after specific Lean approaches or solutions have been identified

# Anexas Consultancy Services

**ANALYZE: Analyze to  
Identify Causes**

Review

# You Are Here—ANALYZE



# ANALYZE Phase Deliverables

- Cause-and-effect analysis to identify potential causes (inputs or X's)
- Analysis of inputs to identify and prioritize the key inputs or X's affecting the process output(s) Y
- Data collected to validate key inputs or X's
- Graphical (and statistical) analysis of data to determine relationships
- Reduced list of potential key inputs that affect the output(s) Y (refined cause-and-effect analysis)

## ANALYZE Phase Deliverables, cont.

- Evidence linking the key inputs to the outputs
- ANALYZE section of storyboard completed
- ANALYZE phase discussed and agreed on with sponsor and coach
- ANALYZE deliverables completed and phase approved

# ANALYZE Checklist

- By the end of the ANALYZE phase, you should be able to show your sponsor which causes you will focus on in IMPROVE by being able to answer the following questions. Please check!:
  - What potential causes (X's) have you identified for each focused problem statement?
  - Which potential causes have you decided to investigate, and why?
  - How have you verified causes?
  - What data has been used? How, when and where was it collected?
  - What is the data showing?

## ANALYZE Checklist, cont.

- By the end of the ANALYZE phase, you should be able to show your sponsor which causes you will focus on in IMPROVE by being able to answer the following questions. Please check!, cont.:
  - Do findings agree with process knowledge?
  - What are the root causes, and how do they affect the process output (Y)?
  - Which of the root causes are within the team's control? Is team makeup still appropriate?
  - What are the proven root causes (critical few) to be tackled in the IMPROVE phase?



# Anexas Consultancy Services

## IMPROVE: Implement Solutions and Evaluate Results

### Overview

# You Are Here—IMPROVE

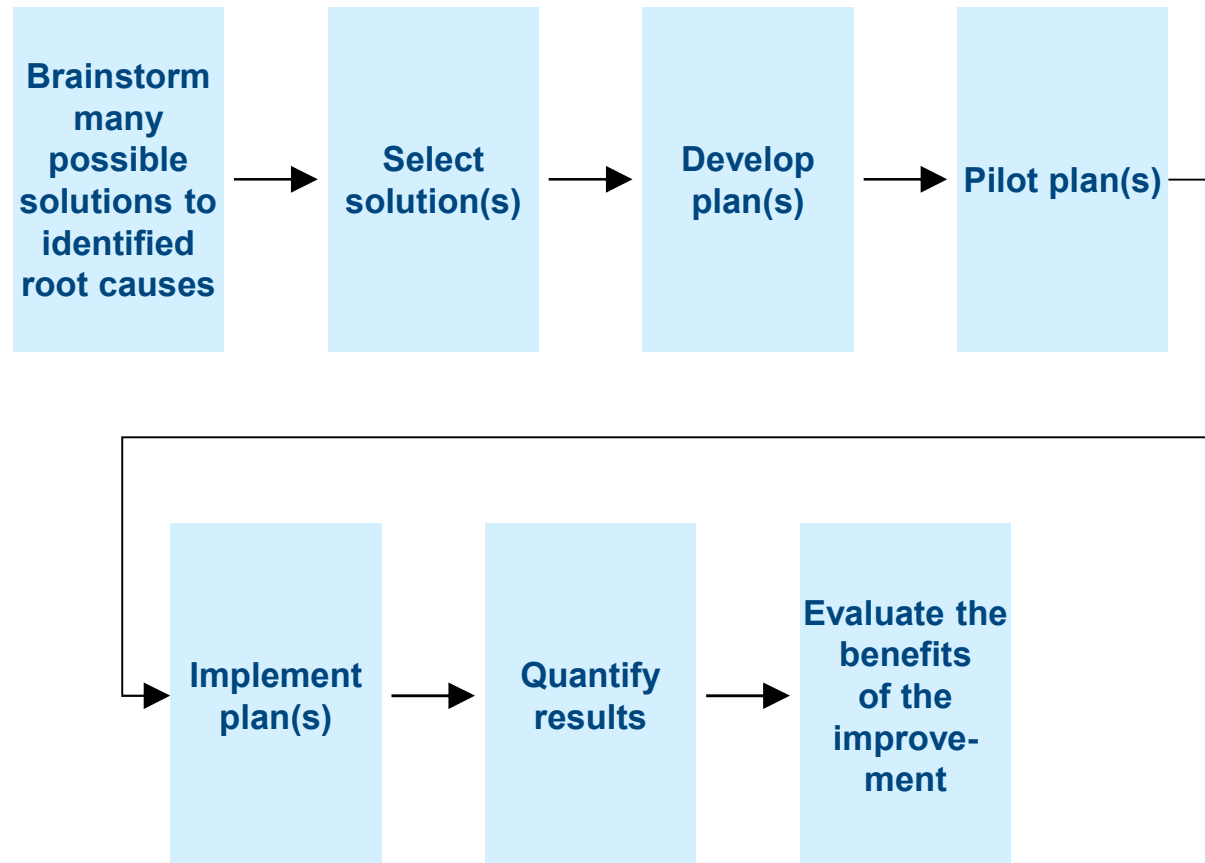


# IMPROVE: Implement Solutions and Evaluate Results

- **Goal:**
  - Develop, try out, and implement solutions that address root causes
  - Use data to evaluate both the solutions and the plans used to carry them out
- **Output:**
  - Planned, tested actions that eliminate or reduce the impact of the identified root causes
  - Before and after data analysis that shows how much of the initial gap was closed
  - A comparison of the plan to actual implementation



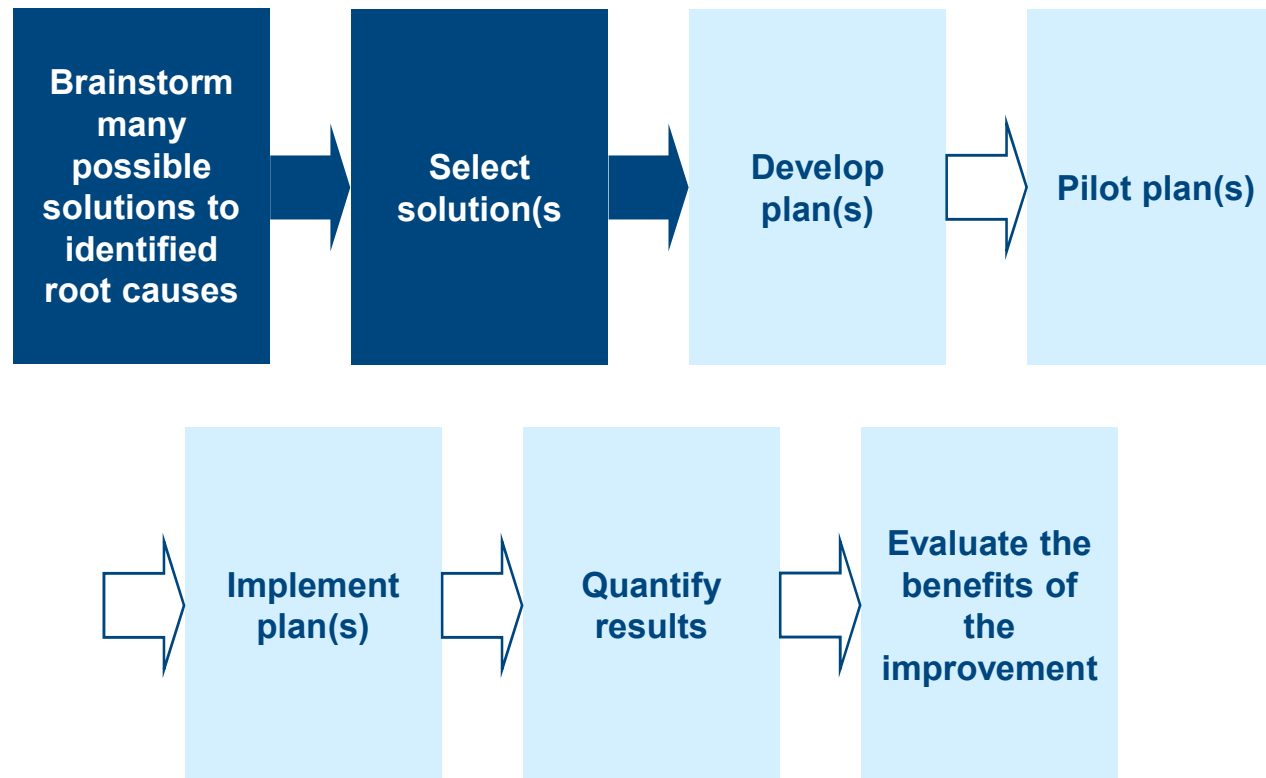
# Approach to IMPROVE



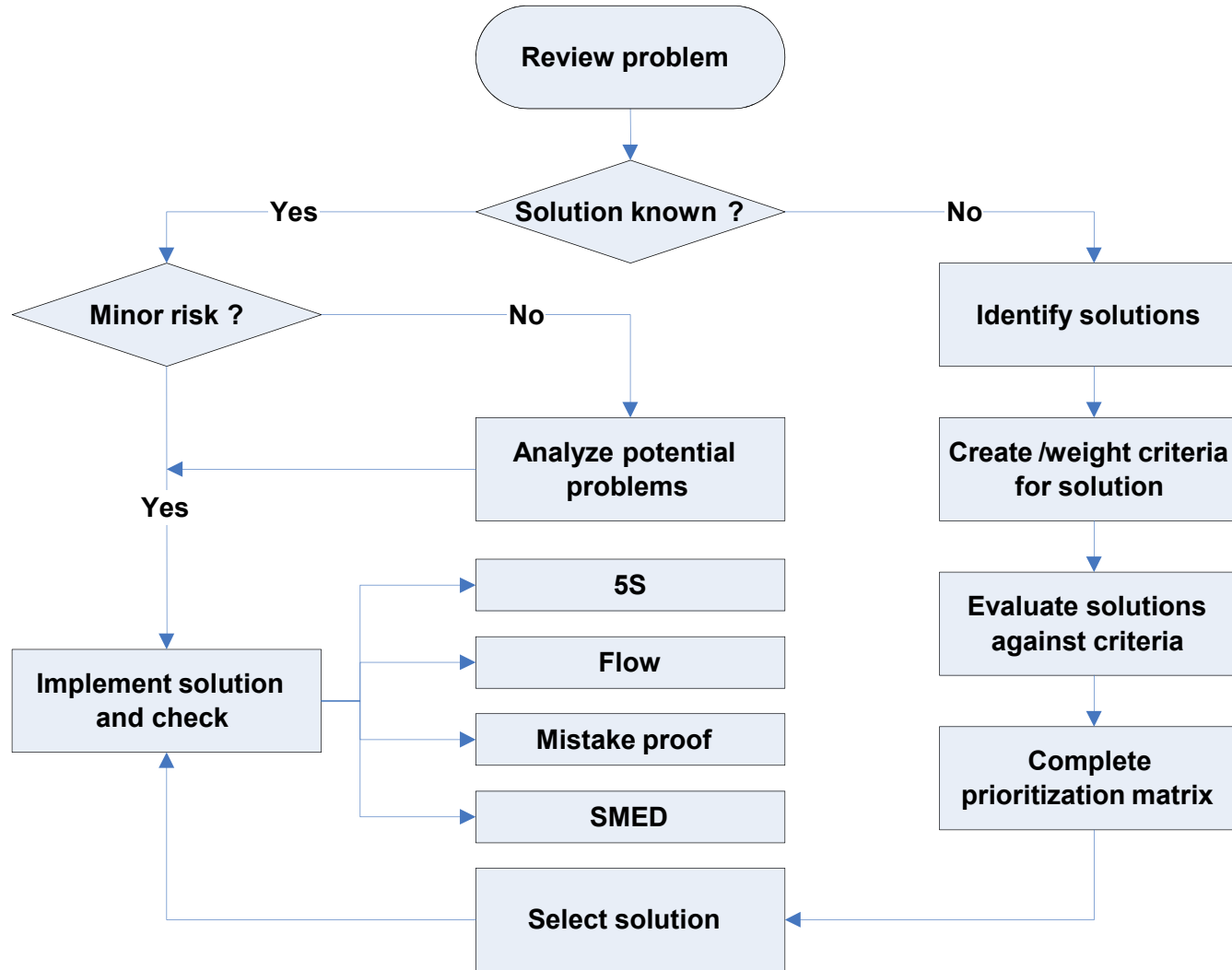
# Anexas Consultancy Services

## Generating and Selecting Solutions

# You Are Here



# Generating and Selecting a Solution Roadmap



# Involving People in Developing Solutions





# Anexas Consultancy Services

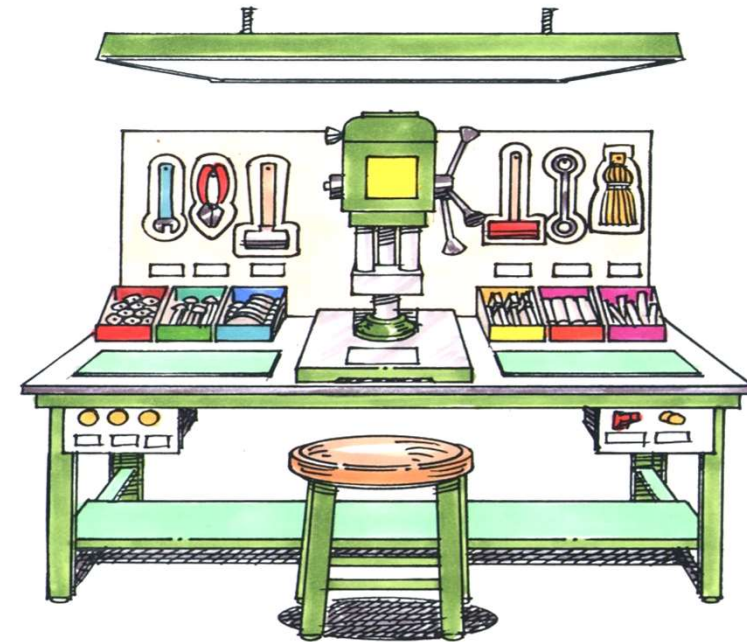
## Lean Solutions

Process Improvements: **5S**, Flow,  
Mistake-Proofing, and SMED

# 5S

- Sort
- Straighten
- Sweep, wash, clean, and tag abnormalities
- Standardize
- Sustain (housekeeping and tagging)

**Keep It Tidy!**



# Workplace Readiness (5S) Actions and Outcomes Plan

<b>Organization and Safety Worksheet                      Workplace Readiness (5S) Actions and Outcomes Plan</b>		
<b>Category</b>	<b>Required Action</b>	<b>Desired Outcomes</b>
<b>Sort</b>	Remove unneeded and unused items from the workplace.	A safe and uncluttered worksite, free of hazards and workarounds.
<b>Straighten</b>	Arrange worksite tools, equipment, and materials in the most convenient location for process use. Identify, label, color code. <b>Note:</b> Follow the <i>Principles of Safe Human Motion</i> .	Tools, equipment, and materials are located within safe and easy reach of use. Waste of motion is at a minimum.
<b>Sweep, Wash, and Clean</b>	Clean the work area, tools, and equipment – tag equipment abnormalities.	The worksite, required tools, equipment, and materials are clean, defect free, and ready for use.
<b>Standardize</b>	Document worksite layout, location of tools, equipment, and materials. Establish a plan and team assignments to maintain operational readiness.	A plan with documented graphic layout of the worksite showing the proper location and amounts of all required tools, equipment, and materials, including visual controls and coding, with required team member actions and assignments.
<b>Sustain</b>	Follow the plan. Improve the plan and worksite.	A continuously ready operational worksite; excellent housekeeping.

# Anexas Consultancy Services

## Lean Solutions

Process Improvements: 5S, **Flow**,  
Mistake-Proofing, and SMED

# Takt Time: The Pulse of One-Piece Flow

- **Takt time** is defined as the amount of time available to produce one unit (one-piece flow is paced to takt time):
  - Usually measured in seconds
  - Used to synchronize the process and establish uniform production flow
  - Based on:
    - Anticipated customer demand
    - Time available or scheduled to produce the right amount of product to meet the sales demand

- Takt time = 
$$\frac{\text{Total production time available}}{\text{Number of units required to meet customer demand}}$$

# Workload Balancing

- The balancing of work performed at and between operations is based on human motion and takt time
- The aim is to:
  - Equalize the workload fairly and safely
  - Eliminate unevenness and overburden
  - Facilitate the ability to easily increase or decrease the flow of production based on customer requirement

# Benefits of a Balanced Workload

- A balanced workload makes redistribution of work safer, easier, and faster:
  - Manpower and cycle time can be increased or decreased while keeping production smooth and uninterrupted
  - Workload balancing facilitates standardization:
    - Where there is no standard, there can be no sustained improvement
  - Balancing provides a safe and consistent workload
  - Supports level production objectives

# ESCAP Analysis

All the process steps are evaluated based ESCAP criteria.

It is checked if following can be done to any of the steps:

- Eliminate
- Simplify
- Combine
- Automate
- Parallel



# Anexas Consultancy Services

## Lean Solutions

Process Improvements: 5S, Flow,  
**Mistake-Proofing**, and SMED

# Continuum of Devices

**Preventative**

**Reactive**

	<b>Control Device</b>	<b>Warning Device</b>	<b>Shutdown Device</b>
<b>Possible Error</b>	Prevents the cause of an error from occurring	Warns that an error has occurred	Shuts process down when defect occurs
Possible error is the wrong password entered for an ATM transaction	ATM machine only accepts a set number of digits	ATM notifies users that password has been incorrectly entered	ATM transaction is deactivated if user incorrectly enters the password more than three times
Cut wood strips don't meet specifications	Wood strip cutting machine has a stop mechanism that prevents overloading feed material	Alarm sounds if overloading occurs	Machine stops if cut length and width deviate from specifications

# Anexas Consultancy Services

## Lean Solutions

Process Improvements: 5S, Flow,  
Mistake-Proofing, and **SMED**

# SMED—Rapid Changeover

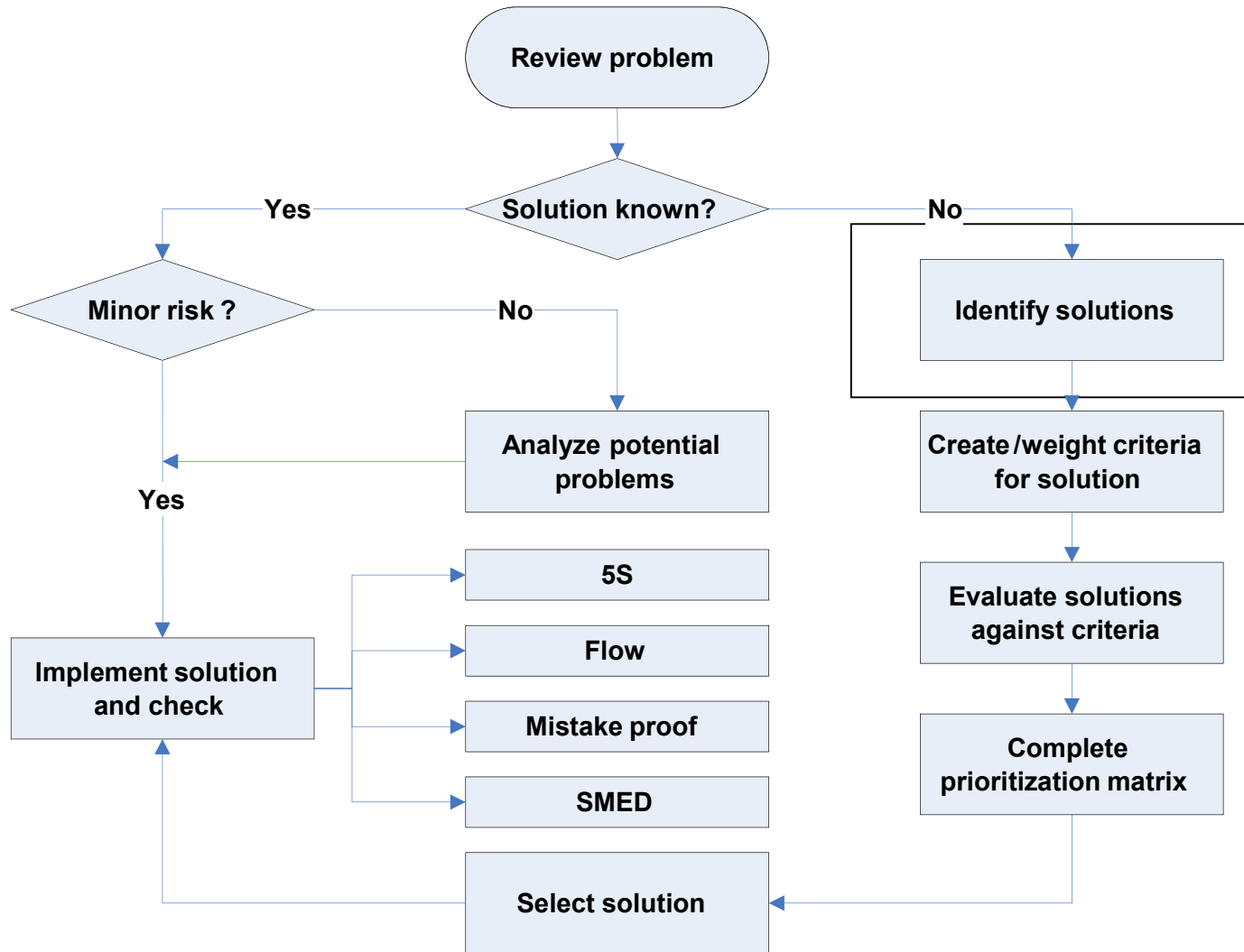
- Rapid changeover means performing changeovers or operations setup in less than 10 minutes
- Sometimes you will hear this referred to as SMED, for Single Minute Exchange of Die
- Although the concept originated in manufacturing, it can also be applied to services



# Anexas Consultancy Services

## Generating Solution Alternatives

# Generating and Selecting Solution Roadmap



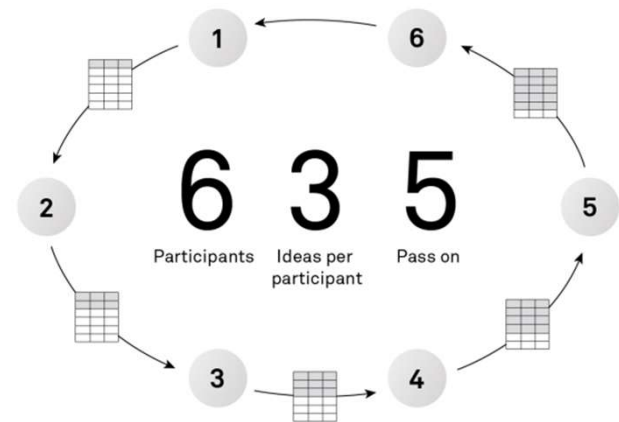
# Generating Solution Ideas

- Review what you know about the process and the verified cause
- Brainstorm solution ideas; use creativity techniques
- Combine ideas into solutions



# Brainwriting 6-3-5

- Team members brainstorm ideas on a written form:
  - Take 5 minutes to write down three solution ideas on the first row of your form
  - Pass your form to the right
  - On the form you have just received from your team member, add another three ideas on the next row
  - Add ideas by:
    - Enhancing an idea already on the sheet
    - Adding a variation of an idea on the sheet
    - Adding a completely new idea
  - Repeat for as many rounds as you have team members





# Round Robin and Anti Solution

## Round Robin

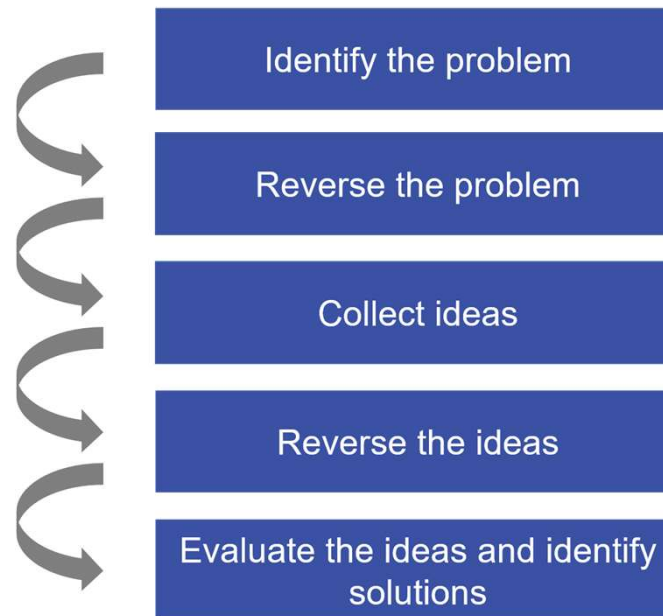
- Everyone gets a chance to put forth his/her idea. If they do not have to contribute an idea, they just say pass.
- This goes on till all the participants have exhausted their ideas.



# Round Robin and Anti Solution

## Anti Solution

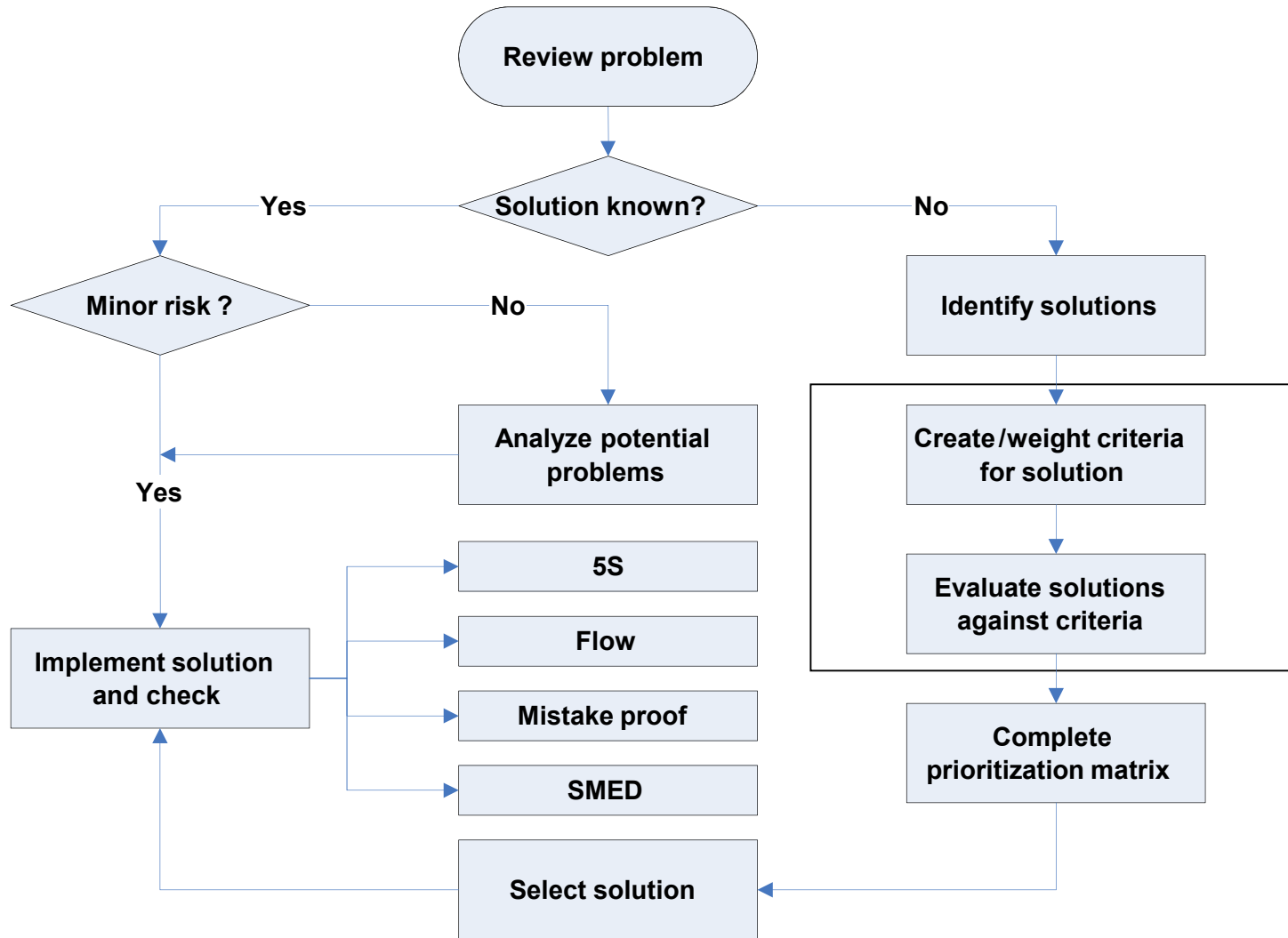
- Team brainstorms on how to increase the problem rather than solving it.
- The brainstormed ideas are reversed to get the solution.



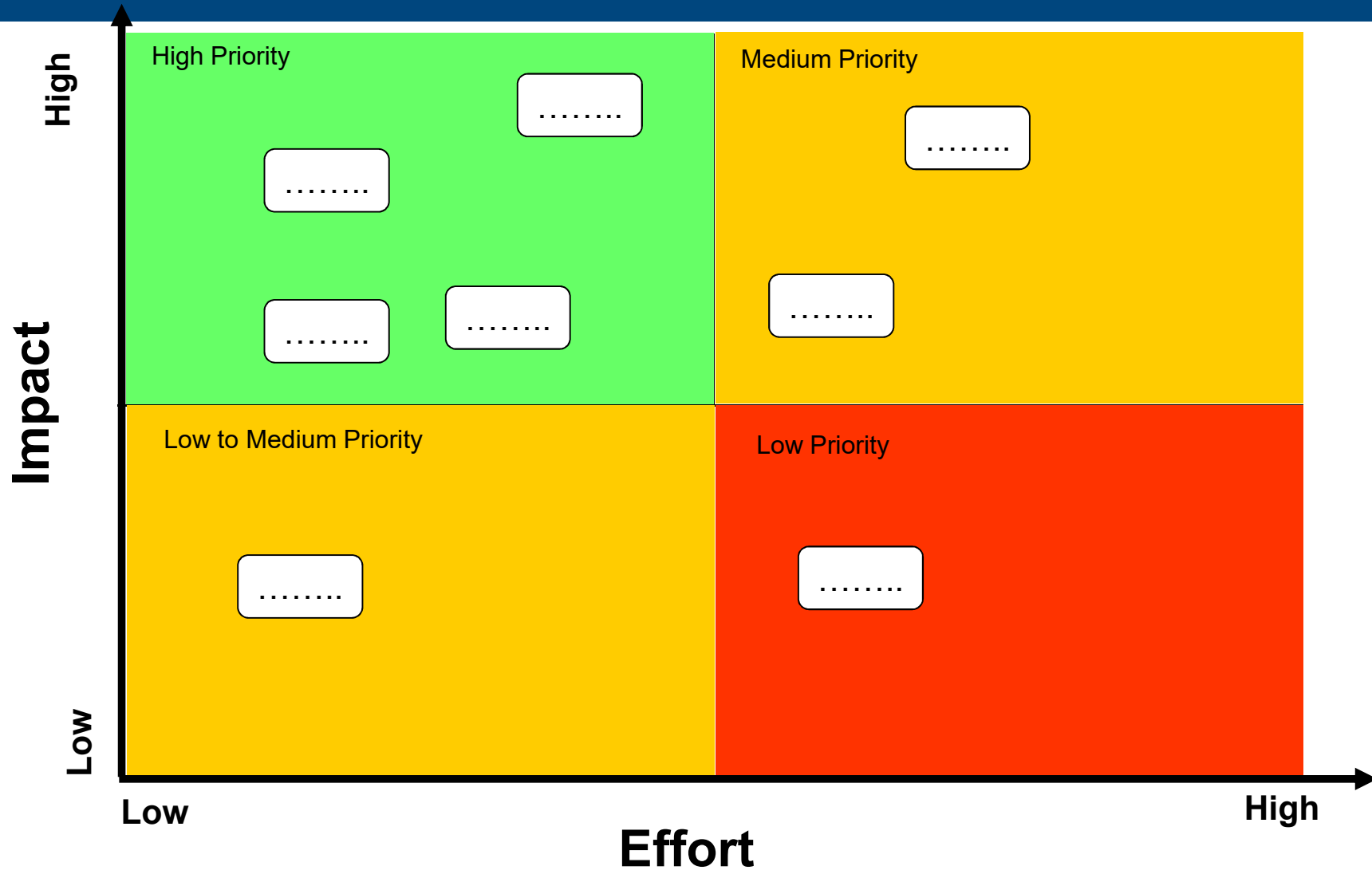
# Anexas Consultancy Services

## Evaluating Solution Alternatives

# Generating and Selecting Solution Roadmap



# Prioritization Tool — Solutions



# Solution Selection Matrix

Criteria		Weight	Solution A		Solution B		Solution C	
			Score	Weighted Score	Score	Weighted Score	Score	Weighted Score
1	cheap solution	3	3	9	9	27	9	27
2	quick to implement	3	9	27	1	3	3	9
3	high impact on CTQs	9	9	81	9	81	9	81
4	compliant	9	1	9	9	81	9	81
5	change management	3	1	3	3	9	9	27
6	resource required	1	9	9	3	3	3	3
<b>TOTAL</b>				<b>138</b>		<b>204</b>		<b>228</b>

Where **weight** and **scores** on following scale : High = 9, Medium = 3 and Low = 1.

## Example :

Solution A = outsource all data processing

Solution B = development of our own software

Solution C = buy a software and adapt to our needs

It seems here that solution C is the most satisfying. B also can be considered as an option.

# Summary

- Involve people
- Use creativity techniques to generate alternatives
- Generate and weight criteria
- Evaluate each alternative against the criteria
- Be creative in finding ways to get additional information about a potential solution
- If no clear choice emerges, use consensus decision making

# Anexas Consultancy Services

## Introduction to the Design of Experiments (DOE)



# Module Objectives

**By the end of this module, the participant will:**

- Understand the strategy behind Design of Experiments (DOE)
- Create a design matrix
- Code factors
- Understand the limitations of OFAT experiments
- Interpret interactions

# Why Learn About Design of Experiments?

- Properly designed experiments will improve
- Efficiency in gathering information
  - Planning
  - Resources
- Predictive knowledge of the process
- Ability to optimize process
  - Response
  - Control input costs
- Capability of meeting customer CTQs

# What is Design of Experiments ?

- A DOE is a planned set of tests on the response variable(s) (KPOVs) with one or more inputs (factors) (PIVs) each at two or more settings (levels) which will:
  - Determine if any factor is significant
  - Define prediction equations
  - Allow efficient optimization
  - Direct activity to rapid process improvements
  - Create significant events for analysis

# DOE Terminology

- Response (Y, KPOV): the process output linked to the customer CTQ
- Factor (X, PIV): uncontrolled or controlled variable whose influence is being studied
- Level: setting of a factor (+, -, 1, -1, hi, lo, alpha, numeric)
- Treatment Combination (run): setting of all factors to obtain a response
- Replicate: number of times a treatment combination is run (usually randomized)
- Repeat: non-randomized replicate
- Inference Space: operating range of factors under study

# DOE Objectives

- Learning the most from as few runs as possible; efficiency is the objective of DOE
- Identifying which factors affect mean, variation, both or none
- Screening a large number of factors down to the vital few
- Modeling the process with a prediction equation:  
$$Y = f(A, B, C \dots)$$
- Optimizing the factor levels for desired response
- Validating the results through confirmation

# Anexas Consultancy Services

## DOE Strategy

# Strategy of Experimentation

## Planning Phase

- Define the problem
- Set the objective
- Select the response(s), Ys
- Select the factors(s), Xs
- Select the factor levels
- Select the DOE design
- Determine sample size
- Run the experiment

## Execution Phase

- Collect data
- Analyze data
- Validate results
- Evaluate conclusions
- Revise SOPs
- Train affected workforce
- Document results
- Consider next experiment

# Anexas Consultancy Services

## Planning Phase

The purpose of an experiment is to better understand the real world, not to understand the experimental data...

William Diamond  
Statistician, IBM



# Define the Problem

## Step 1

- The problem statement should:
- Be complete and detailed
- Contain no conclusions
- Contain no solutions
- Contain no causes
- Be specific

**Well begun is half done...**

Mary Poppins

# Issues With Problem Definition

- Common problem definition issues
- Response not well defined
- Response not quantifiable
- Response does not link to CTQs
- Specifications not customer focused
- Quantification not based upon data
- Stated as a predetermined solution

**Guide your team to unambiguous,  
executable experiments**

# Set the Objectives

## Step 2

- The upfront objectives should
- Define what you want to learn from the experiment
- Support screening, modeling or both
- Look at main effects, interactions or both
- Set boundaries for optimization
- Know resource requirements

**Do not use all of your resources on  
the first experiment**

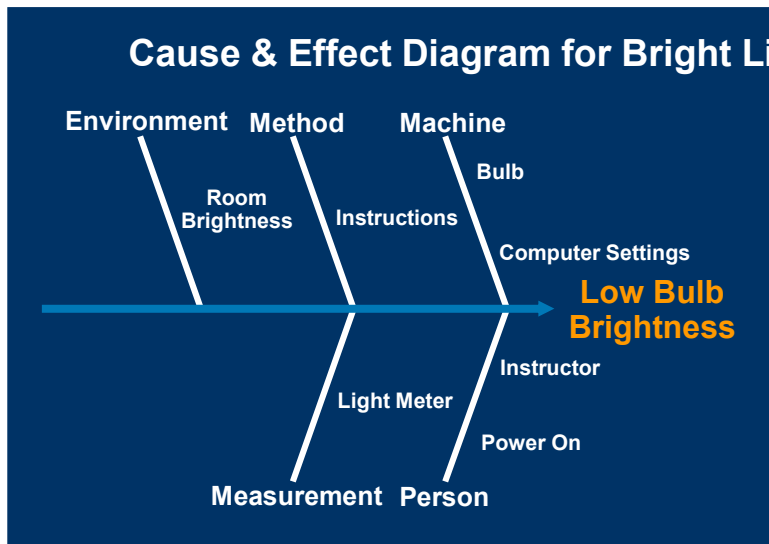
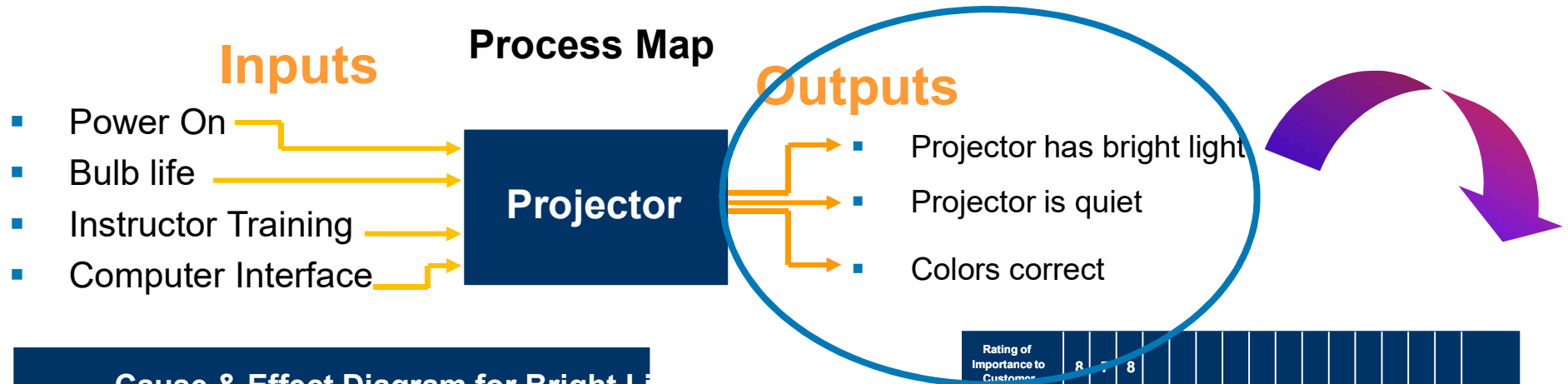
# Selecting the Response

## Step 3

A response is the output linked to the customer CTQ

- Usually quantitative (continuous)
  - Yield
  - Level of purity
  - Effort Variance
  - Energy consumed
- May be qualitative (discrete)
  - Bubbles in glass plate
  - Defective Coding
- Comes from Define and Measure Phase
  - Voice of the Customer
  - C&E, FMEA, process map and other

# Response Relationship to Other Tools



		Rating of Importance to Customer															Total						
		8	7	6	5	4	3	2	1	1	2	3	4	5	6	7		8					
Process Inputs	Bright Light																						
	Quiet																						
	Color Correct																						
1	Power on	9	3	2																			119
2	Bulb Life	8	1	3																			95
3	Instructor	1	1	1																			23
4	Computer	2	1	9																			95
5																							
6																							
7																							

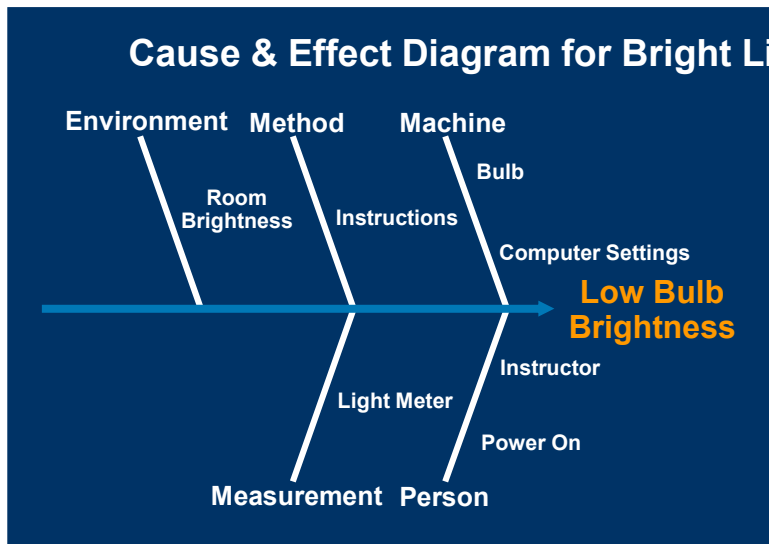
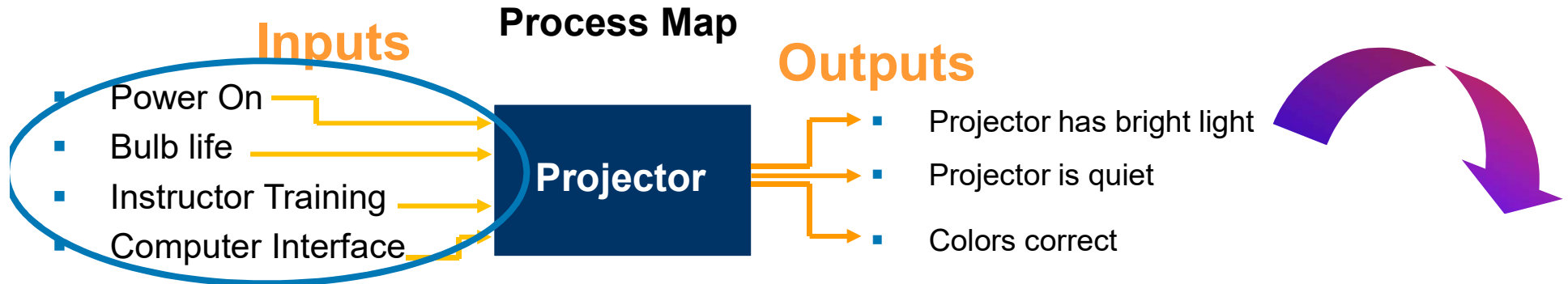
# Selecting the Factors

## Step 4

A factor is one of the inputs suspected of influencing the response

- May be qualitative (discrete)
  - Different operators
  - Different shifts
  - Different Resources
- May be quantitative (continuous)
  - Pressure
  - Pull back angle
- Comes from Measure and Analyze Phase
  - C&E, FMEA, process map and other
  - Hypothesis testing

# Response Relationship to Other Tools



		Rating of Importance to Customer														
		8	7	8												
Process Inputs		Bright Light	Quiet	Color Correct												Total
1	Power on	9	3	2												119
2	Bulb Life	8	1	3												95
3	Instructor	1	1	1												23
4	Computer	2	1	9												95
5																
6																
7																

# Selecting the Factor Levels

## Step 5

The factor levels are the range of values of the input under study

- Qualitative (discrete) inputs
  - operator 1, operator 2, operator 3...
  - shift 1, shift 2, shift 3 ...
  - pin loc 1, 2
- Quantitative (continuous) inputs
  - 50 , 75, 100 psi
  - 160, 170, 180 degrees
- Process knowledge/inference space

**Consider what works, what could work and what won't hurt**



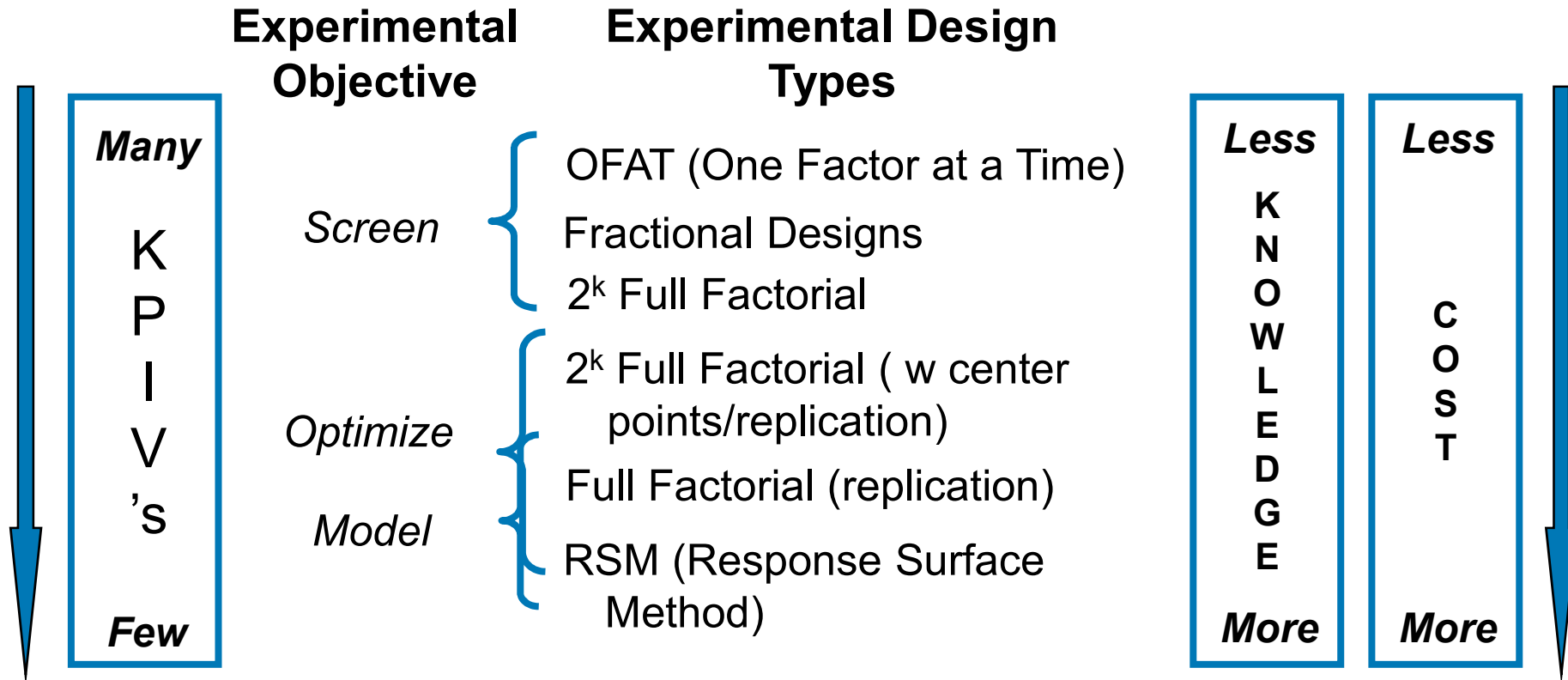
# Select the DOE Design

## Step 6

The selected design should align with the objectives of the experiment and resource commitment

- OFAT (One Factor at a Time)
- 2k-n Fractional Factorial Designs
- 2k Full Factorial
- Full Factorial (replication)
- RSM (response surface method)

# Experimental Design Considerations



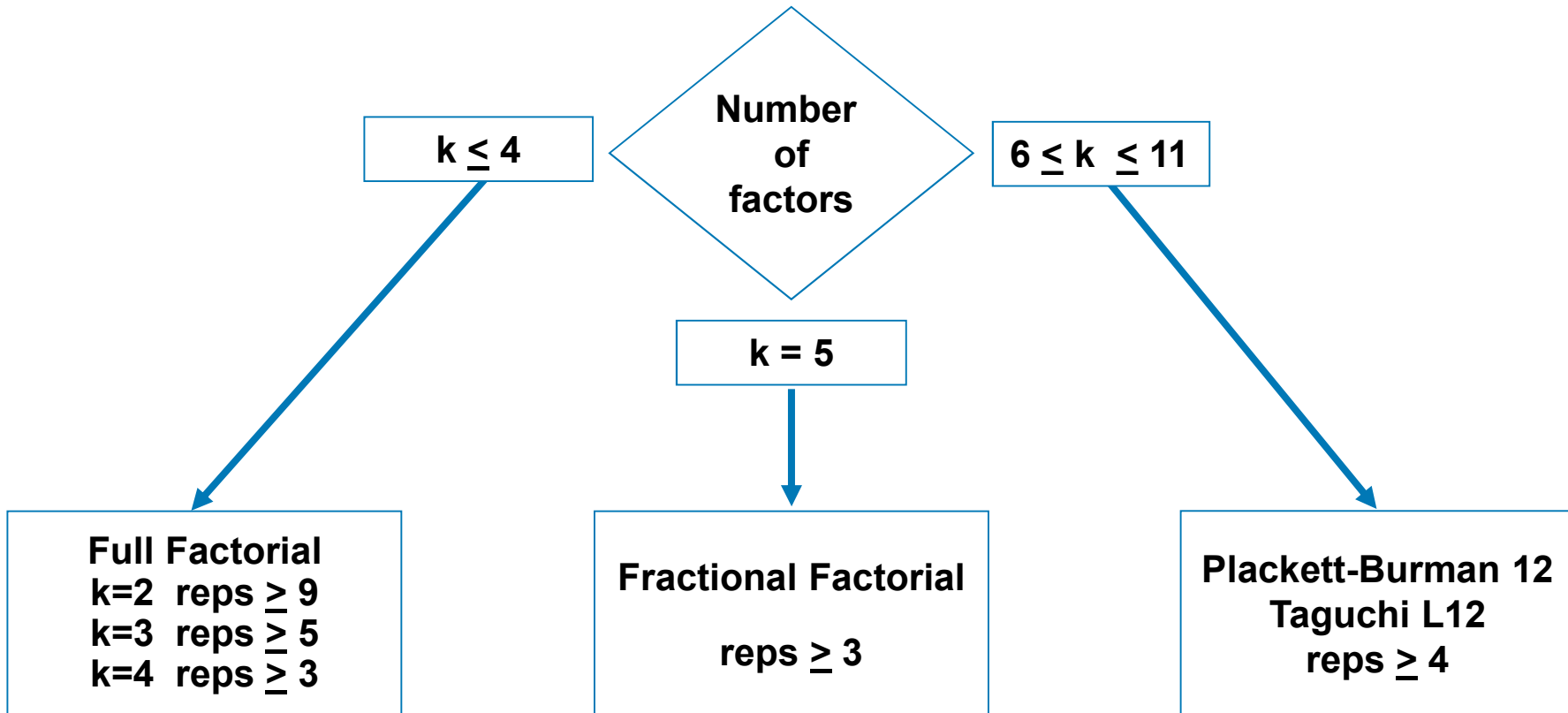
**Higher complexity designs offer greater knowledge at a higher price**

# Determine Sample Size

## Step 7

- The type of design determines minimum number of samples (replicates/repeats). Considerations for replicates
  - Multiple runs help identify data and measurement errors
  - Screening for main effects requires fewer replicates
  - Modeling designs require more replicates to have confidence in prediction models
  - Detectable effect
  - Resources

# Two Level Sample Size



**Provides greater than 99% confidence in mean and 95% confidence in variance prediction models**

# Run the Experiment

## Step 8

- Manage resources
  - Assign tasks
  - Stage supplies
- Have contingency plan
  - Personnel changes
  - Business changes
  - Botched runs
- Statistical common sense
  - Randomize runs if possible
  - Replicate and/or repeat

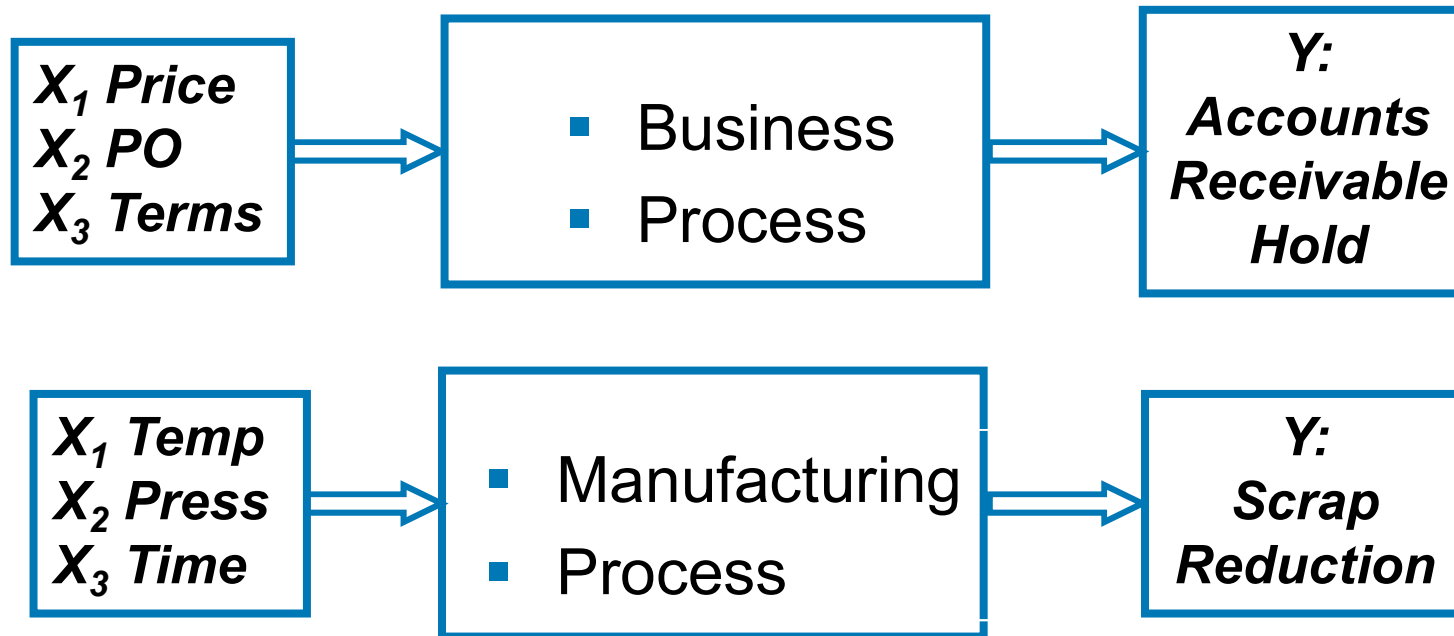
# DOE Strategy Exercise

- Break into table teams. Select two projects at each table (one industrial process and one business process). Using flip charts list the Planning Phase elements for each project.
- Be prepared to discuss your learnings.

# Anexas Consultancy Services

Factors

# Factors and the Process



**DOE can be applied to both business and industrial processes**



# Organizing the Factors

- The best conceived experiment will fail if executed poorly. Defining the factor settings and capturing the process response(s) properly is paramount to a successful DOE.

	INPUT FACTORS			OUTPUT RESPONSE REPLICATES					STATISTCAL ANALYSIS	
RUN	X <sub>1</sub>	X <sub>2</sub>	X <sub>3</sub>	Y <sub>1</sub>	Y <sub>2</sub>	-	-	Y <sub>n</sub>	Ybar	s
1										
2										
3										
4										
-										
-										
-										
n										

**A matrix that clearly defines the input settings to the output response showing the flow of the experiment is a must**

# Factors at Two Levels

Many experiments will be quite successful when only two levels of each factor are studied.

	INPUT FACTORS			OUTPUT RESPONSE REPLICATES					STATISTCAL ANALYSIS	
RUN	X <sub>1</sub>	X <sub>2</sub>	X <sub>3</sub>	Y <sub>1</sub>	Y <sub>2</sub>	-	-	Y <sub>n</sub>	Ybar	s
1	234	1	0.0345						11	1.46
2	116	1	0.0737						14	2.03
3	234	2	0.0737						9	2.18
4	116	2	0.0345						16	4.22
-										
-										
-										
n										

**Experimental Data Placed Here**

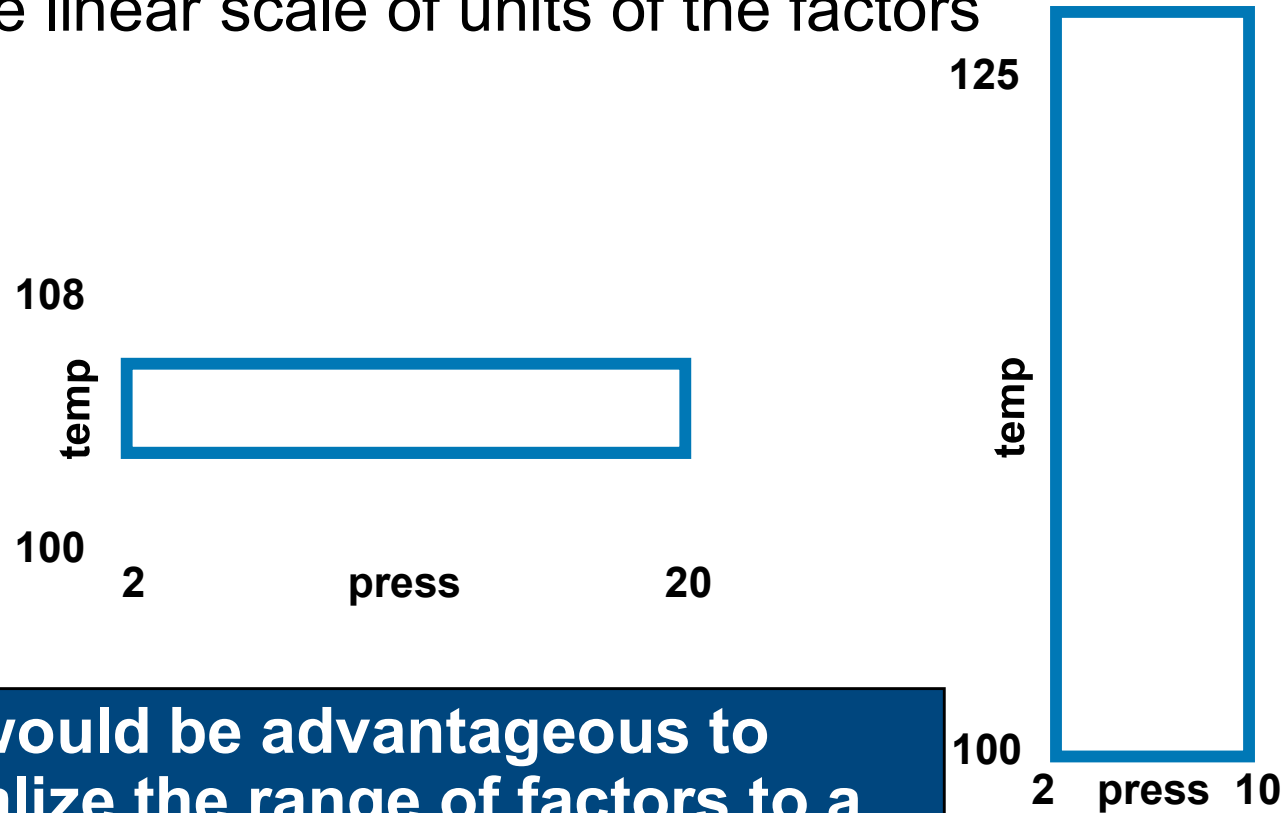
**The matrix of inputs should be arranged to reduce or eliminate correlation between factors**

# Anexas Consultancy Services

Coding

# Factors and Scale of Units

Representation of the inference space is often related to the linear scale of units of the factors



**It would be advantageous to normalize the range of factors to a common scale**

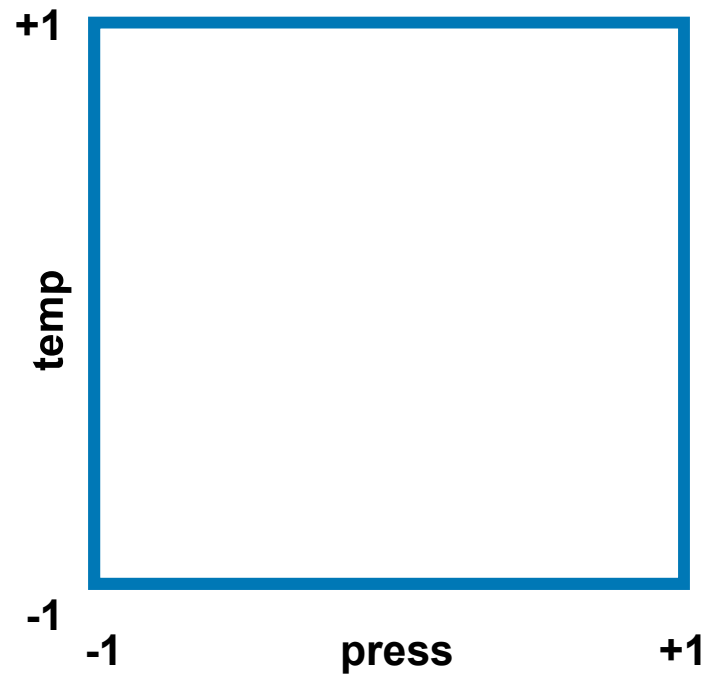
# Concept of Coding

**Low**  
**-1**

**High**  
**+1**

**Press = 2**  
**Temp = 10**

**Press = 10**  
**Temp = 125**



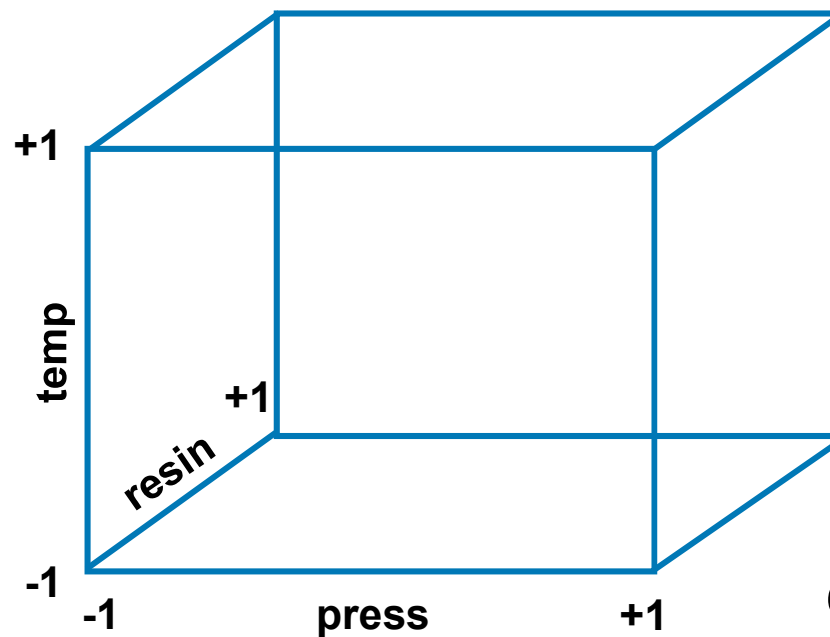
# Coding Three Factors

**Low**  
**-1**

**High**  
**+1**

**Press = 2**  
**Temp = 10**  
**Resin = 56**

**Press = 10**  
**Temp = 125**  
**Resin = 3560**



**Coding convention often uses “+” for high and “-” for low**

# Class Coding Exercise

**Transform the uncoded factor levels to coded factor levels using “+” and “-”**

RUN	UNCODED INPUT FACTORS			CODED INPUT FACTORS			OUTPUT RESPONSE			STATISTCAL ANALYSIS	
	X <sub>1</sub>	X <sub>2</sub>	X <sub>3</sub>	X <sub>1</sub>	X <sub>2</sub>	X <sub>3</sub>	Y <sub>1</sub>	Y <sub>2</sub> ...	Y <sub>n</sub>	Ybar	s
1	234	1	0.0345							11	1.46
2	116	1	0.0737							14	2.03
3	116	2	0.0737							9	2.18
4	234	2	0.0345							16	4.22
-											
-											
-											
n											

# Why Code?

- Allows experimenter to focus on the design
- Shows the design generation clearly
- Permits easier mathematical analysis
- Produces fewer data entry errors
- Reduces analysis errors
- Manages magnitude of scale

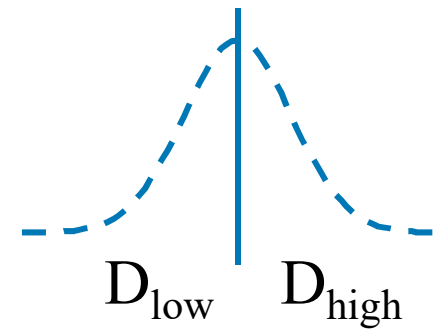
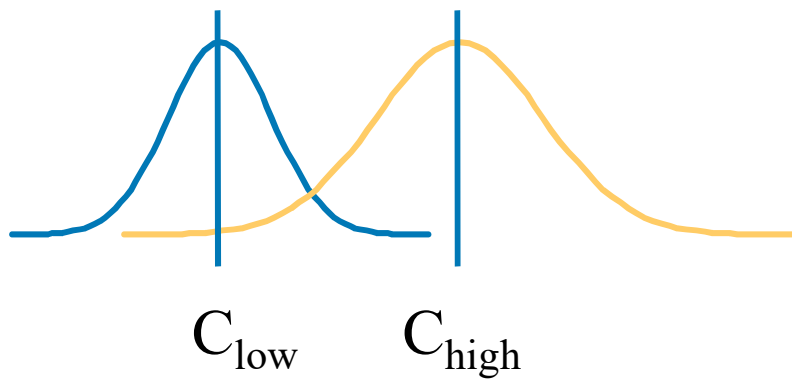
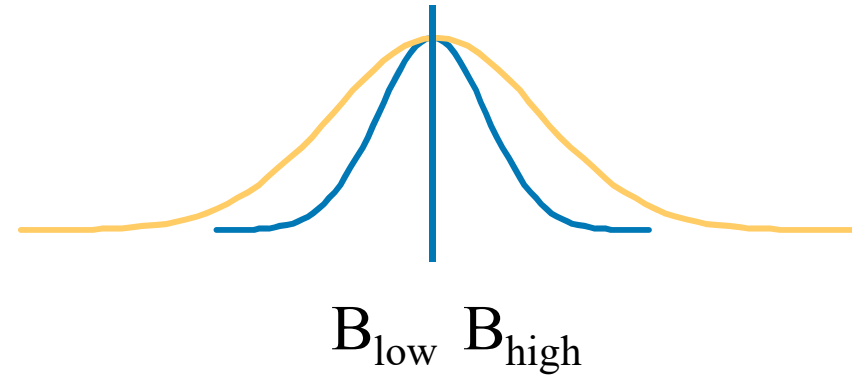
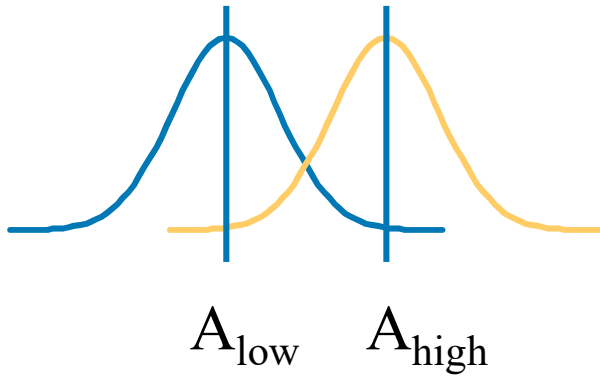
**Coding is the standard practice for  
experimental design**



# Anexas Consultancy Services

## Exploring Factor Effects

# How Factors Affect a Response



**Which factors affect mean or variation?**

# Anexas Consultancy Services

One Factor at a Time  
(OFAT) Experiments

# One Factor at a Time

- A process has two variable inputs: flow and temperature. Historically the process has run with flow = 18 and temp = 160 with a yield of 81.7%.

Flow	Temp	Yield
18	140	71.4
18	145	74.8
18	150	78.3
18	155	79.4
18	160	81.7
18	165	82.8
18	170	84.0
18	175	84.5
18	180	84.2
18	185	83.3
18	190	80.5
18	195	78.3
18	200	76.0

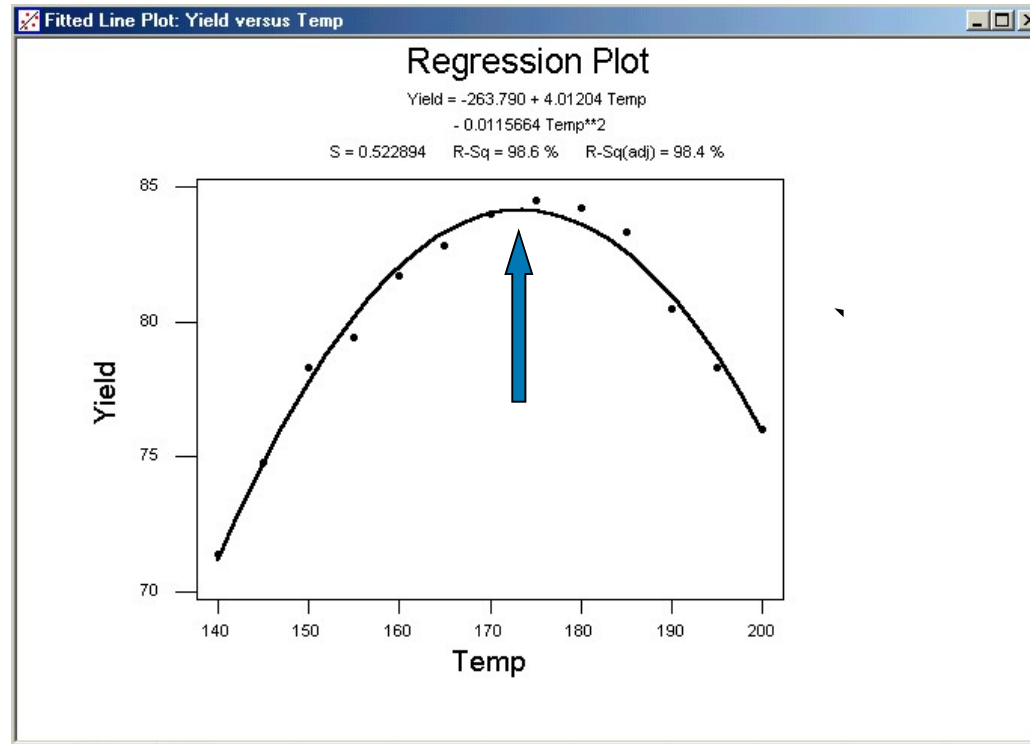
# One Factor at a Time Data Analysis First Step

	Flow	Temp	Yield
	18	140	71.4
	18	145	74.8
	18	150	78.3
Old setting	18	155	79.4
→	18	160	81.7
New setting	18	165	82.8
→	18	170	84.0
	18	175	84.5
	18	180	84.2
	18	185	83.3
	18	190	80.5
	18	195	78.3
	18	200	76.0

**Higher yield is obtainable, but at a higher temperature (cost)**

**Data is used for a regression analysis**

# One Factor at a Time Data Analysis Second Step



Regression analysis provides an equation which can be solved for temperature

$$Y = -263.79 + 4.012Temp - 0.011566Temp^2$$

Another test is planned to study flow at the “optimum” temperature of 174.4°C

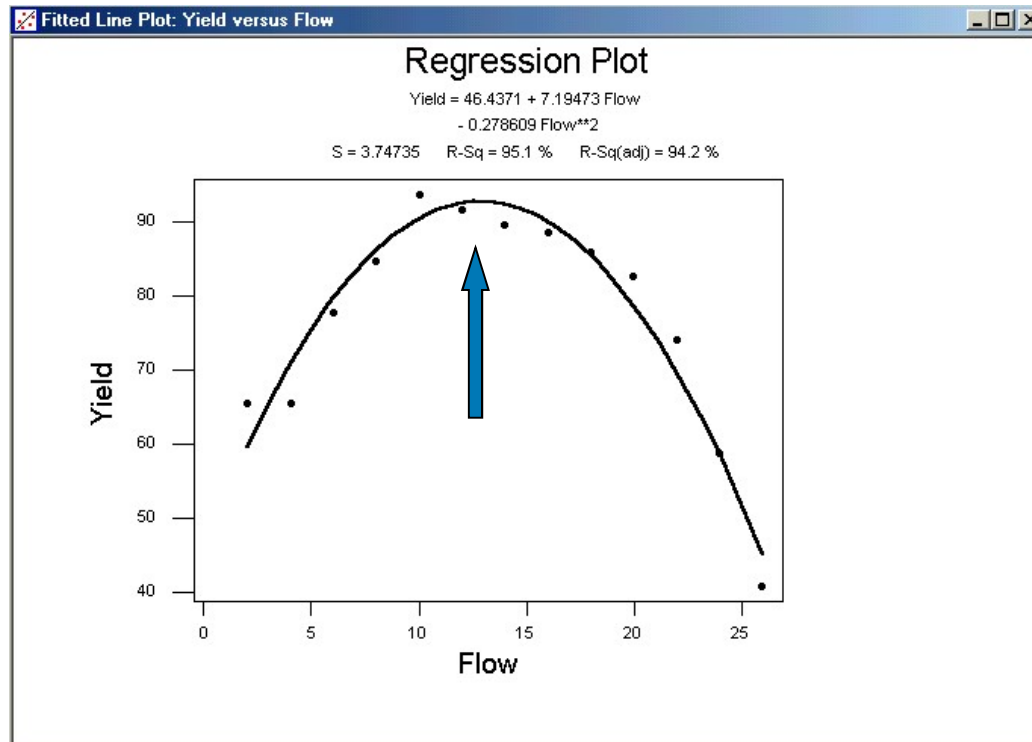
# One Factor at a Time Data Analysis Third Step

	Flow	Temp	Yield
	2	174.4	65.5
	4	174.4	65.5
	6	174.4	77.8
	8	174.4	84.8
New setting	10	174.4	90.8
→	12	174.4	91.8
	14	174.4	89.8
Old setting	16	174.4	88.8
→	18	174.4	84.5
	20	174.4	82.8
	22	174.4	74.2
	24	174.4	58.8
	26	174.4	40.8

**Higher yield at lower flow is found, but is this the true optimum?**

**Data is used for a regression analysis**

# One Factor at a Time Data Analysis Fourth Step



Regression analysis provides equation which can be solved for flow

$$Y = 46.437 + 7.195Flow - 0.2786Flow^2$$

**Another test is planned to study temp at the “optimum” flow of 12.9 lpm**



## One Factor at a Time Data Analysis Fifth Step

- Repeat steps 1 – 4 until satisfied “optimum” is found
- Each cycle will produce an equation for yield versus temperature
- Each cycle will produce an equation for yield versus flow
- Solve equations iteratively
- Repeat cycle, obtaining NEW equations

# OFAT Equations

- Each cycle produces a set of independent, simultaneous equations describing the process
- First cycle:

$$Y = -263.79 + 4.012Temp - 0.011566Temp^2$$

$$Y = 46.437 + 7.195Flow - 0.2786Flow^2$$

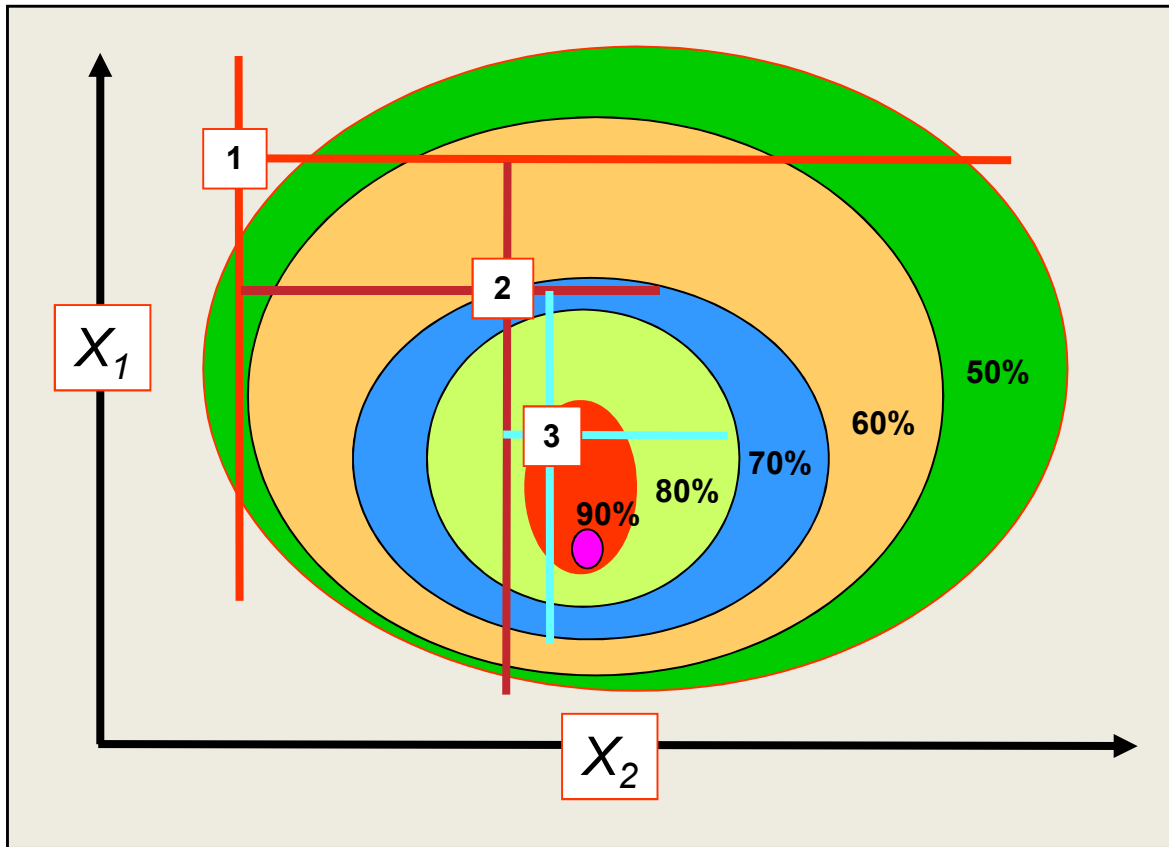
- Subsequent cycles:

$$Y = f(Temp)$$

$$Y = f(Flow)$$

**OFAT will generate a family of independent prediction equations that do not readily lead to the optimum solution**

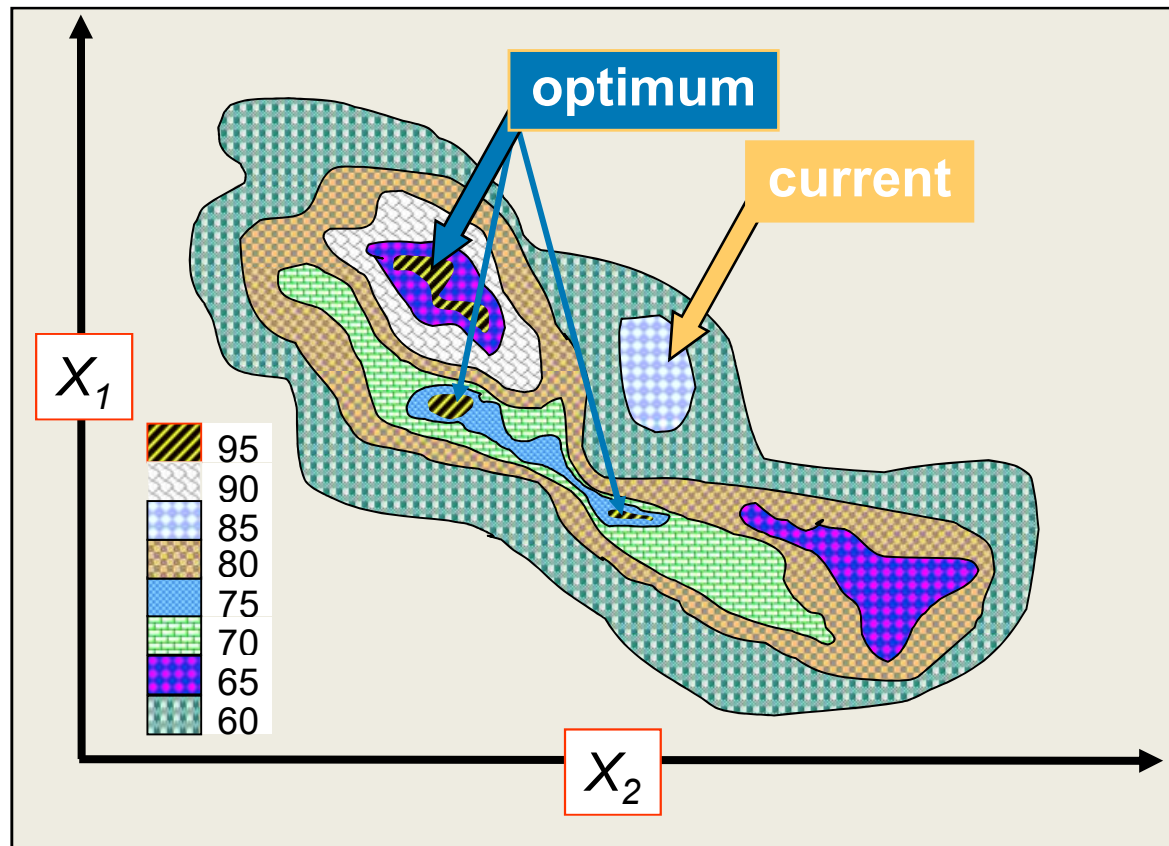
# OFAT Prediction Contours



One factor at a time presumes a nested family of responses

**Factor interactions are missed with OFAT**

# Process Response Contours



More often than not, the response contours of a process contain many peaks and valleys created by the interaction of factors

**OFAT optimization can become inefficient and sometimes misleading**

# Prediction Equations

- More useful and efficient is one equation that predicts the output as a function of all factors

$$\hat{Y} = f(temp, press)$$

$$\hat{Y} = f(A, B, C...)$$

- This type of equation has multiple factors. It will contain:
  - Main effects
  - Interactions

# General Form of DOE Prediction Equations

- The general full equation has all combinations and power terms

$$\hat{Y} = b_0 + b_1 Flow + b_2 Temp + b_3 Flow * Flow + b_4 Flow^2 + \dots$$

- Most DOE experiments will use the linear solution of the general equation to define the response

$$\hat{Y} = b_0 + b_1 A + b_2 B + b_3 C + b_4 AB + b_5 AC + \dots + b_7 ABC$$

**DOE equations will deal with a linear subset of the general equation**

# Capturing Interactions

Statistically based designs provide information about factors and interactions

- 2k-n Fractional Factorial Designs
- 2k Full Factorial
- Full Factorial (replication)
- RSM (response surface method)

**Revealing information about main effects and interactions is key to process understanding**

# Prepared to Explore DOE

DoE 3 Factor 2 Level ***								
↓	C1	C2	C3	C4	C5	C6	C7	C8
	StdOrder	RunOrder	CenterPt	Blocks	A	B	C	
1	6	1	1	1	1	-1	1	
2	3	2	1	1	-1	1	-1	
3	2	3	1	1	1	-1	-1	
4	1	4	1	1	-1	-1	-1	
5	8	5	1	1	1	1	1	
6	7	6	1	1	-1	1	1	
7	4	7	1	1	1	1	-1	
8	5	8	1	1	-1	-1	1	
9								

Minitab will be the tool for creating and analyzing DOE experiments in the Breakthrough Strategy™



# Key Learning Points

- 
- 
- 
- 
- 
- 
-

# Objectives Review

**By the end of this module, the participant should:**

- Understand the strategy behind Design of Experiments (DOE)
- Create a design matrix
- Code factors
- Understand the limitations of OFAT experiments
- Interpret interactions

# Anexas Consultancy Services

## Full Factorial Design of Experiments

# Module Objectives

By the end of this module, the participant will:

- Generate a full factorial design
- Look for factor interactions
- Develop coded orthogonal designs
- Write process prediction equations (models)
- Set factors for process optimization
- Create and analyze designs in Minitab™
- Evaluate residuals
- Develop process models from Minitab™ analysis
- Determine sample size

# Why Learn About Full Factorial DOE?

- A Full Factorial Design of Experiment will
- Provide the most response information about
  - Factor main effects
  - Factor interactions
- Provide the process model's coefficients for
  - All factors
  - All interactions
- When validated, allow process to be optimized

# What is a Full Factorial DOE

- A full factorial DOE is a planned set of tests on the response variable(s) (KPOVs) with one or more inputs (factors) (PIVs) with all combinations of levels
- ANOVA analysis will show which factors are significant
- Regression analysis will provide the coefficients for the prediction equations
  - Mean
  - Standard deviation
- Residual analysis will show the fit of the model

# DOE Terminology

- **Response** (Y, KPOV): the process output linked to the customer CTQ
- **Factor** (X, PIV): uncontrolled or controlled variable whose influence is being studied
- **Level**: setting of a factor (+, -, 1, -1, hi, lo, alpha, numeric)
- **Treatment Combination** (run): setting of all factors to obtain a response
- **Replicate**: number of times a treatment combination is run (usually randomized)
- **Repeat**: non-randomized replicate
- **Inference Space**: operating range of factors under study

# Full Factorial DOE Objectives

- Learning the most from as few runs as possible..
- Identifying which factors affect mean, variation, both or have no effect
- Modeling the process with prediction equations,

$$\hat{Y} = f(A, B, C...)$$

$$\hat{s} = f(A, B, C...)$$

- Optimizing the factor levels for desired response
- Validating the results through confirmation



# Anexas Consultancy Services

## Linear Combinations of Factors for Two Levels

## Combinations of Factors and Levels

- A process whose output Y is suspected of being influenced by three inputs A, B and C. The SOP ranges on the inputs are
  - A 15 through 25, by 1
  - B 200 through 300, by 2
  - C 1 or 2
- A DOE is planned to test all combinations

**Is testing all combinations possible, reasonable and practical?**

# Combinations of Factors and Levels cont'd

- Setting up a matrix for the factors at all possible process setting levels will produce a really large number of tests.
- The possible levels for each factor are
  - A = 11
  - B = 51
  - C = 2
- How many combinations are there?

A	B	C
15	200	1
16	200	1
17	200	1
18	200	1
19	200	1
20	200	1
21	200	1
22	200	1
23	200	1
24	200	1
25	200	1
15	202	1
16	202	1
17	202	1
.	.	.
.	.	.
.	.	.
.	.	.
22	300	2
23	300	2
24	300	2
25	300	2

**We must make assumptions about the response in order to manage the experiment**

## Linear Response for Factors at Two Levels

- The team decides, from process knowledge, that the response is close to being linear throughout the range of factor level settings (inference space).
- A reasonable assumption for most processes
- The levels of the factors for the test would then be
  - A 15 and 25
  - B 200 and 300
  - C 1 and 2

**The design becomes much more manageable!**

## The Three Factor Design at Two Levels

- The revised experiment consists of all possible combinations of A, B and C each at the chosen low and high settings:

A	B	C
15	200	1
15	200	2
15	300	1
15	300	2
25	200	1
25	200	2
25	300	1
25	300	2

**This is a  $2^3$  full factorial design (pronounced two to the three). It consists of all combinations of the three factors each at two levels**

# Naming Conventions

- The naming convention for full factorial designs has the level raised to the power of the factor:

(factor)  
***level***

- and is called “a (level) to the (factor) design”
- What would a two level, four factor design be called?
- How many combinations (runs) are in a 2<sup>3</sup> design?

# Class Exercise

- Write the total number of combinations for the following designs

$$2^3$$

$$2^4$$

A	B	C	D

- Assume factors are named A, B, C, D, etc. and the levels are low “-” and high “+”.

**Did we all generate the same designs?**

# Anexas Consultancy Services

## The Yates Standard Order

A method to generate experimental designs in a consistent and logical fashion was developed by Frank Yates.



# The Yates Standard Order

## Step 1

Create a matrix with factors along the top, runs down the left side ( $2^3$  example shown)

	Factors		
Runs	A	B	C
1			
2			
3			
4			
5			
6			
7			
8			

For a  $2^3$   
there will  
be 8 runs

# The Yates Standard Order

## Step 2

Starting with the first factor, insert its low value in the first row followed by its high value in the second row. Repeat through the last row.

Runs	Factors		
	A	B	C
1	15		
2	25		
3	15		
4	25		
5	15		
6	25		
7	15		
8	25		

# The Yates Standard Order

## Step 3

Move to the next factor and place its low value in the first two rows, followed by its high value in the next two rows. Repeat through the last row.

Runs	Factors		
	A	B	C
1	15	200	
2	25	200	
3	15	300	
4	25	300	
5	15	200	
6	25	200	
7	15	300	
8	25	300	

# The Yates Standard Order

## Step 4

Move to the next factor and place its low value in the first four rows, followed by its high value in the next four rows. Repeat through the last row.

Runs	Factors		
	A	B	C
1	15	200	1
2	25	200	1
3	15	300	1
4	25	300	1
5	15	200	2
6	25	200	2
7	15	300	2
8	25	300	2

# The Yates Standard Order Step N

Continue the pattern until all factors are included

## Yates Design Generator

Factor	Row	Pattern
1st	1 low	1 high
2nd	2 low	2 high
3rd	4 low	4 high
	•	
	•	
nth	$2^{n-1}$ low	$2^{n-1}$ high

# The Yates Standard Order Summary

- The Yates design generator yields all possible combinations of factor levels.
- This is a full factorial two level design for three factors.

	Factors		
Runs	A	B	C
1	15	200	1
2	25	200	1
3	15	300	1
4	25	300	1
5	15	200	2
6	25	200	2
7	15	300	2
8	25	300	2

## Class Exercise

- Write the total number of combinations for the following designs using the Yates Standard order design generator.
- $2^3$
- $2^4$
- Assume factors are named A, B, C, D, etc. and the levels are low = “-” and high = “+”.

**Did we all generate the same designs?**

# Anexas Consultancy Services

Creating a Full Factorial Design in  
Minitab®

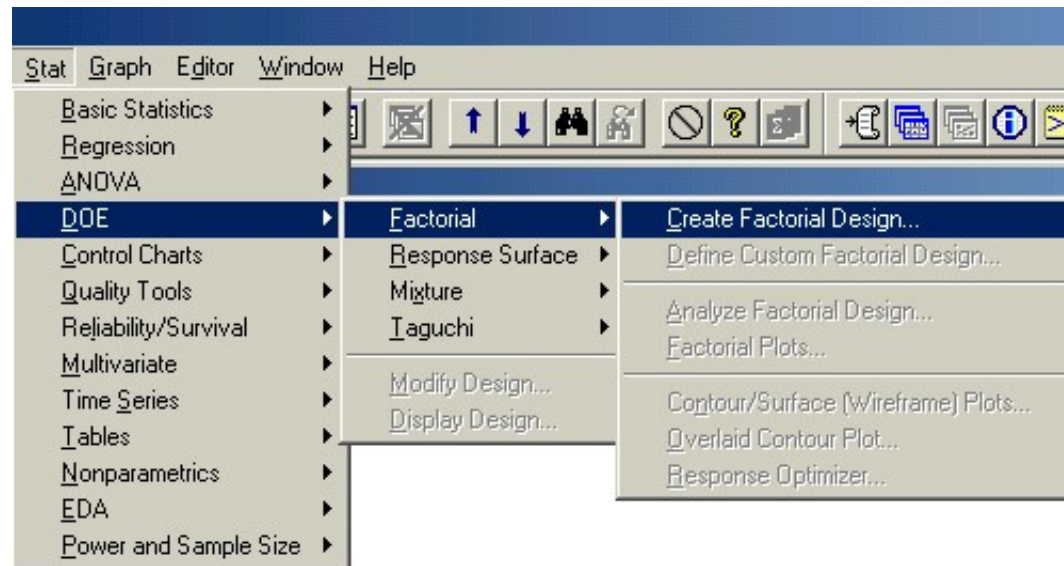


# Creating a Full Factorial Design in Minitab Step 1

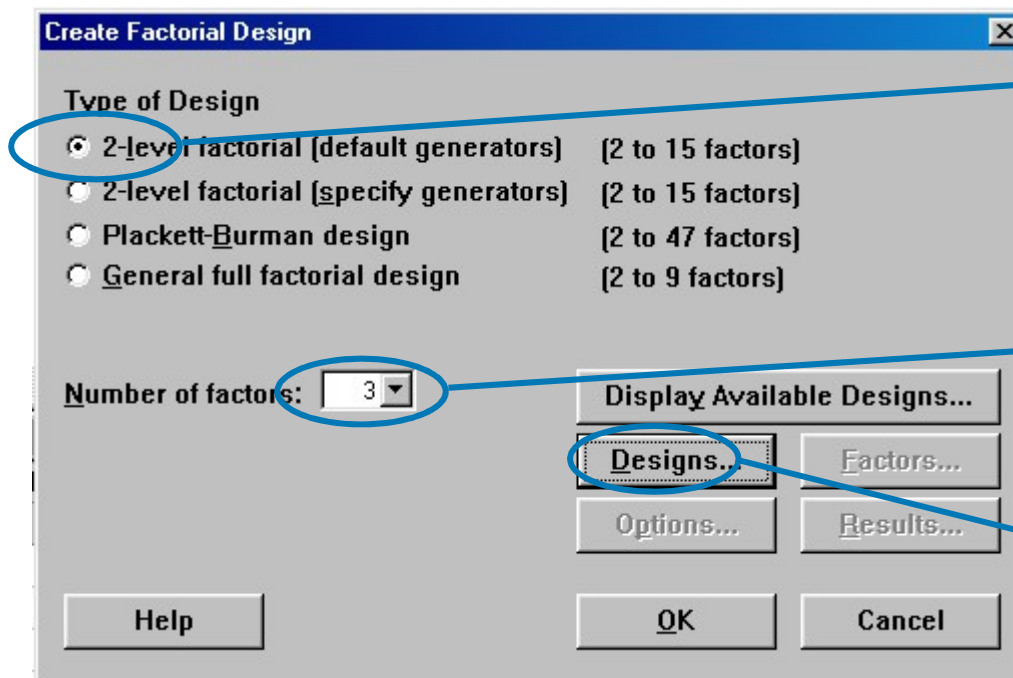
Tool Bar Menu > Stat > DOE > Factorial > Create Factorial Design

- Create a Minitab® design for
  - A - pressure
  - B - temperature
  - C - material vendor

$2^3$  design



# Creating a Full Factorial Design in Minitab Step 2

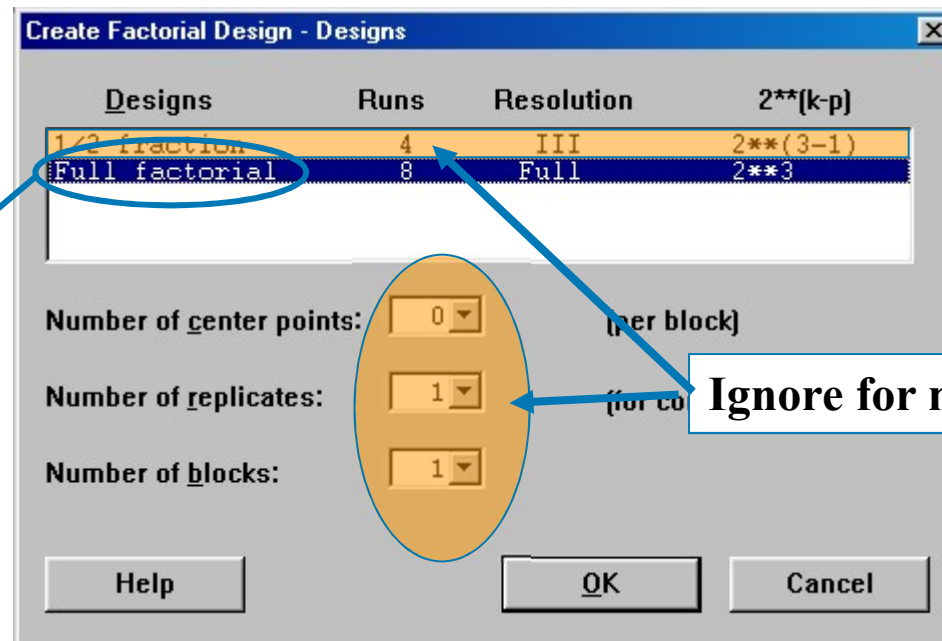


1. Select default generators (Yates is default)

2. Set number of factors to three

3. Go to Designs

# Creating a Full Factorial Design in Minitab Step 3



Select Full factorial

Ignore for now

# Creating a Full Factorial Design in Minitab Step 4

The image shows two overlapping Minitab dialog boxes. The top box is 'Create Factorial Design' and the bottom box is 'Create Factorial Designs - Options'. Annotations include a callout '1. Select Options' pointing to the 'Options...' button in the top dialog, and another callout '2. Un-check Randomize runs' pointing to the 'Randomize runs' checkbox in the bottom dialog.

**Create Factorial Design**

Type of Design

- 2-level factorial (default generators) (2 to 15 factors)
- 2-level factorial (specify generators) (2 to 15 factors)
- Plackett-Burman design (2 to 47 factors)
- General full factorial design (2 to 9 factors)

Number of factors: 3

Display Available

Designs... Options... OK Help

**Create Factorial Designs - Options**

Fold Design

- Do not fold
- Fold on all factors
- Fold just on factor: [ ]

Fraction

- Use principal fraction
- Use fraction number: [ ]

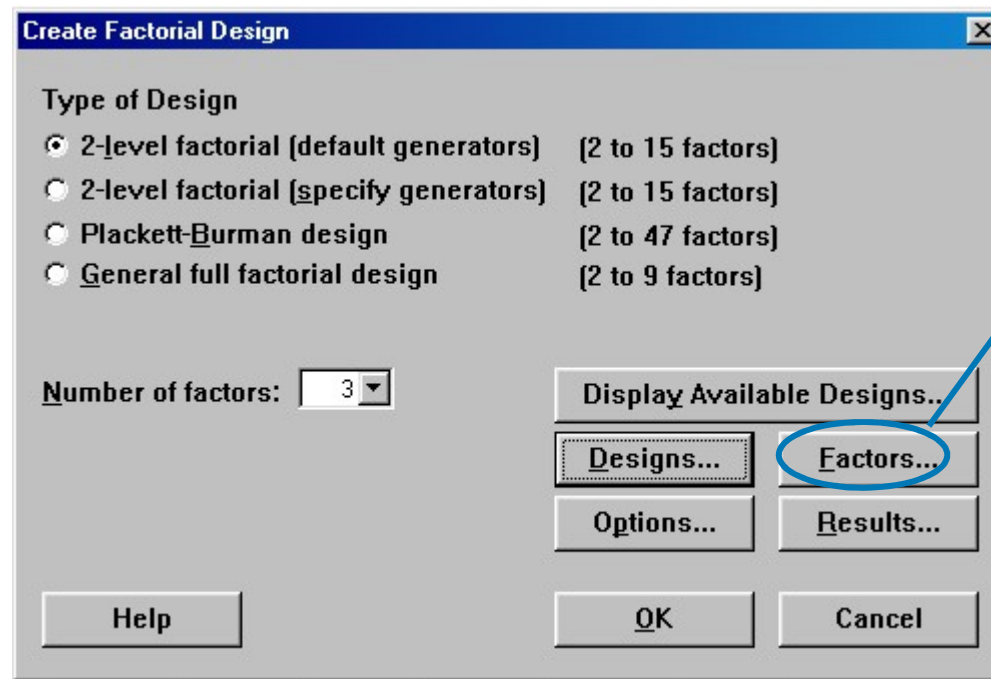
Randomize runs

Base for random data generator [ ]

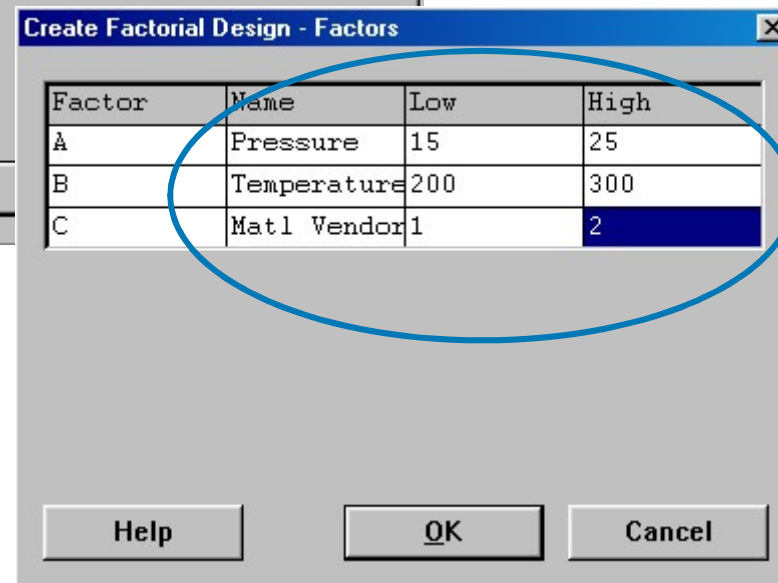
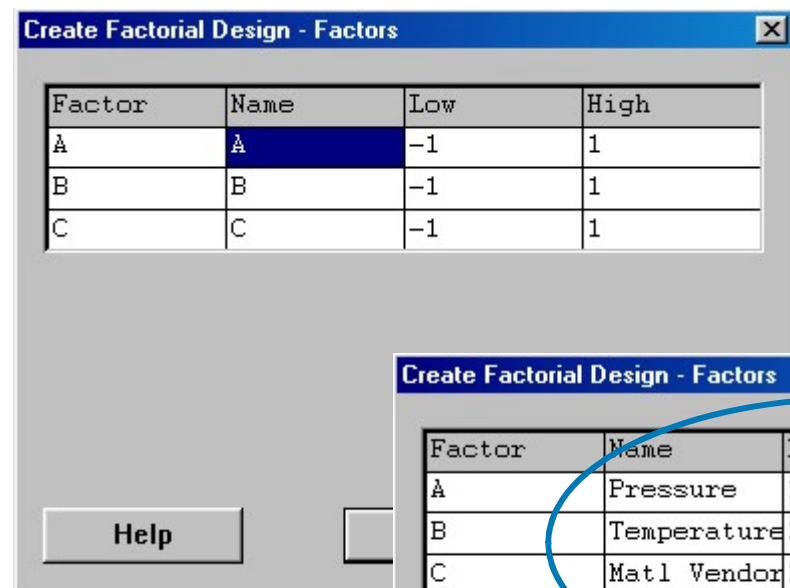
- Store design in worksheet

Help OK Cancel

# Creating a Full Factorial Design in Minitab Step 5

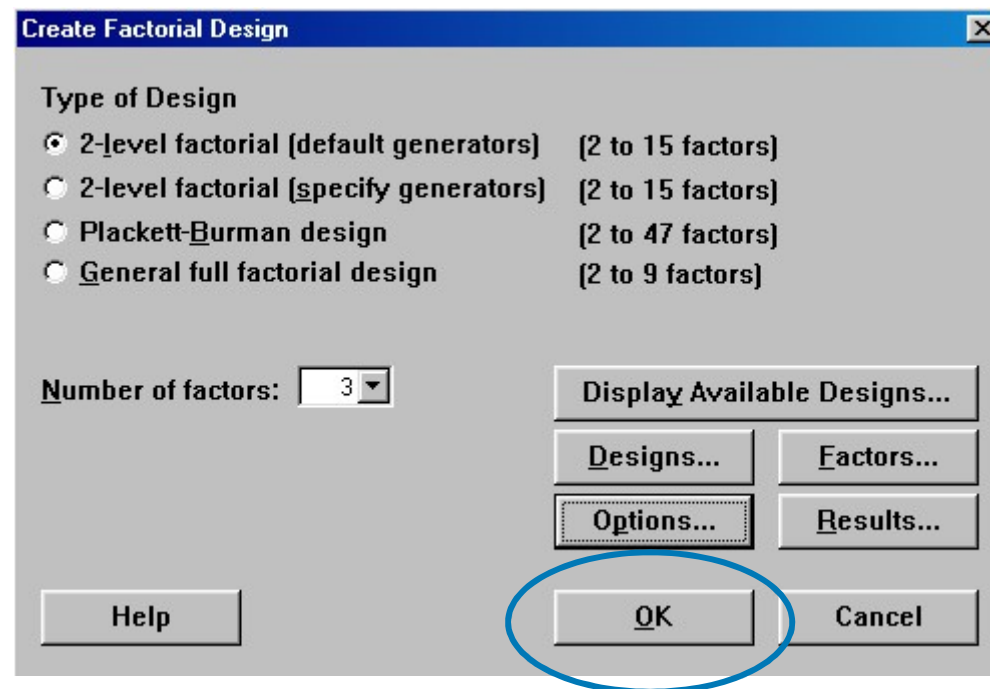


# Creating a Full Factorial Design in Minitab Step 6



**Input the Factor Names and Levels**

# Creating a Full Factorial Design in Minitab Step 7



**Design parameters are entered; ready to create the design in the worksheet.**

# Creating a Full Factorial Design in Minitab Step 8

Worksheet 1 \*\*\*

+	C1	C2	C3	C4	C5	C6	C7	C8
	StdOrder	RunOrder	CenterPt	Blocks	Pressure	Temperature	Matl Vendor	
1	1	1	1	1	15	200	1	
2	2	2	1	1	25	200	1	
3	3	3	1	1	15	300	1	
4	4	4	1	1	25	300	1	
5	5	5	1	1	15	200	2	
6	6	6	1	1	25	200	2	
7	7	7	1	1	15	300	2	
8	8	8	1	1	25	300	2	
9								

**This is the Yates Standard order for a  $2^3$  uncoded design**



# Std Order Column of Minitab Design

Worksheet 1 \*\*\*

↓	C1	C2	C3	C4	C5	C6	C7	C8
	StdOrder	RunOrder	CenterPt	Blocks	Pressure	Temperature	Matl Vendor	
1	1	1	1	1	15	200	1	
2	2	2	1	1	25	200	1	
3	3			1	15	300	1	
4	4			1	25	300	1	
5	5			1	15	200	2	
6	6			1	25	200	2	
7	7			1	15	300	2	
8	8	8	1	1	25	300	2	
9								

**The StdOrder Column is the Yates Standard Order**

# Run Order Column of Minitab Design

Worksheet 1 \*\*\*

↓	C1	C2	C3	C4	C5	C6	C7	C8
	StdOrder	RunOrder	CenterPt	Blocks	Pressure	Temperature	Matl Vendor	
1	1	1	1	1	15	200	1	
2	2	2			25	200	1	
3	3	3			15	300	1	
4	4	4			25	300	1	
5	5	5			15	200	2	
6	6	6			25	200	2	
7	7	7			15	300	2	
8	8	8			25	300	2	
9								

**RunOrder  
Column is  
the  
sequence  
of the runs**

**StdOrder is created by the design choice RunOrder is created by randomize runs choice**

# Class Exercise Part 1

- Generate the previous design with the randomize runs tic box selected
- Compare results with your neighbors

## Class Exercise Part 2

Create a non random Minitab design for the following:

The factors are

	NAME	LEVEL	
		(Low	High)
A	Customer Type	Retail	Consumer
B	System	Legacy	SAP
C	Warehouse	Atlanta	Dallas
D	Overtime Hours	0	100

The response is on-time delivery.

Be prepared to present your results.

# Anexas Consultancy Services

## Replicates and Repeats

# What are Replicates and Repeats?

- Replicate
- Total run of all treatment combinations
  - Usually in random order
- Requires factor level change between runs
- All experiments will have one replicate
  - Two replicates are two complete experiment runs
- Statistically best experimental scenario
- Repeat (also repetition)
- Additional run without factor level change

# Minitab Design Replication

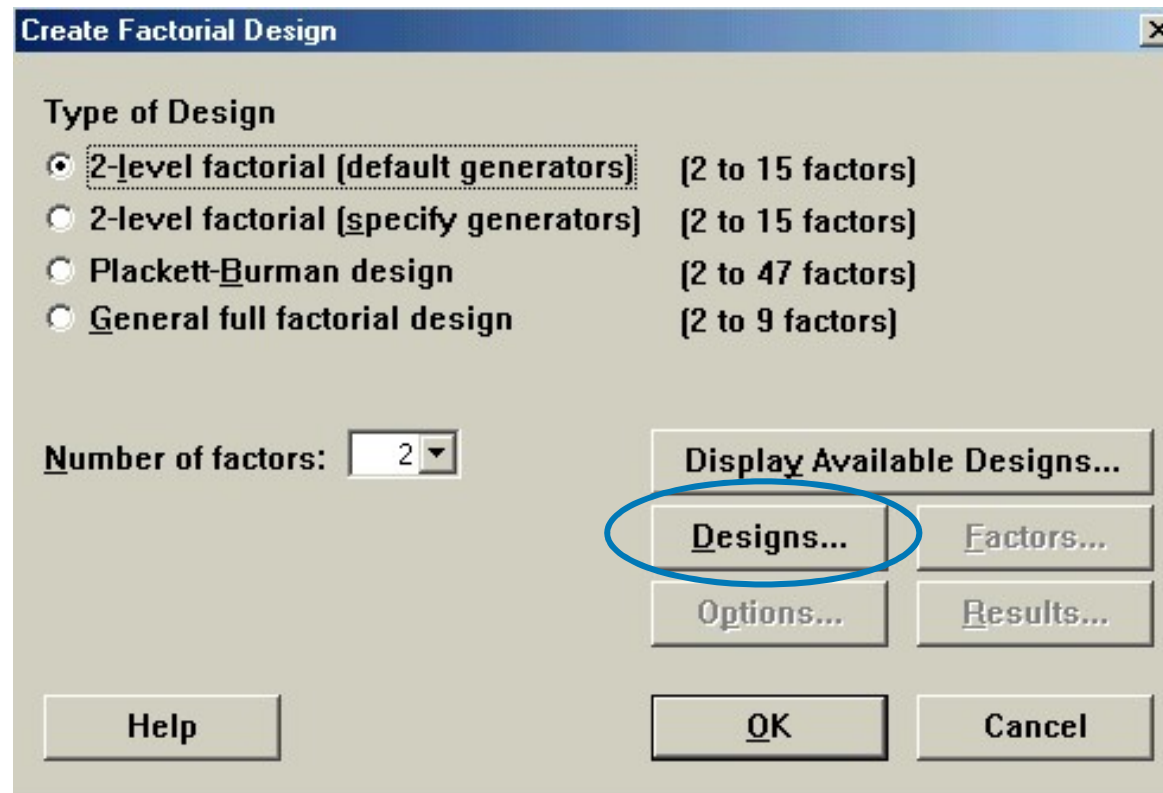
- Minitab easily handles replicating the design
- Replicate or repeat is treated same in design
- Actual factor level change between runs is at the discretion of the experimenter
  - Minitab provides treatment combination
  - Randomization or information needed is part of strategy of experiment

# Replication in Minitab

## Step 1

Tool Bar Menu > Stat > DOE > Factorial > Create Factorial Design

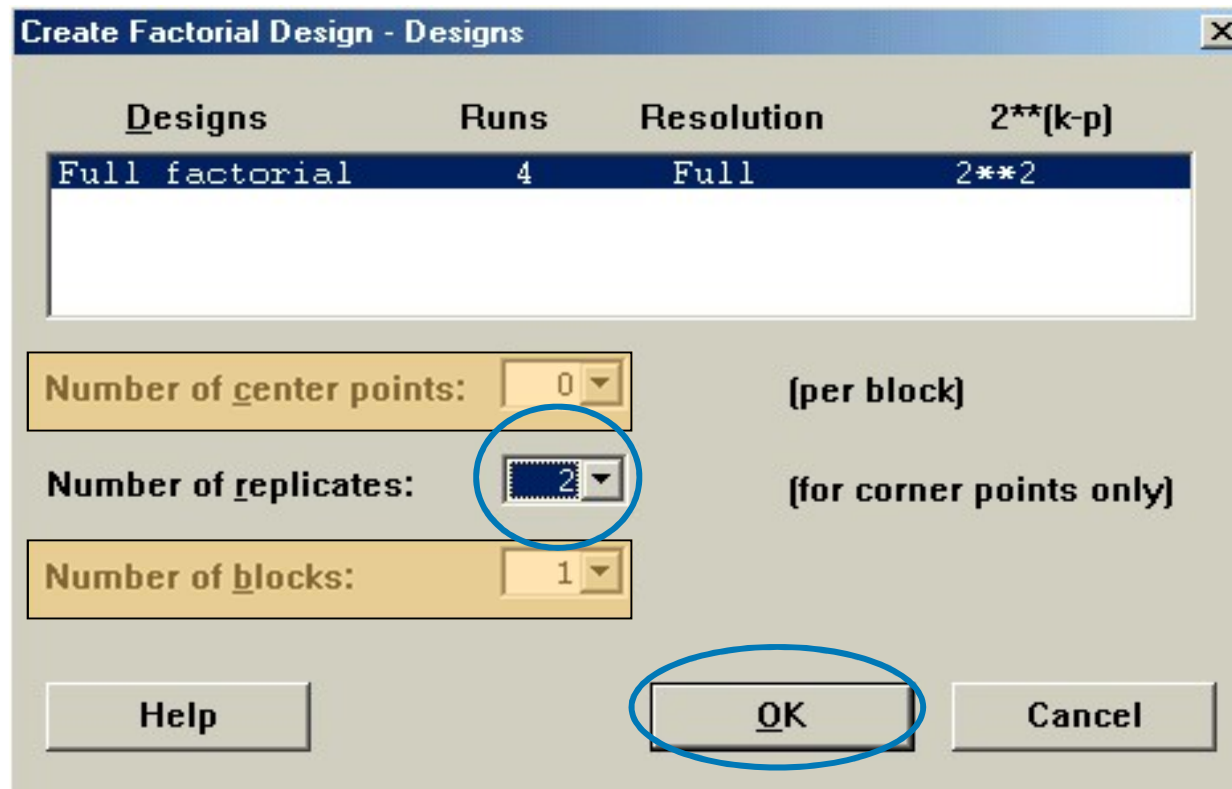
Create a 2<sup>2</sup> with two non randomized replicates





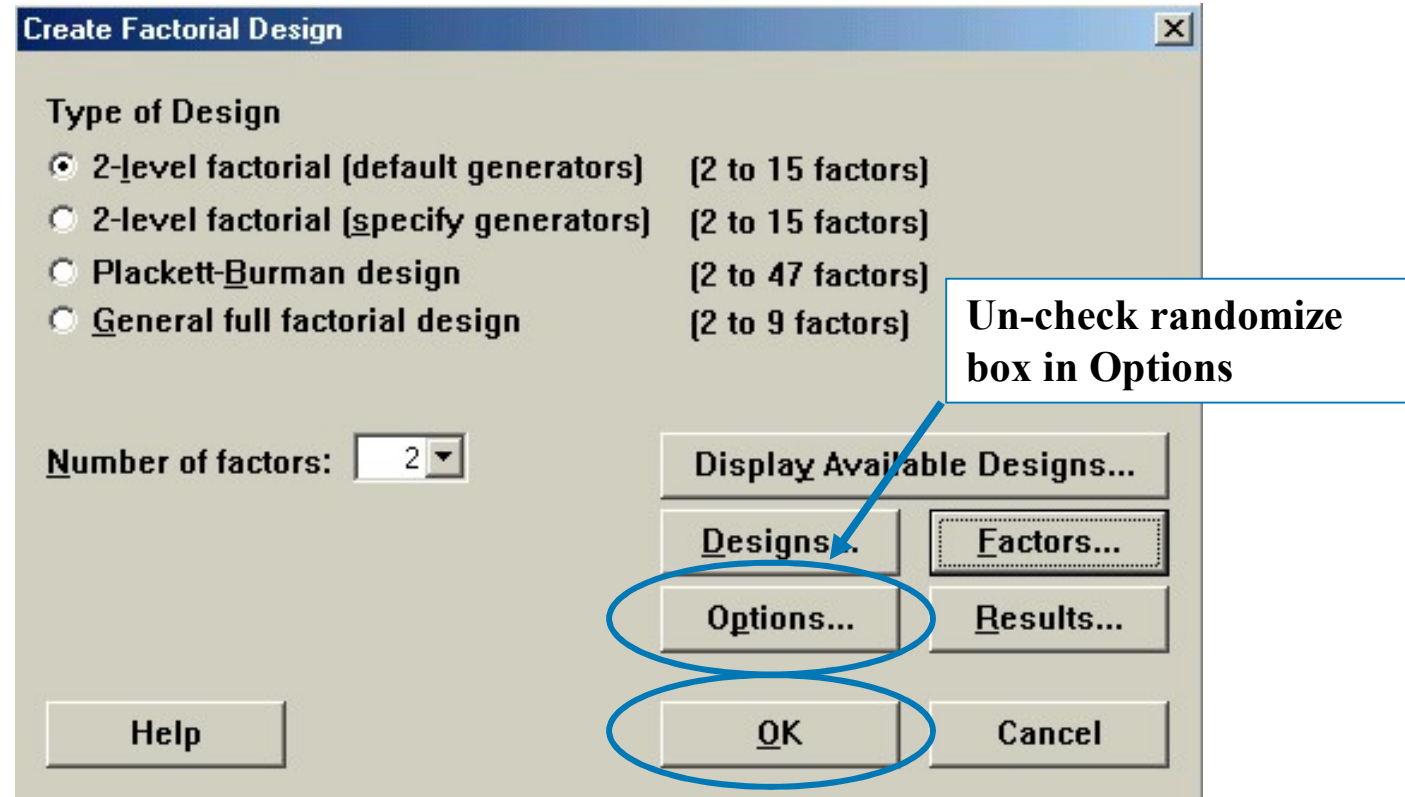
# Replication in Minitab

## Step 2



# Replication in Minitab

## Step 3



**Skip the other dialog options**

# Replication in Minitab

## Step 4

Worksheet 1 ***								
+	C1	C2	C3	C4	C5	C6	C7	
	StdOrder	RunOrder	CenterPt	Blocks	A	B		
1	1	1	1	1	-1	-1		
2	2	2	1	1	1	-1	First Replicate	
3	3	3	1	1	-1	1		
4	4	4	1	1	1	1		
5	5	5	1	1	-1	-1		
6	6	6	1	1	1	-1	Second Replicate	
7	7	7	1	1	-1	1		
8	8	8	1	1	1	1		
9								

**Response Y would be placed in C7**

# Randomized Replication in Minitab for Step 4

Worksheet 2 \*\*\*

↓	C1	C2	C3	C4	C5	C6	C7
	StdOrder	RunOrder	CenterPt	Blocks	A	B	
1	7	1	1	1	-1	1	
2	4	2	1	1	1	1	
3	5	3	1	1	-1	-1	
4	3	4	1	1	-1	1	
5	8	5	1	1	1	1	
6	2	6	1	1	1	-1	
7	1	7	1	1	-1	-1	
8	6	8	1	1	1	-1	
9							

**Response Y would be placed in C7**

# Anexas Consultancy Services

## Coding the Design

Coding the design by transforming the low factor level to a “-1” and the high factor level to a “+1” offers analysis advantages

# Coding Review Exercise

**Fill in the coded design based upon the uncoded design**

	Uncoded Factors			Coded Factors		
Runs	A	B	C	A	B	C
1	15	200	1			
2	25	200	1			
3	15	300	1			
4	25	300	1			
5	15	200	2			
6	25	200	2			
7	15	300	2			
8	25	300	2			

**Any uncoded design can be transformed into a coded design**

# Requirement of Factor Independence

- Factors are mathematically independent when only the response is a function of the factors
- A factor is not a function of another factor
- The coded design is orthogonal
  - Factors will be independent

**DOE analysis requires that the factors be independent**

# Anexas Consultancy Services

## Orthogonal Designs



# Determining Orthogonality

Consider the  $2^2$  design

	Coded Factors		OutPut
Runs	$X_1$	$X_2$	Y
1	-1	-1	$(y_1)$
2	1	-1	$(y_2)$
3	-1	1	$(y_3)$
4	1	1	$(y_4)$

The run outputs,  $Y_n$ , can be described by

where  $b$  and  $c$  are **coded settings** of the factors

It can be shown that for  $X_1$  to be independent of  $X_2$

$$\sum_{i=1}^n b_i c_i = 0$$

# Calculating Orthogonality

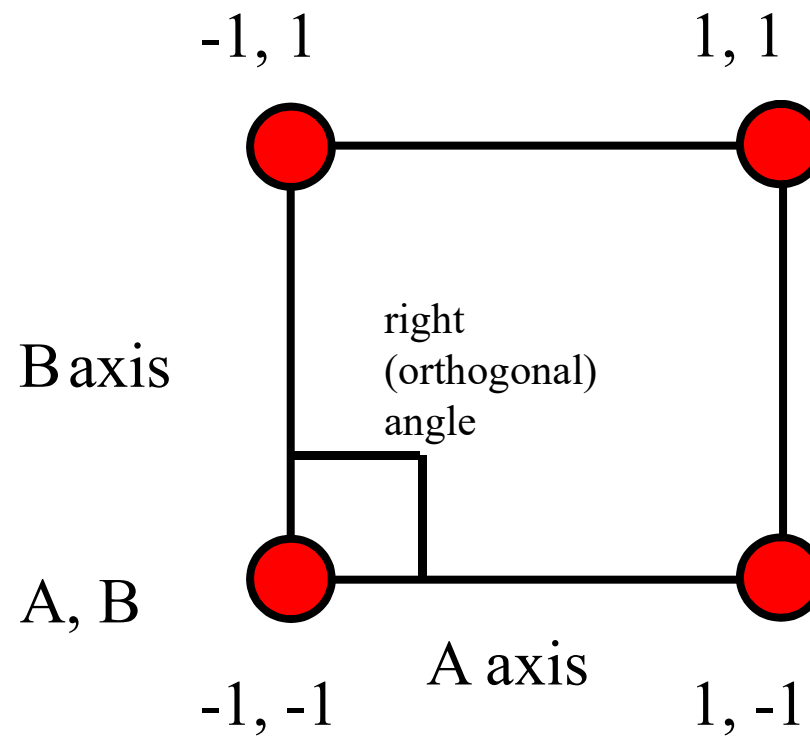
	Coded Factors		Coef1*coeff2
Runs	X1	X2	X1*X2
1	-1	-1	1
2	1	-1	-1
3	-1	1	-1
4	1	1	1
		$\Sigma$	0

Satisfies

$$\sum_{i=1}^n b_i c_i = 0$$

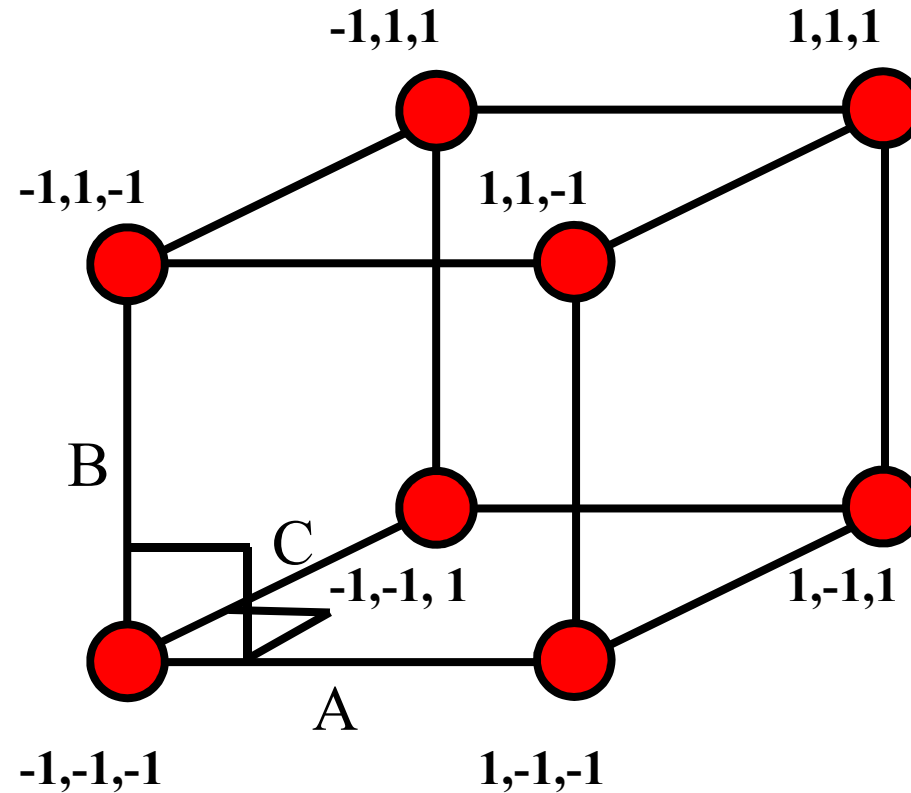
Therefore  
orthogonal and  
independent

# Coding and Orthogonality for a $2^2$ Design



**Orthogonal designs can be represented as a geometric (mathematical) figure**

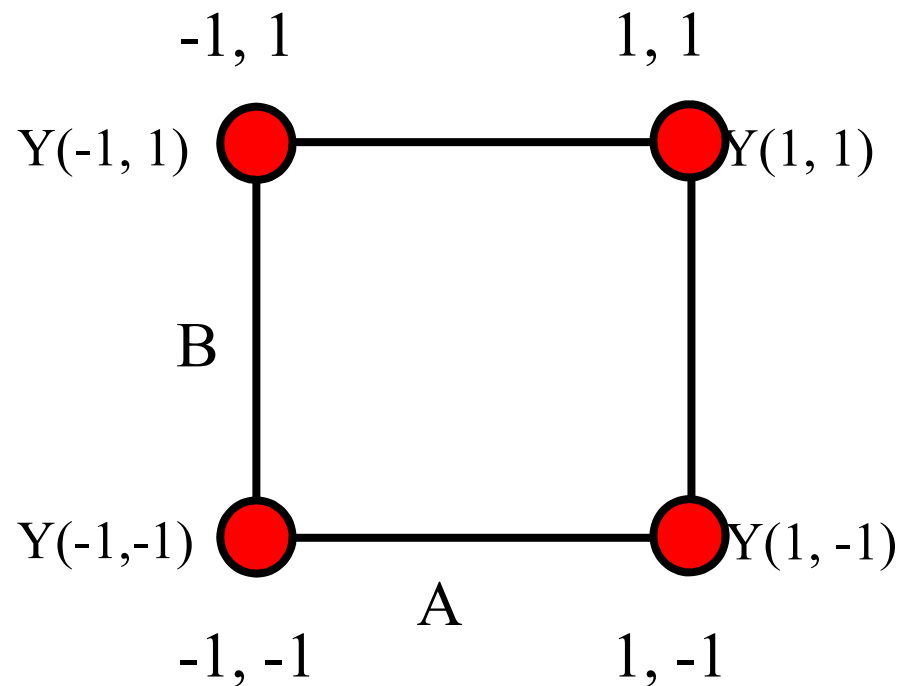
# Coding and Orthogonality for a $2^3$ Design



The vertices are the response measurement points; the volume within is the inference space

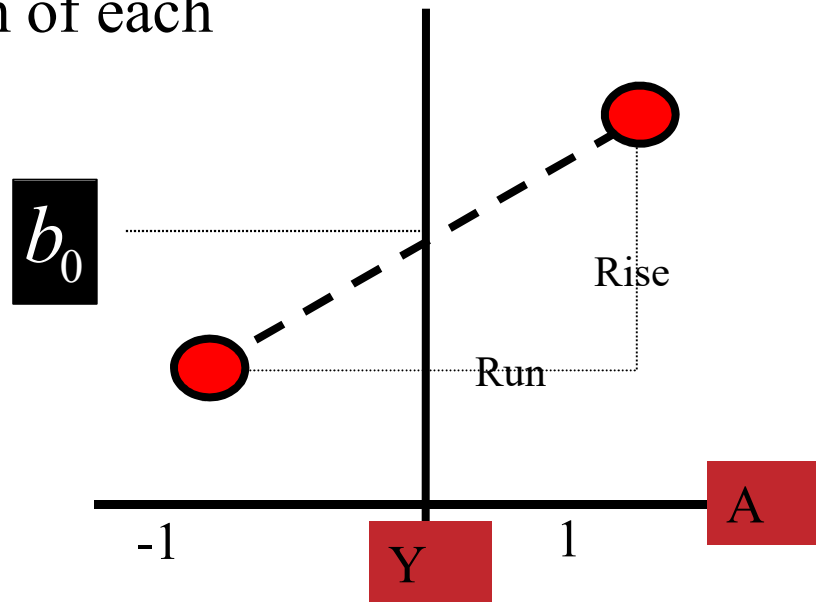
# Response and Orthogonality

**The response can be measured at each corner of the design, as represented by  $Y(A,B)$ .**



# Coding and the Linear Model

Since the factors are orthogonal,  $Y$  is a linear combination of each factor



The linear equation,

$$Y = b_0 + b_1 A$$

describes the line between the points

$b_0 =$  Intercept

$b_1 =$  Rise/Run

Fitting a linear model to the coded design becomes very easy

# Anexas Consultancy Services

## Main Effects and Interactions

# Calculating Main Effects

**A DOE is run:**

	Coded Factors		Response
	A	B	Y
	-1	-1	48
	1	-1	96
	-1	1	72
	1	1	36
<b>Y<sub>ave</sub> at FACTOR<sub>high</sub></b>	<b>66</b>	<b>54</b>	
<b>Y<sub>ave</sub> at FACTOR<sub>low</sub></b>	<b>60</b>	<b>72</b>	
<b>Effect</b>	<b>6</b>	<b>-18</b>	

$$\frac{96 + 36}{2} = 66$$

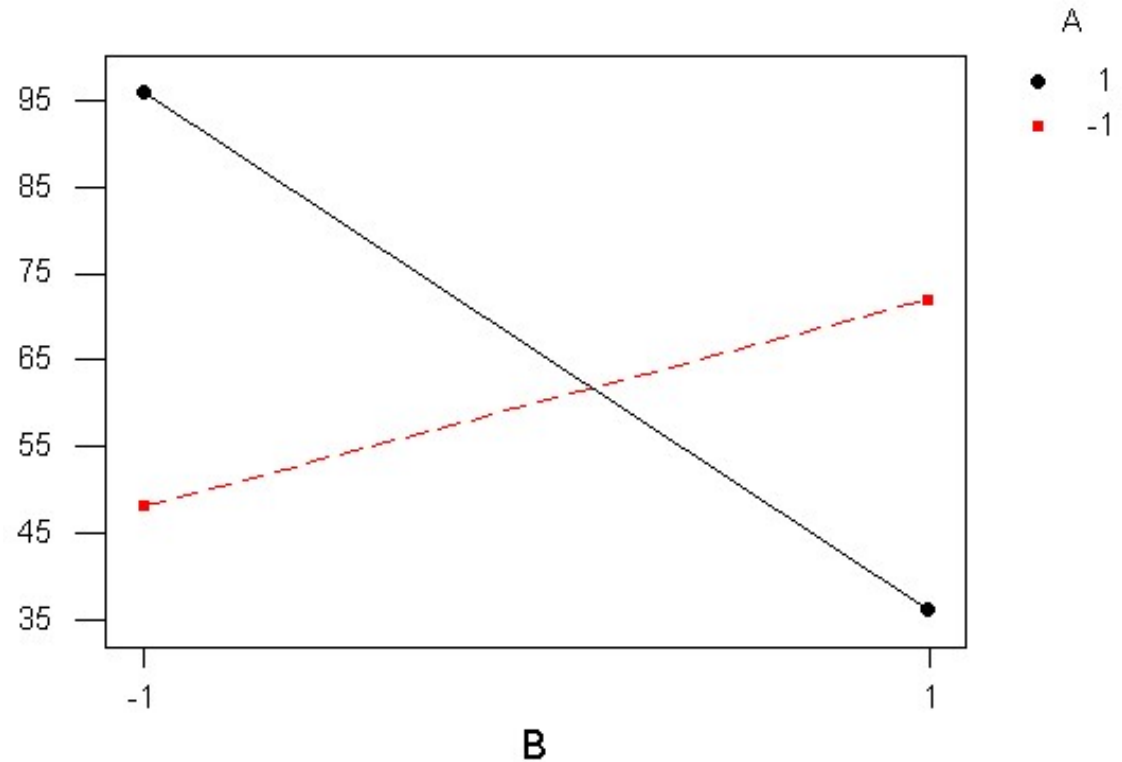
What does a non-zero effect mean?

$$effect = \bar{Y}(@ factorhigh) - \bar{Y}(@ factorlow)$$

**The factor (or main) effects are easily calculated**



# Discovering Interactions



**A response change due to both A and B changing is called an interaction**

# Linear Prediction Equation With Interaction Term

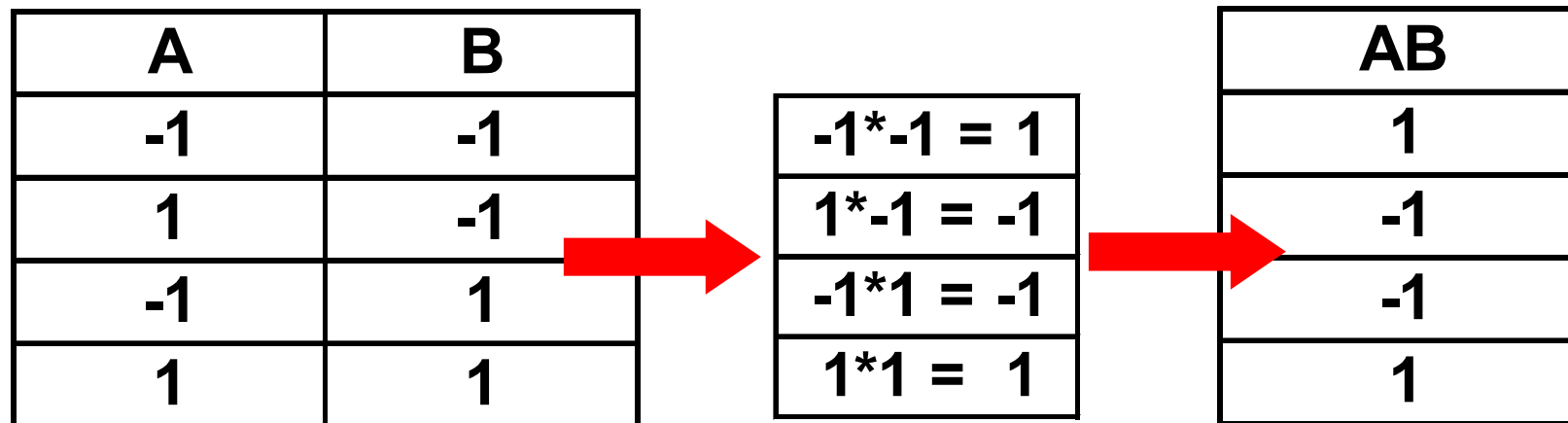
- The interaction will add a term to the linear equation

$$Y = b_0 + b_1 A + b_2 B + b_3 AB$$

- A and B are the main effects
- AB is the interaction
- $b_0$  is the grand mean (intercept)
- $b_1$ ,  $b_2$ , and  $b_3$  are the term coefficients

# Coded Values for Interactions


The coded value matrix for an interaction is the product of the factor coded values:



**Finding the interaction levels in a coded design is as simple as multiplying one times one**

# Calculating Interaction Effects

## The DOE again

	Coded Factors		Interaction	Response
This is the Yates Standard Order for a $2^2$ coded design with interaction	A	B	AB	Y
	-1	-1	1	48
	1	-1	-1	96
	-1	1	-1	72
	1	1	1	36
<b>Y<sub>ave</sub> at FACTOR<sub>high</sub></b>	<b>66</b>	<b>54</b>	<b>42</b>	
<b>Y<sub>ave</sub> at FACTOR<sub>low</sub></b>	<b>60</b>	<b>72</b>	<b>84</b>	
<b>Effect</b>	<b>6</b>	<b>-18</b>	<b>-42</b>	

The interaction is set by the design (math) based upon the factor settings

# Anexas Consultancy Services

## Main Effects, Interactions and Cube Plots in Minitab

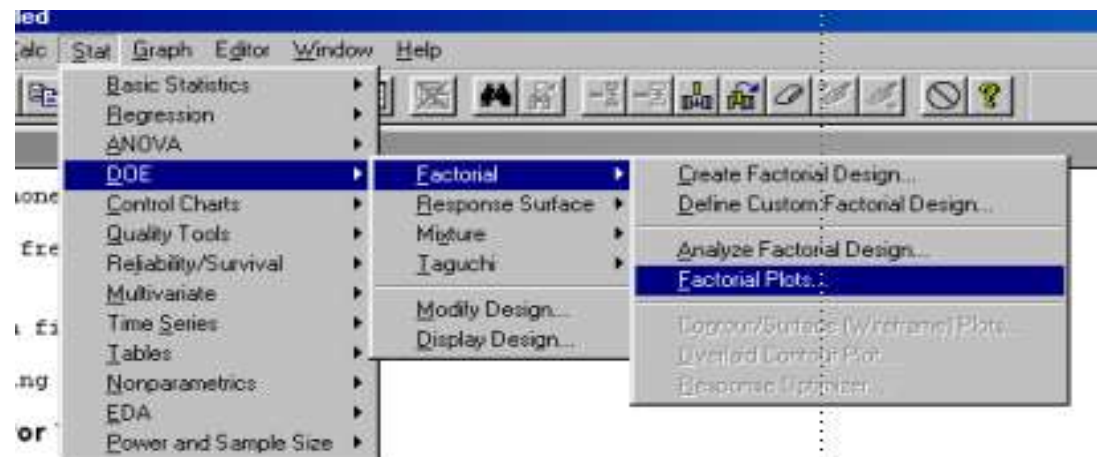
# Create the Experiment

Tool Bar Menu > Stat > DOE > Factorial > Factorial Plots

Create a 2<sup>2</sup> coded design for factors A and B.

	C1	C2	C3	C4	C5	C6	C7	C8
	StdOrder	RunOrder	CenterPt	Blocks	A	B	Y	
1	1	1	1	1	-1	-1	48	
2	2	2	1	1	1	-1	96	
3	3	3	1	1	-1	1	72	
4	4	4	1	1	1	1	36	
5								

Input the Y response



# Factorial Plots in Minitab

## Step 1

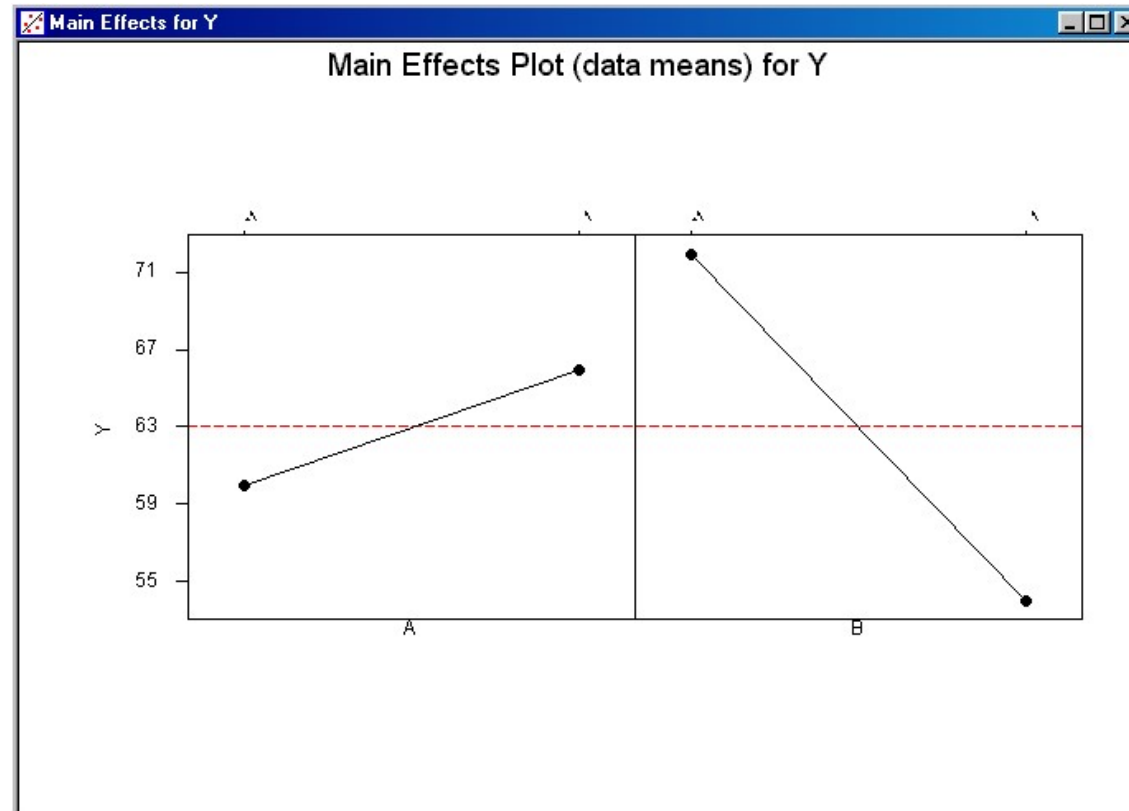
The image shows two Minitab dialog boxes for creating factorial plots. The first dialog, 'Factorial Plots', has three checked options: 'Main Effects Plot', 'Interaction Plot', and 'Cube Plot'. A red arrow points from the 'Main Effects Plot' option to the second dialog. The second dialog, 'Factorial Plots - Main Effects', has 'Y' entered in the 'Responses:' field. In the 'Factors to Include in Plots' section, 'A:A' and 'B:B' are listed in the 'Selected:' list. A red arrow points from the 'Available:' list to the 'Selected:' list.

Select the type of factorial plot desired

Similarly setup interaction and cube plots

Select the response column and the factor columns

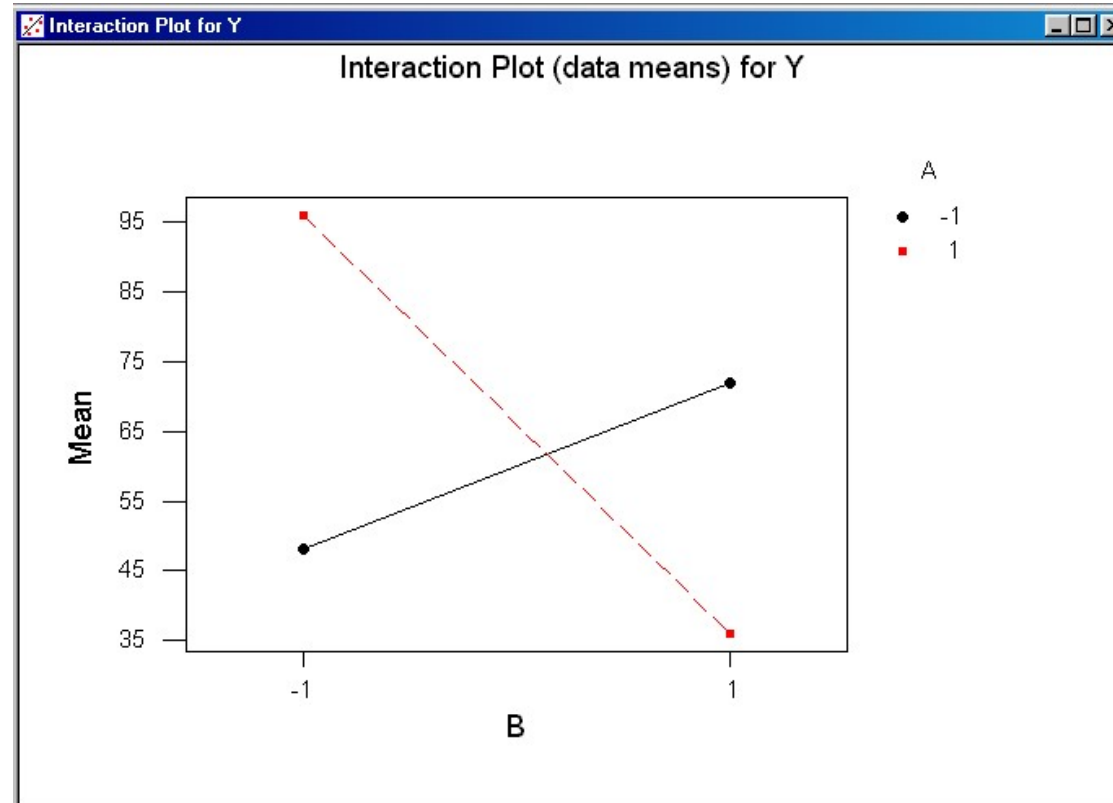
# Main Effects Plot



**The response is plotted by factor from low level to high level**

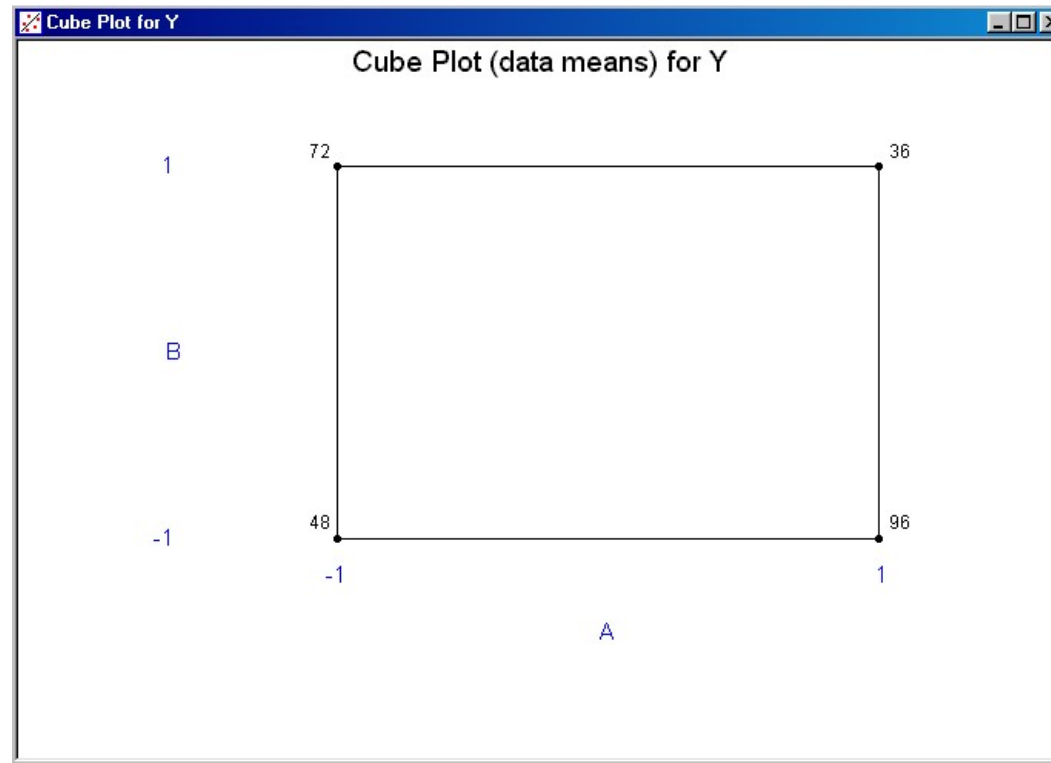


# Interaction Plot



**Parallel lines indicate no interaction; the less parallel, the higher the degree of interaction**

# Cube Plot



**The response is plotted on the orthogonal factor axis**

# Class Exercise

**Using the file *TWO CUBE EXERCISE.MTW* create all three factorial plots.**

**Be prepared to discuss your results.**

# Anexas Consultancy Services

## The Prediction Equations

# Developing the Prediction Equation

**Knowing the design, responses and effect analysis the prediction equation can be found.**

	Coded Factors		Interaction	Response
	A	B	AB	Y
	-1	-1	1	48
	1	-1	-1	96
	-1	1	-1	72
	1	1	1	36
<b>Y<sub>ave</sub> at FACTOR<sub>high</sub></b>	<b>66</b>	<b>54</b>	<b>42</b>	
<b>Y<sub>ave</sub> at FACTOR<sub>low</sub></b>	<b>60</b>	<b>72</b>	<b>84</b>	
<b>Effect</b>	<b>6</b>	<b>-18</b>	<b>-42</b>	

$$\hat{Y} = b_0 + b_1A + b_2B + b_3AB$$

# The Intercept – Constant – Grand Mean Relationship

$$Y = b_0 + b_1 A + b_2 B + b_3 AB$$

If we set all (coded) factors to equal zero, the equation becomes:

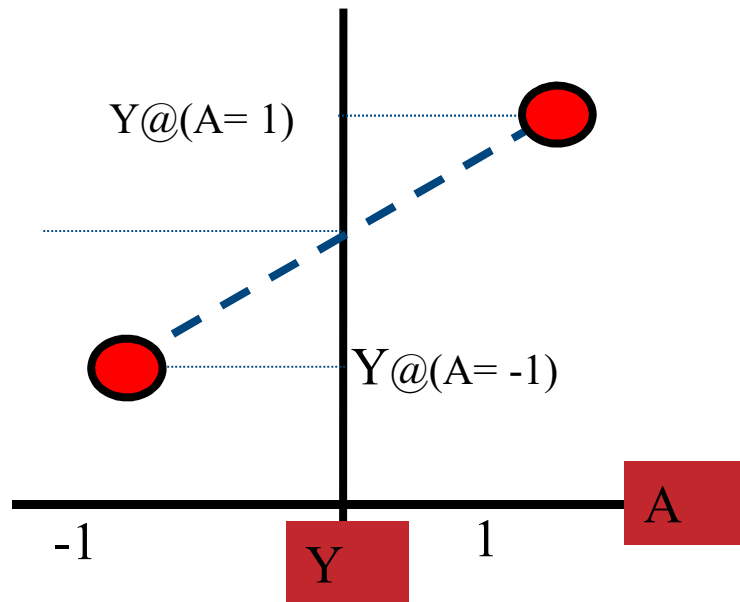
$$Y = b_0$$

which is the overall average, or grand mean. The grand mean of the response is 63.

**The regression constant is the grand mean of the responses for a coded design**

# Finding the Coefficients

$$\hat{Y} = 63 + b_1 A + b_2 B + b_3 AB$$



Looking at  $b_1 A$  in coded units

when  $A = 1$ ,  $Y_{ave} = 66$

and

when  $A = -1$ ,  $Y_{ave} = 60$

The rise from  $-1$  to  $+1$  is 6 The run is 2

The slope is  $6/2 = 3$

The regression coefficients are the factor effects divided by two for a coded design

# Finishing the Matrix

	Coded Factors		Interaction		Response
	A	B	AB		Y
	-1	-1	1		48
	1	-1	-1		96
	-1	1	-1		72
	1	1	1		36
<b>Y<sub>ave</sub> at FACTOR<sub>high</sub></b>	<b>66</b>	<b>54</b>	<b>42</b>	<b>Grand Mean</b>	<b>63</b>
<b>Y<sub>ave</sub> at FACTOR<sub>low</sub></b>	<b>60</b>	<b>72</b>	<b>84</b>		
<b>Effect</b>	<b>6</b>	<b>-18</b>	<b>-42</b>		
<b>Coefficient</b>	<b>3</b>	<b>-9</b>	<b>-21</b>		

$$Y = 63 + 3A - 9B - 21 AB$$

**Verify the equation for the coded A and B values in the matrix**



# Anexas Consultancy Services

## DOE Analysis in Minitab™

# Create the Experiment

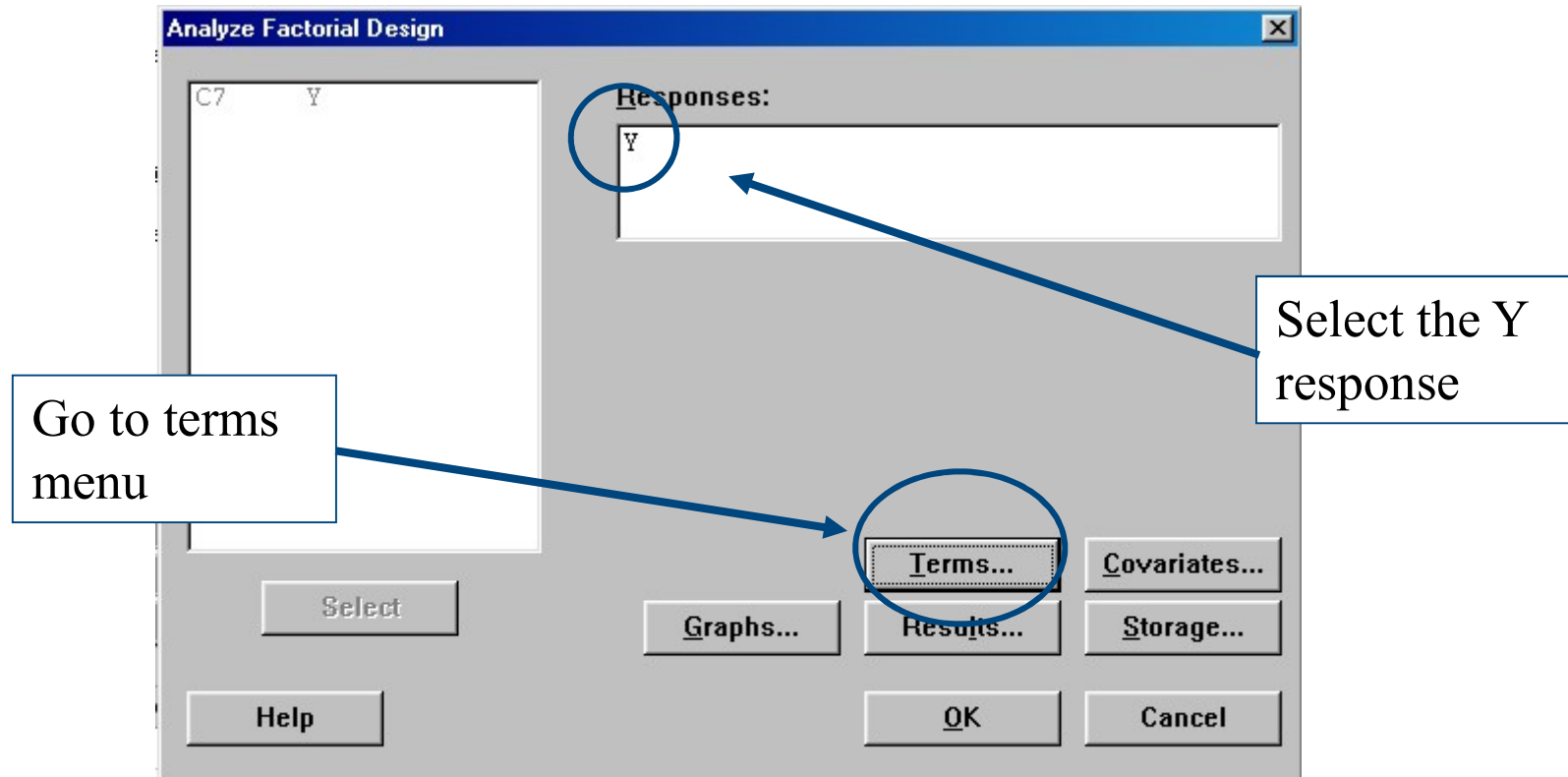
Create or recall the 2<sup>2</sup> coded design for factors A and B.

Worksheet 4 ***								
↓	C1	C2	C3	C4	C5	C6	C7	C8
	StdOrder	RunOrder	CenterPt	Blocks	A	B	Y	
1	1	1	1	1	-1	-1	48	
2	2	2	1	1	1	-1	96	
3	3	3	1	1	-1	1	72	
4	4	4	1	1	1	1	36	
5								

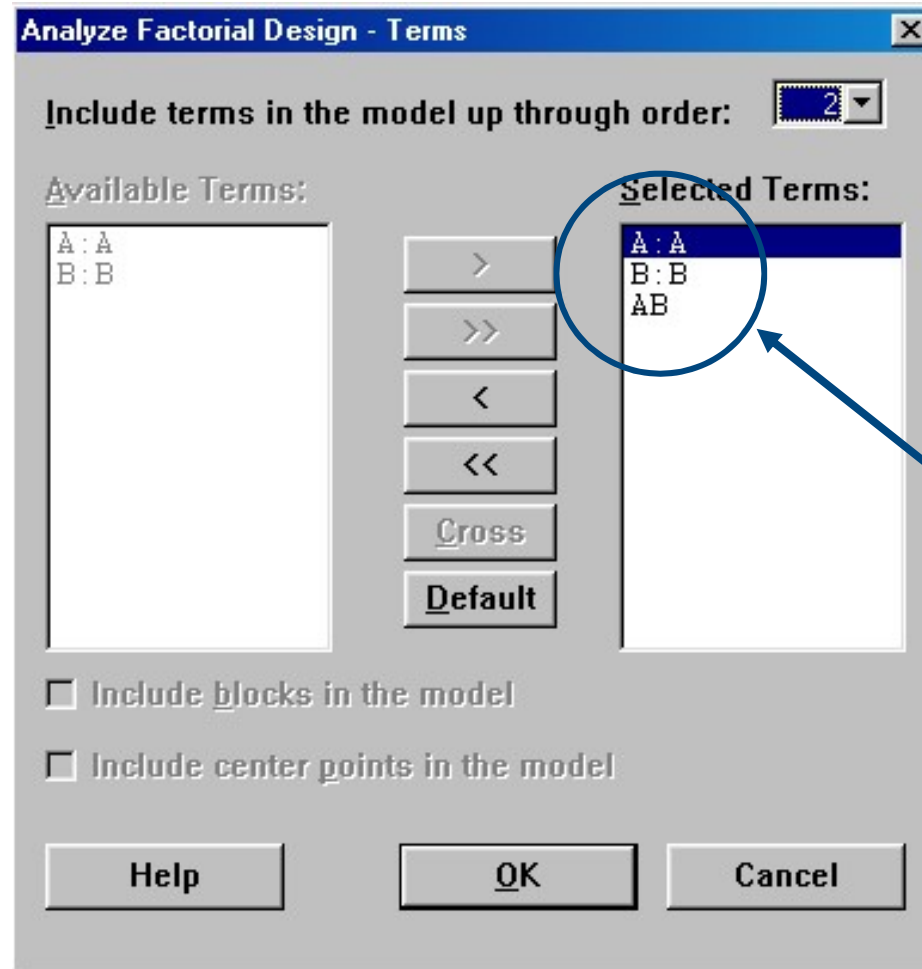
Input the Y response

# Setting Up the Analysis

**Tool Bar Menu > Stat > DOE > Factorial > Analyze Factorial Design**



# Selecting Terms



Select all of the available terms note the AB interaction!

# The Analysis

## Fractional Factorial Fit: Y versus A, B

Estimated Effects and Coefficients for Y (coded units)

Term	Effect	Coef
Constant		63.00
A	6.00	3.00
B	-18.00	-9.00
A*B	-42.00	-21.00

Analysis of Variance for Y (coded units)

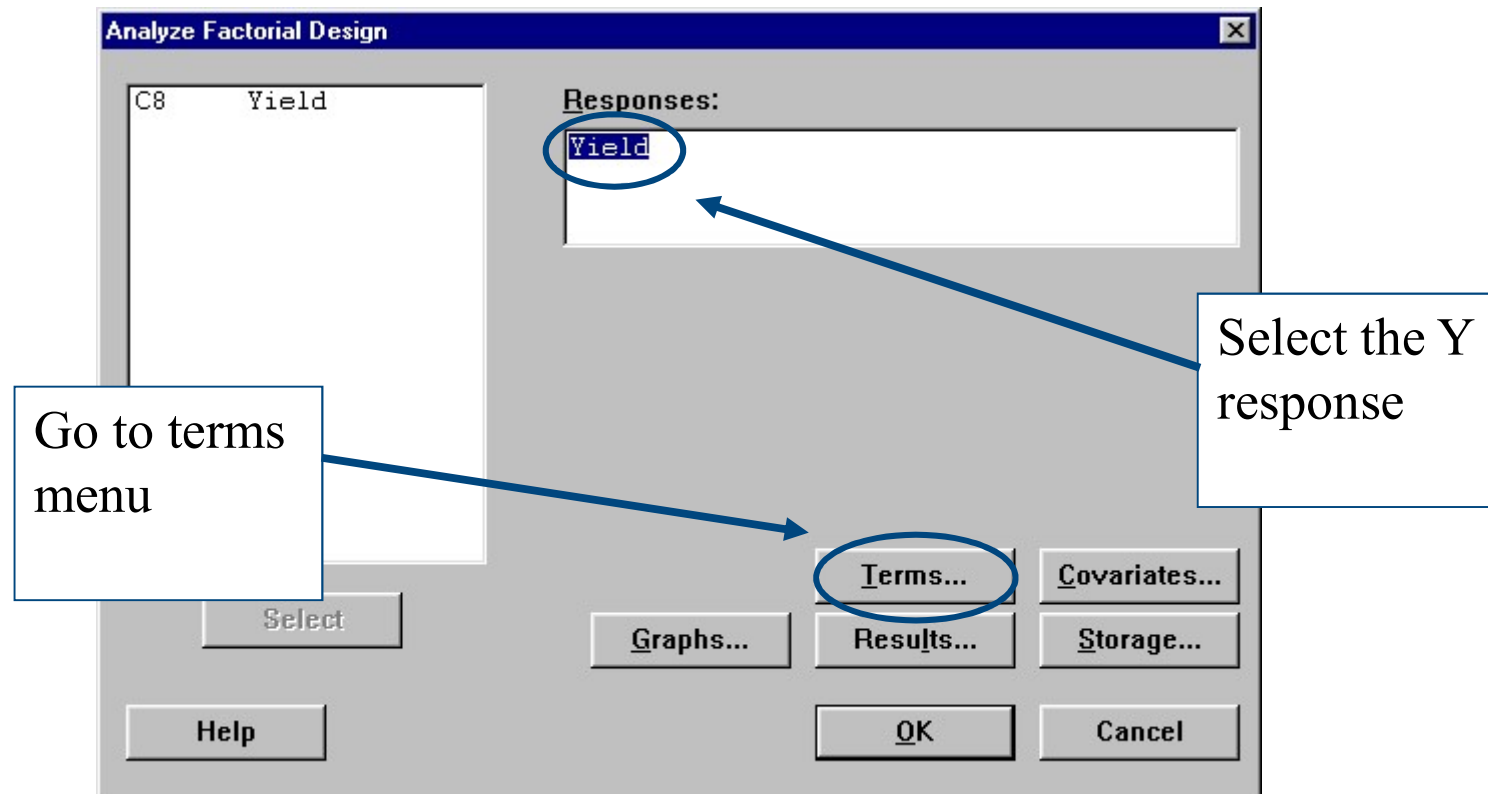
Source	DF	Seq SS	Adj SS	Adj MS	F	P
Main Effects	2	360.0	360.0	180.0	*	*
2-Way Interactions	1	1764.0	1764.0	1764.0	*	*
Residual Error	0	0.0	0.0	0.0		
Total	3	2124.0				

**The coefficients are identical to manual analysis**

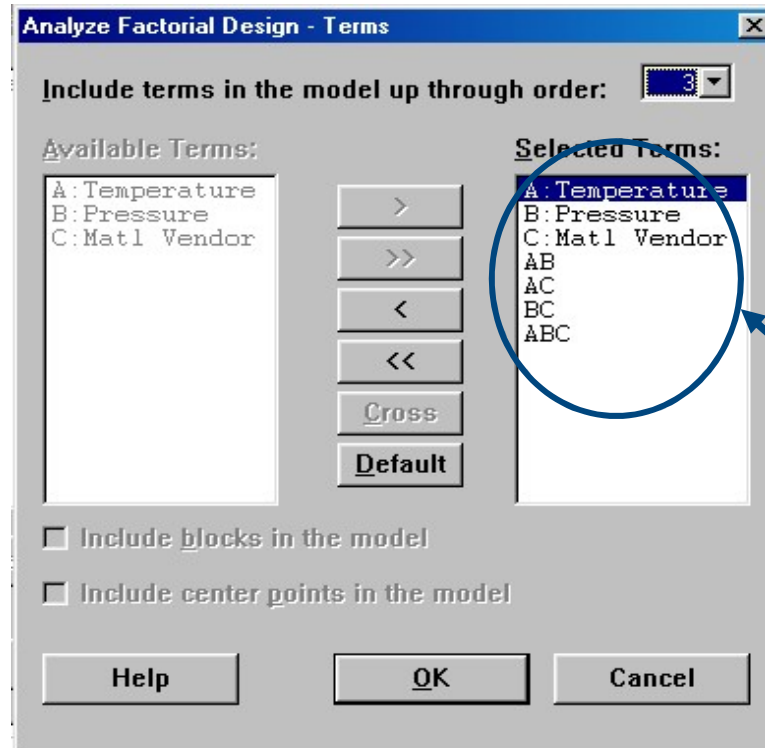
# Another Analysis

Tool Bar Menu > Stat > DOE > Factorial > Analyze Factorial Design

Recalling the *TWO CUBE EXERCISE.MTW* file



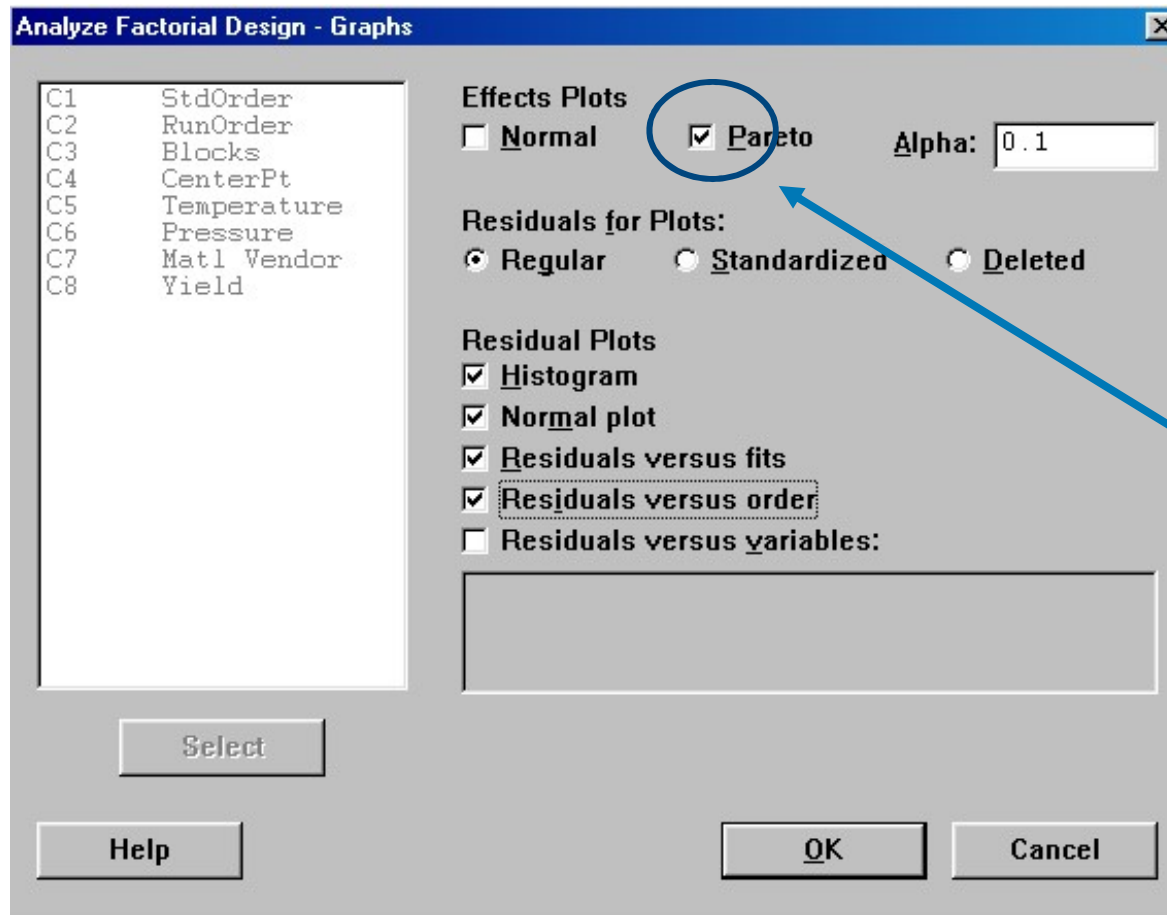
# Setting Up the Analysis



Select all of the available terms  
note interaction terms

**What can be estimated with this model?**  
**How many terms in the prediction equation?**

# Selecting Graph Options



From the Graphs menu select the graph shown



# The ANOVA Results

**Results for: TWO CUBE EXERCISE.MTW**

**Fractional Factorial Fit: Yield versus Temperature, Pressure, ...**

Estimated Effects and Coefficients for Yield (coded units)

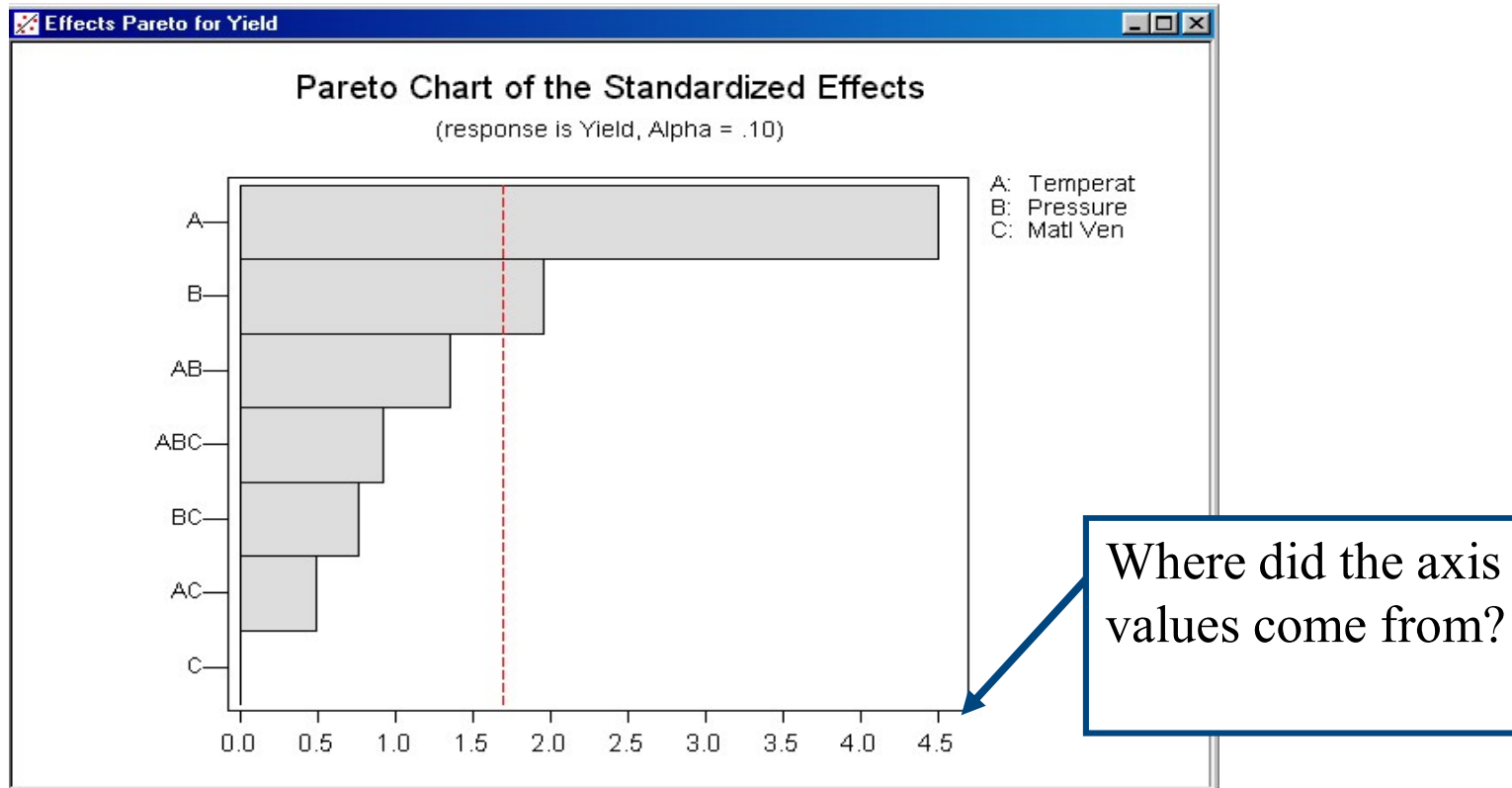
Term	Effect	Coef	SE Coef	T	P
Constant		0.66432	0.01356	49.01	0.000
Temperat	-0.12226	-0.06113	0.01356	-4.51	0.000
Pressure	-0.05303	-0.02651	0.01356	-1.96	0.059
Matl Ven	-0.00000	-0.00000	0.01356	-0.00	1.000
Temperat*Pressure	0.03682	0.01841	0.01356	1.36	0.184
Temperat*Matl Ven	0.01326	0.00663	0.01356	0.49	0.628
Pressure*Matl Ven	-0.02062	-0.01031	0.01356	-0.76	0.452
Temperat*Pressure*Matl Ven	0.02504	0.01252	0.01356	0.92	0.363

Analysis of Variance for Yield (coded units)

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Main Effects	3	0.177592	0.177592	0.059197	8.05	0.000
2-Way Interactions	3	0.019571	0.019571	0.006524	0.89	0.458
3-Way Interactions	1	0.006271	0.006271	0.006271	0.85	0.363
Residual Error	32	0.235199	0.235199	0.007350		
Pure Error	32	0.235199	0.235199	0.007350		
Total	39	0.438632				

The ANOVA Table shows temperature and pressure are significant; regression coefficients are listed

# The Pareto of Standardized Effects



**The Pareto of effects shows the ranking of the variation identified by each source; note the “line of significance”**

# The Prediction Equation from Minitab™

Term	Effect	Coef
Constant		0.66432
Temperat	-0.12226	-0.06113
Pressure	-0.05303	-0.02651
Matl Ven	-0.00000	-0.00000
Temperat*Pressure	0.03682	0.01841
Temperat*Matl Ven	0.01326	0.00663
Pressure*Matl Ven	-0.02062	-0.01031
Temperat*Pressure*Matl Ven	0.02504	0.01252

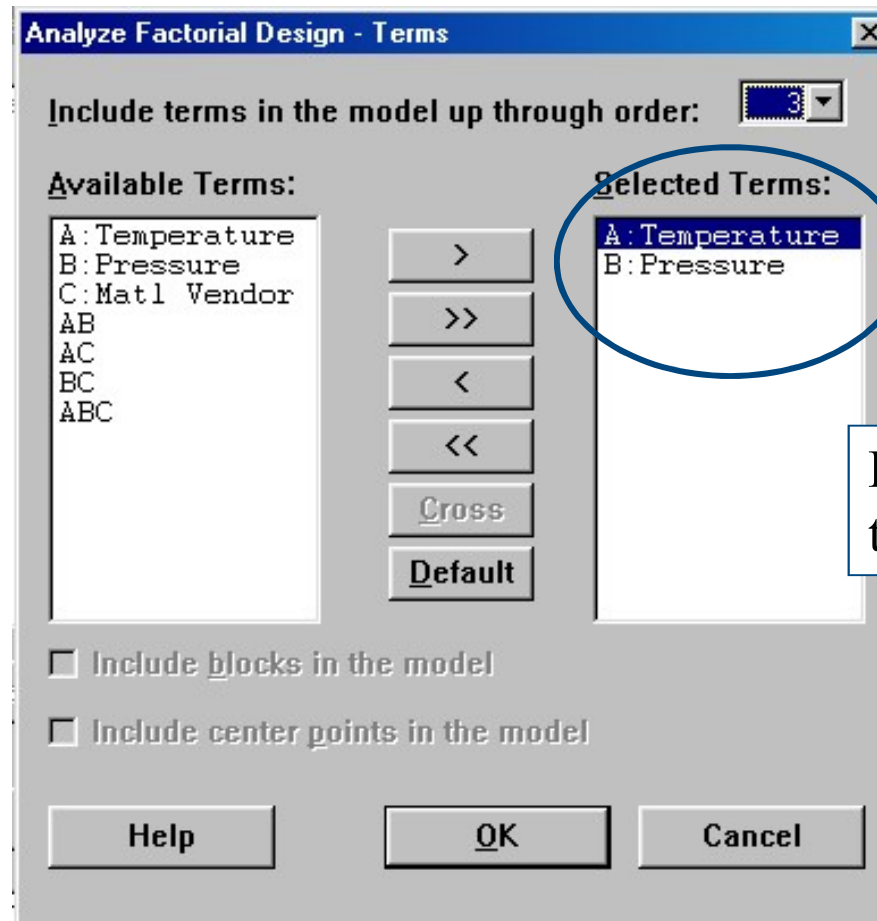
- The ANOVA table coefficients column feeds the prediction equation

$$Y = b_0 + b_1 A + b_2 B + b_3 C + b_4 AB + b_5 AC + b_6 BC + b_7 ABC$$

- The ANOVA table p-value column identifies the significant effects, or terms of the regression; to simplify the prediction equation the number of terms must be reduced

# Removing Model Terms in Minitab™

**Tool Bar Menu > Stat > DOE > Factorial > Analyze Factorial Design > Terms**



Remove insignificant terms

# Removing Terms Stepwise

- When reducing the regression model do not remove all insignificant terms at once
- Removed terms variance go into error, along with their degrees of freedom
- Significance levels can change as terms are removed.
- Start with higher order terms and remove no more than two at a time.
- Run analysis and check for changes in significance

# Re-run Reduced Model in Minitab™

## Fractional Factorial Fit: Yield versus Temperature, Pressure

Estimated Effects and Coefficients for Yield (coded units)

Term	Effect	Coef	SE Coef	T	P
Constant		0.66432	0.01328	50.02	0.000
Temperat	-0.12226	-0.06113	0.01328	-4.60	0.000
Pressure	-0.05303	-0.02651	0.01328	-2.00	0.053

Analysis of Variance for Yield (coded units)

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Main Effects	2	0.17759	0.17759	0.088796	12.59	0.000
Residual Error	37	0.26104	0.26104	0.007055		
Lack of Fit	1	0.01356	0.01356	0.013561	1.97	0.169
Pure Error	36	0.24748	0.24748	0.006874		
Total	39	0.43863				

$$Y = 0.664 - 0.061A - 0.026 B$$

Where do you set the factors to optimize this process?

# Class 2<sup>3</sup> DOE Exercise

## Part 1

- Your company has a global operation. Times are tough and a restructuring of your world wide product centers is needed. Your team is required to develop the restructuring model based upon three core global business metrics: exchange rate, customs hold time and cross border banking fees. The goal is to find the combination of these criteria that minimizes the variance to the operating plan for the business.
- Exchange rate is felt to be unimportant due to currency stability in the countries you deal in. Cross border fees are a matter of local control and can be greatly reduced by moving cost centers into the countries, thus keeping money transfers to a minimum. The team agrees that the real problem is custom hold time – which can be expedited by additional cost, but will reduce the supply side pressure.
- What do you recommend to the CEO about the importance of exchange rates, cross border fees and custom hold times as she makes her decision - in the next 20 minutes?

# Class 2<sup>3</sup> DOE Exercise

## Part 2

- Using the file ***TWO CUBED CLASS.XLS***
- Create the design in Minitab
- Import the data from Excel™
- Analyze the factorial plots
- Analyze the ANOVA
- Select significant terms
- Reduce the model
- Determine the final prediction equation
- Optimize the process by selecting factor levels
- Be prepared to present your results



# Anexas Consultancy Services

## The General Linear Model

# Why GLM?

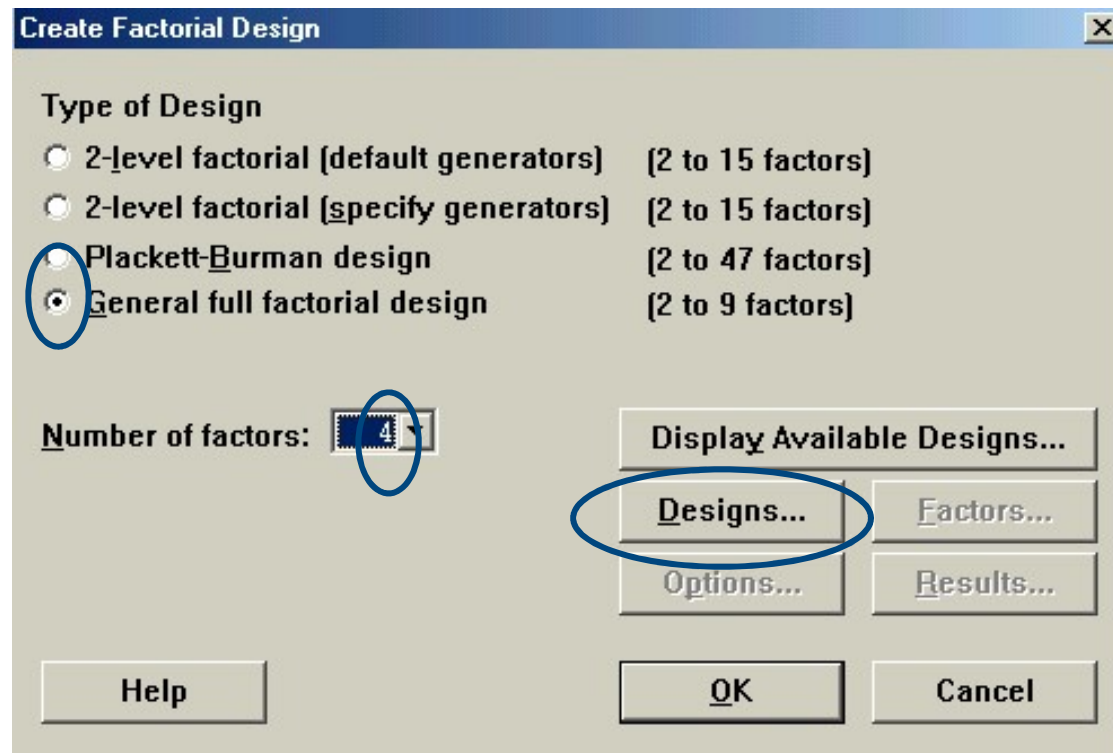
- The General Linear Model
- Allows more flexible design
- Allows multiple levels
- Does not require factors to have same number of levels
- Is well suited for business process problems

# Setting up a GLM Design

- Account receivables lockup, where payments are withheld, is thought to be caused by four factors
- SPAs (Special Pricing Agreements) -4 categories
- Market sector – 3 demographics
- Sales region – 6 regional centers
- Performance to contract – 3 levels
  
- Design a DOE to study the problem

# Setting up GLM

**Tool Bar Menu > Stat > DOE > Factorial > Create Factorial Design**



# Factor Name and Number of Level Assignments

The image displays two instances of the 'Create Factorial Design - Designs' dialog box. The left instance shows a table with four factors (A, B, C, D) and their corresponding names and levels. The right instance shows the same dialog box but with specific values assigned to each factor. The 'OK' button in the right instance is circled in blue.

Factor	Name	Number of Levels
A	A	
B	B	
C	C	
D	D	

Number of replicates: 1  
 Block on replicates  
 Help OK Cancel

Factor	Name	Number of Levels
A	SPA	4
B	Market Sector	3
C	Sales Region	6
D	Performance	3

Number of replicates: 1  
 Block on replicates  
 Help OK Cancel

# Factor Level Assignments

The image shows two overlapping dialog boxes from Minitab. The top dialog is 'Create Factorial Design' and the bottom one is 'Create Factorial Design - Factors'.

**Create Factorial Design Dialog:**

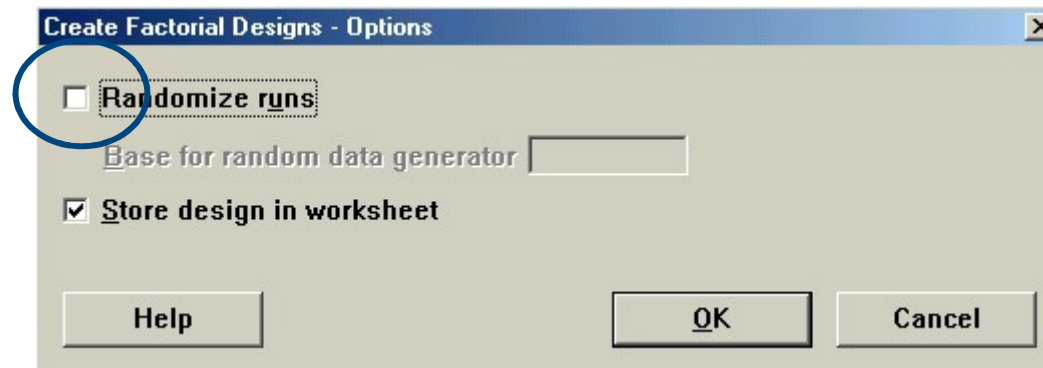
- Type of Design:
  - 2-level factorial (default generators) [2 to 15 factors]
  - 2-level factorial (specify generators) [2 to 15 factors]
  - Plackett-Burman design [2 to 47 factors]
  - General full factorial design [2 to 9 factors]
- Number of factors: 4
- Buttons: Display Available Designs..., Designs..., Factors... (circled), Options..., Results..., Help, OK, Cancel.

**Create Factorial Design - Factors Dialog:**

Factor	Name	Level Values	Levels
A	SPA	1 2 3 4	4
B	Market	C I M	3
C	Sales	R N S E W Eu AP	6
D	Perform	H M L	3

Buttons: Help, OK, Cancel.

# Options



# GLM Generated Worksheet

C1	C2	C3	C4	C5-T	C6-T	C7-T	C8
StdOrder	RunOrder	Blocks	SPA	Market Sector	Sales Region	Performance	AR Held
1	1	1	1 C	C	N	H	
2	2	1	1 C	C	N	M	
3	3	1	1 C	C	N	L	
4	4	1	1 C	C	S	H	
5	5	1	1 C	C	S	M	
6	6	1	1 C	C	S	L	
7	7	1	1 C	C	E	H	
8	8	1	1 C	C	E	M	
9	9	1	1 C	C	E	L	
10	10	1	1 C	C	W	H	
11	11	1	1 C	C	W	M	
12	12	1	1 C	C	W	L	
13	13	1	1 C	C	Eu	H	
14	14	1	1 C	C	Eu	M	
15	15	1	1 C	C	Eu	L	
16	16	1	1 C	C	AP	H	
17	17	1	1 C	C	AP	M	
18	18	1	1 C	C	AP	L	
19	19	1	1 I	I	N	H	
20	20	1	1 I	I	N	M	
21	21	1	1 I	I	N	L	
22	22	1	1 I	I	S	H	

C1	C2	C3	C4	C5-T	C6-T	C7-T	C8
StdOrder	RunOrder	Blocks	SPA	Market Sector	Sales Region	Performance	AR Held
196	196	1	4 I	I	AP	H	
197	197	1	4 I	I	AP	M	
198	198	1	4 I	I	AP	L	
199	199	1	4 M	M	N	H	
200	200	1	4 M	M	N	M	
201	201	1	4 M	M	N	L	
202	202	1	4 M	M	S	H	
203	203	1	4 M	M	S	M	
204	204	1	4 M	M	S	L	
205	205	1	4 M	M	E	H	
206	206	1	4 M	M	E	M	
207	207	1	4 M	M	E	L	
208	208	1	4 M	M	W	H	
209	209	1	4 M	M	W	M	
210	210	1	4 M	M	W	L	
211	211	1	4 M	M	Eu	H	
212	212	1	4 M	M	Eu	M	
213	213	1	4 M	M	Eu	L	
214	214	1	4 M	M	AP	H	
215	215	1	4 M	M	AP	M	
216	216	1	4 M	M	AP	L	

This is a BIG test – but Minitab easily handles it.



# Objectives Review

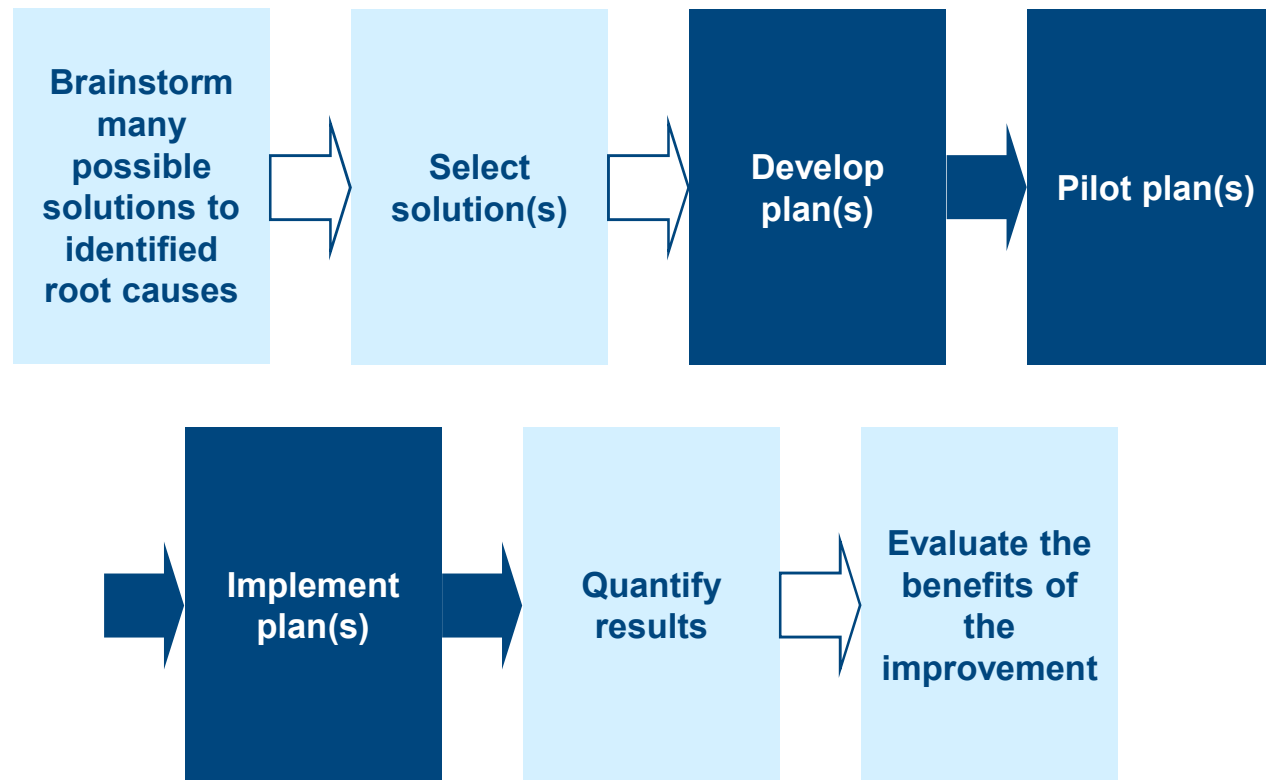
By the end of this module, the participant should:

- Generate a full factorial design
- Look for factor interactions
- Develop coded orthogonal designs
- Write process prediction equations (models)
- Set factors for process optimization
- Create and analyze designs in Minitab™
- Evaluate residuals
- Develop process models from Minitab™ analysis
- Determine sample size

# Anexas Consultancy Services

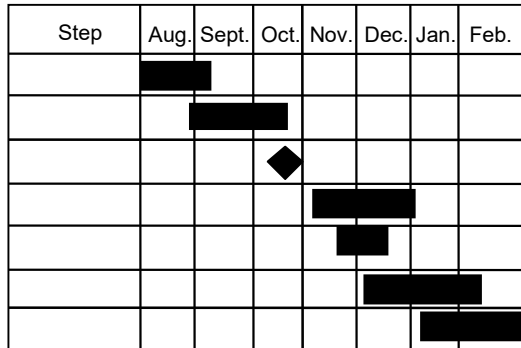
## Implementing Solutions

# You Are Here

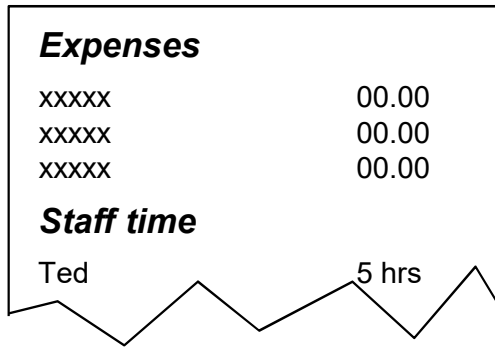


# Elements of a Plan

## Tasks and Timeline



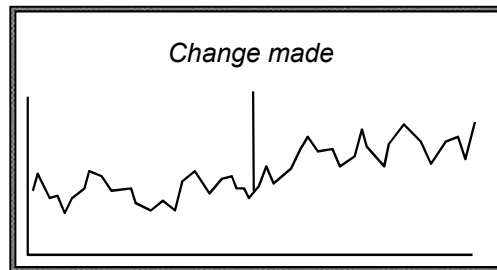
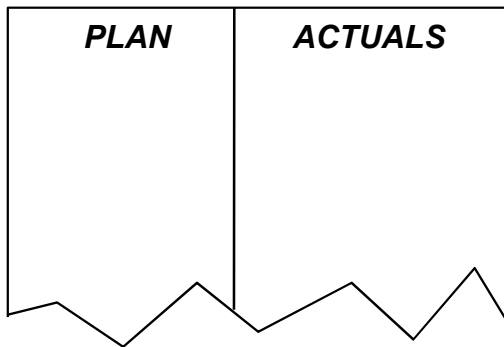
## Budget and Resources



## Stakeholders

PERSON or GROUP	Communication and Participation
Finance	
Sales	
IS	

## How to Check



## Potential Problems

Step	Pot. Failure	Pot. Cause	Counter-measures

# Anexas Consultancy Services

FMEA



# Anexas Consultancy Services

**Piloting**

# Benefits of Piloting

- Improved solution
- Improved implementation plan
- Increased buy-in
- Get some of the benefits of the improvement quickly
- Reduced risk of failure or unknown complications
- Ability to confirm assumed cause-and-effect relationships
- Increased ability to quantify costs and benefits
- Overall benefit is a better solution with fewer surprises



# Anexas Consultancy Services

## Implementing the Plans

# Implementation Steps

- Flowchart new procedures if you have not already
- Revise job instructions
- Prepare data collection forms
- Provide training
- Test on a small scale first, if at all possible:
  - Revise procedures based on the test
- Do full-scale implementation
- Monitor the implementation

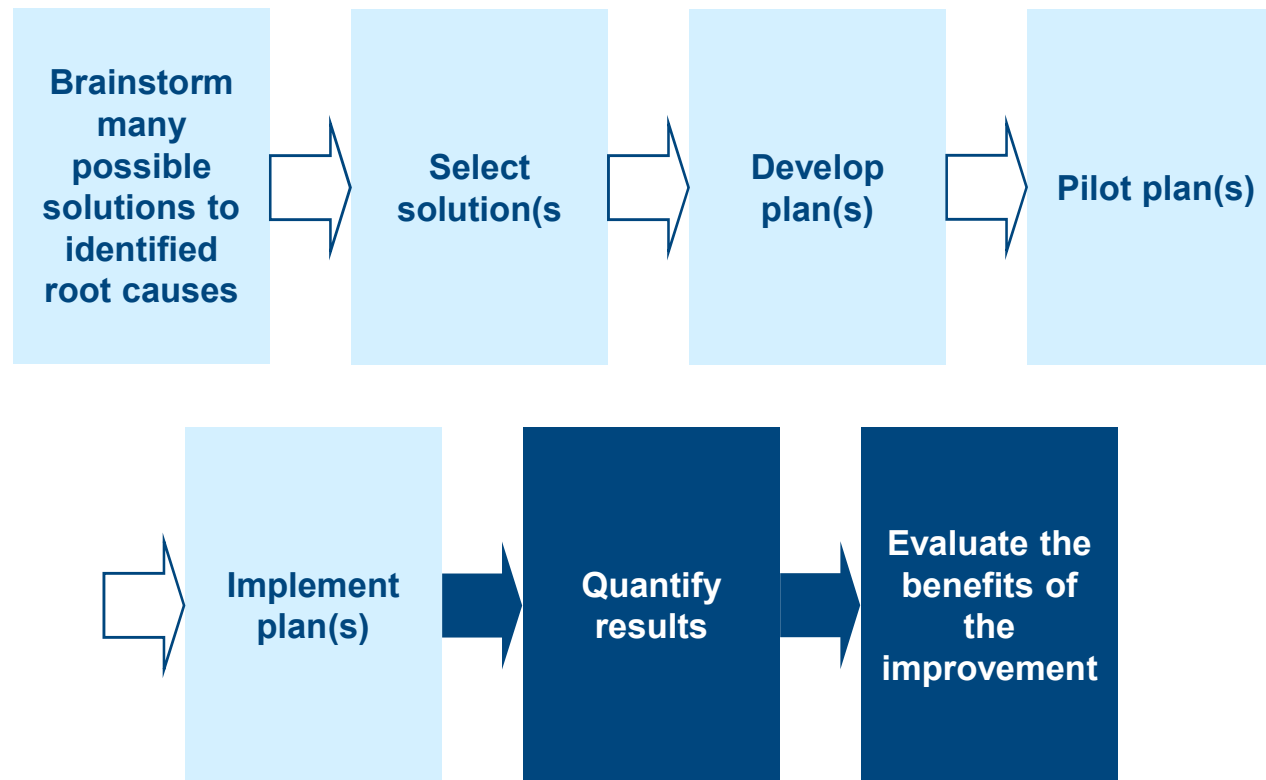
# Implementing Solutions: Review

- Prepare detailed plans that include:
  - Tasks and timelines
  - Budgets and resources
  - Stakeholder involvement
  - Plans for checking
  - Failure prevention
  - Implement on small scale first, then move to full scale

# Anexas Consultancy Services

## Evaluating Results

# You Are Here

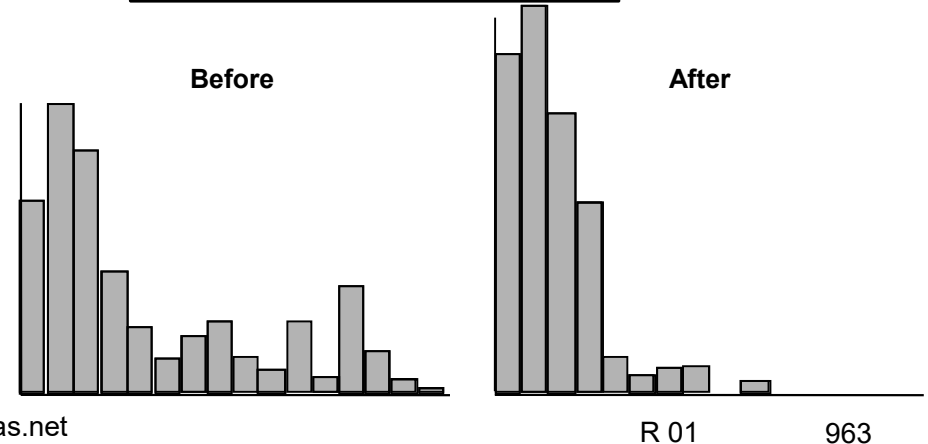
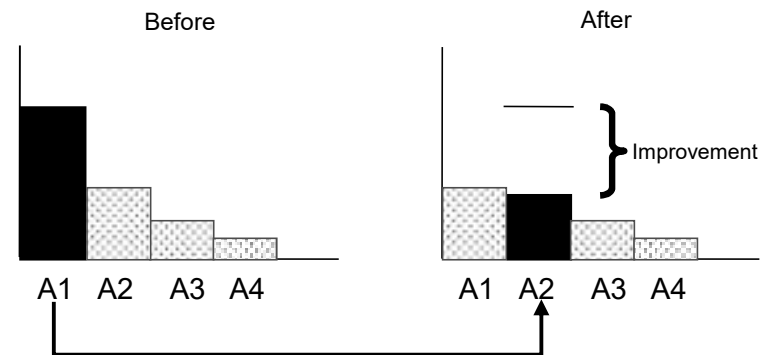
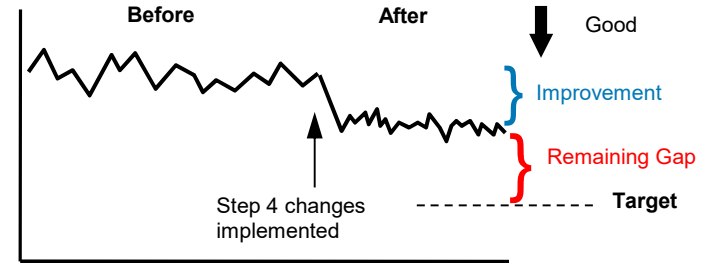


# Anexas Consultancy Services

## Quantifying Results

# Displaying Before and After Data

- Add more data to an existing run chart or control chart
- Prepare new Pareto charts for those you created in MEASURE:
  - Make scale and dimensions the same so you can more accurately judge degree of improvement
- Draw new frequency plots on the same scale as the original plots



# Recalculate Process Sigma

Step	Units	Opportunities	BEFORE	AFTER
1	40	2 x 40 = 80	61	17
2	40	1 x 40 = 40	37	1
3	40	3 x 40 = 120	113	12
4	40	1 x 40 = 40	34	0
5	40	4 x 40 = 160	142	8
6	40	1 x 40 = 40	38	0
7	40	2 x 40 = 80	69	3
8	40	4 x 40 = 160	143	37
9	40	1 x 40 = 40	37	0
10	40	2 x 40 = 80	64	5
11	40	2 x 40 = 80	72	2
12	40	1 x 40 = 40	35	1
		<b>960</b>	<b>846</b>	<b>86</b>

**BEFORE**

$$\begin{aligned} \text{Yield} &= 1 - \frac{846}{960} \\ &= 1 - .88 \\ &= \mathbf{12\%} \\ \text{Sigma} &= \mathbf{0.3} \end{aligned}$$

**AFTER**

$$\begin{aligned} \text{Yield} &= 1 - \frac{86}{960} \\ &= 1 - .09 \\ &= \mathbf{91\%} \\ \text{Sigma} &= \mathbf{2.8} \end{aligned}$$



# Financial Results

- Estimate the financial benefits of the project:
  - Cost/benefit ratio
  - Payback cycle time
- Order-of-magnitude estimates usually enough:
  - Benefits of scrap or rework reduction are straightforward
  - Benefits of cycle time reduction, etc. are more difficult to quantify
- Consult with the Financial Controller to validate the estimate



# Anexas Consultancy Services

**IMPROVE: Implement Solutions  
and Evaluate Results**

Review

# You Were Here—IMPROVE



# IMPROVE Phase Deliverables

- List of potential improvements
- Optimum solutions identified and selected
- Proof of impact of solutions
- Failure mode effects analysis (FMEA)
- Cost benefit analysis
- Before versus after analysis
- Revised process map
- Implementation plan

## IMPROVE Phase Deliverables, cont.

- Revised stakeholder matrix and communication plan
- IMPROVE section of storyboard completed
- IMPROVE phase discussed and agreed with sponsor and coach
- IMPROVE deliverables completed and phase approved

# IMPROVE Checklist

- By the end of the IMPROVE phase, you should be able to answer the following questions. Please check!
  - What solutions have you identified? How do they affect the root causes?
  - What criteria have you used to select a solution?
    - This includes criteria such as how the solution was linked to the verified root cause(s) identified in the ANALYZE phase
  - How do the various alternatives score against those criteria?
  - What are the results of designed experiments (if applicable) to find optimal settings?
  - How have you tested the solutions, and what were the results of these tests?
  - How do timing and results compare to plans and expected results?

## IMPROVE Checklist, cont.

- By the end of the IMPROVE phase, you should be able to answer the following questions. Please check!, cont.:
  - What is your plan for detailed implementation, including training?
  - What process/areas are to be improved? Who are the process owners? Who else is affected?
  - What is the data showing about the solution's effectiveness?
  - What is the actual benefit? Was it calculated and approved by finance?
  - Is additional work required to refine the solution, such as collecting additional data, revisiting an earlier step, etc.?
  - Why are you confident that the current solution should be standardized?

# You Are Here—CONTROL





# CONTROL: Standardize and Make Future Plans

- **Goal:**

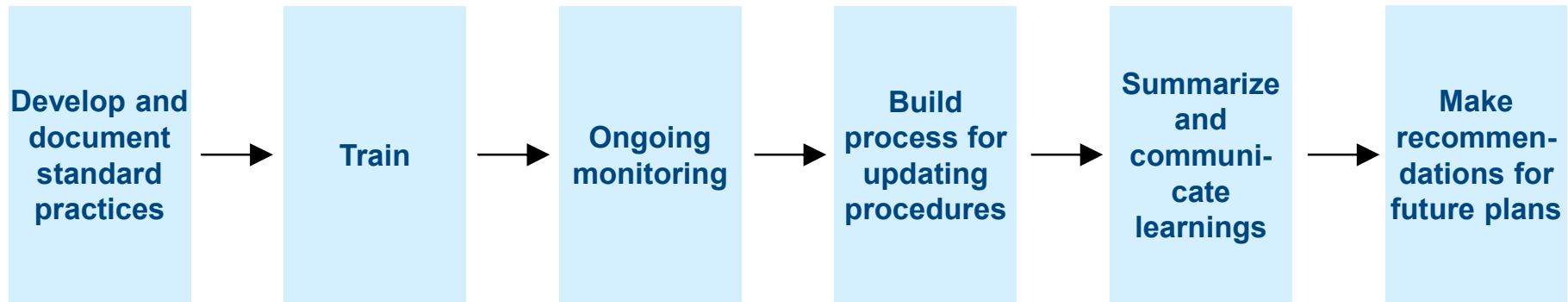
- Maintain the gains by standardizing work methods or processes
- Anticipate future improvements and preserve the lessons from this effort

- **Output:**

- Documentation of the new method
- Training in the new method
- A system for monitoring its consistent use and for checking the results
- Completed documentation and communication of results, learnings, and recommendations



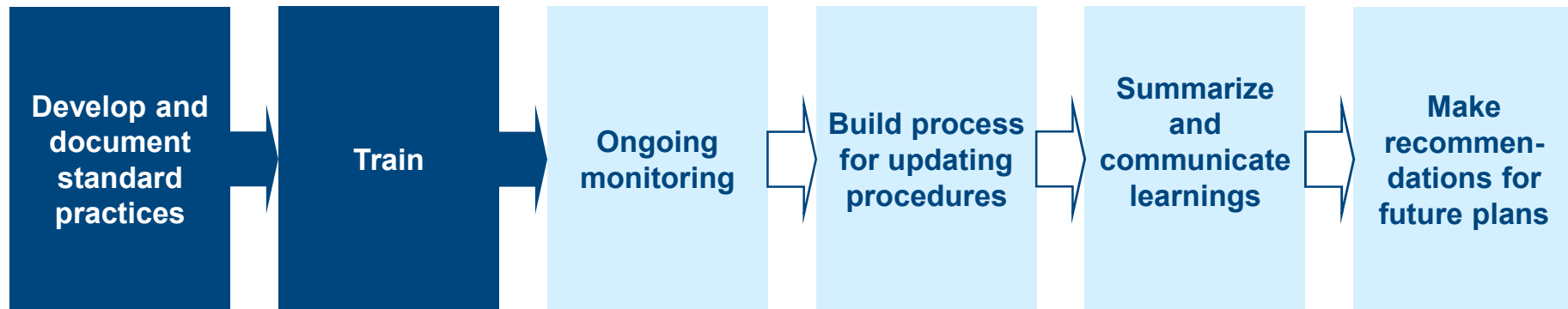
# Approach to CONTROL



# Anexas Consultancy Services

**Standardization:  
Methods and Training**

# You Are Here



# What Is Standardized Work?

- Standardized work is the most efficient and effective combination of people, material, and equipment that is presently possible:
  - Sets the baseline, which allows:
    - Measurement of performance
    - Identification of improvements
    - Assessment of changes

# Anexas Consultancy Services

Training

# Training

- When you have completed the documentation, you need to make sure that everyone using the process is trained in the new methods
- Even experienced employees need to be trained in the new methods

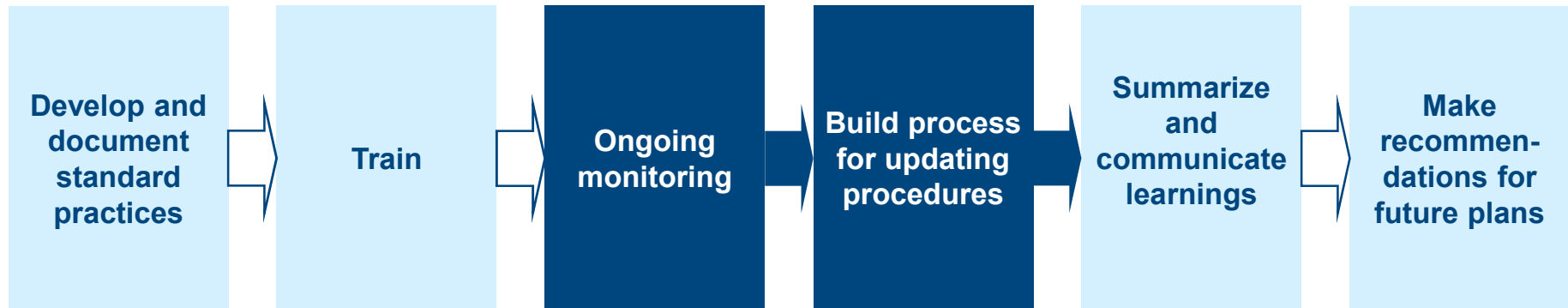


# Anexas Consultancy Services

## Process Monitoring and Control



# You Are Here



# Goals

- Understand what is meant by the term process management
- Create a process management chart using the process management chart format
- Understand what is meant by the term visual controls
- Match the following charts with situations in which each is most appropriate: Xbar, R-chart, pchart, npchart, cchart, and uchart
- Apply control charts to the monitoring plan for your project

# Anexas Consultancy Services

## Methods for Monitoring and Control



# Process Management Chart Features

The plan is typically captured as a process map.

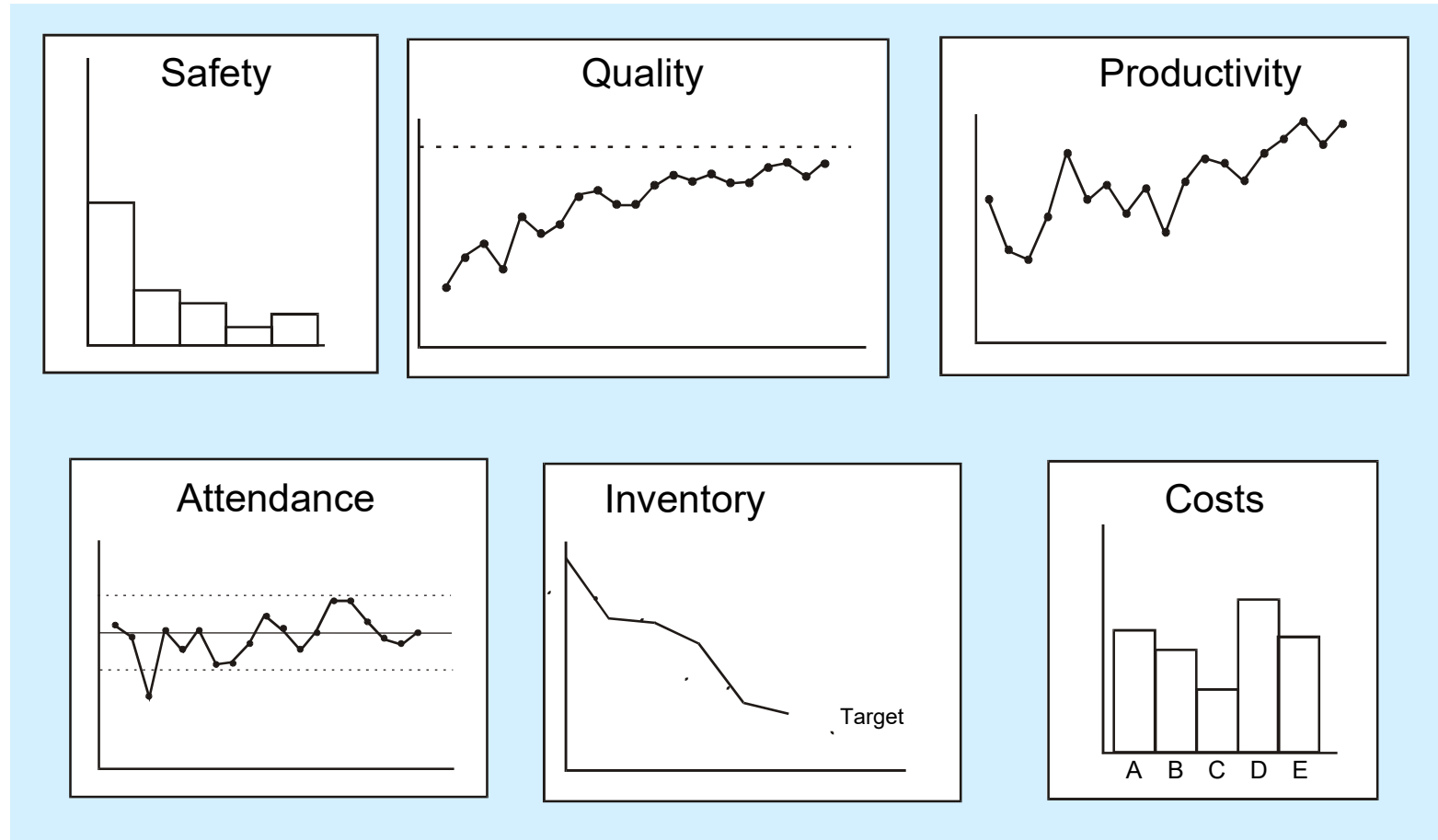
The middle column describes what you will check in the process to monitor its quality.

PLAN/DO	CHECK	ACT
<p><b>Process Map</b></p>	<p><b>Indicators</b></p> <p><i>Plot time on each order; should be ≤ 2 hours; check for special causes</i></p> <p><i>Count errors</i></p>	<p><b>Corrective Actions</b></p> <p><i>Alert Sam immediately; organize investigation</i></p> <p><i>If more than 1 per order, stop process, contact Sam</i></p>

The third column describes how the process operators should react depending on what they find in the measures.

# Visual Controls: Communication Board

## Visual Controls



# Anexas Consultancy Services

Ongoing Data Collection  
and Process Control

# Ongoing Data Collection

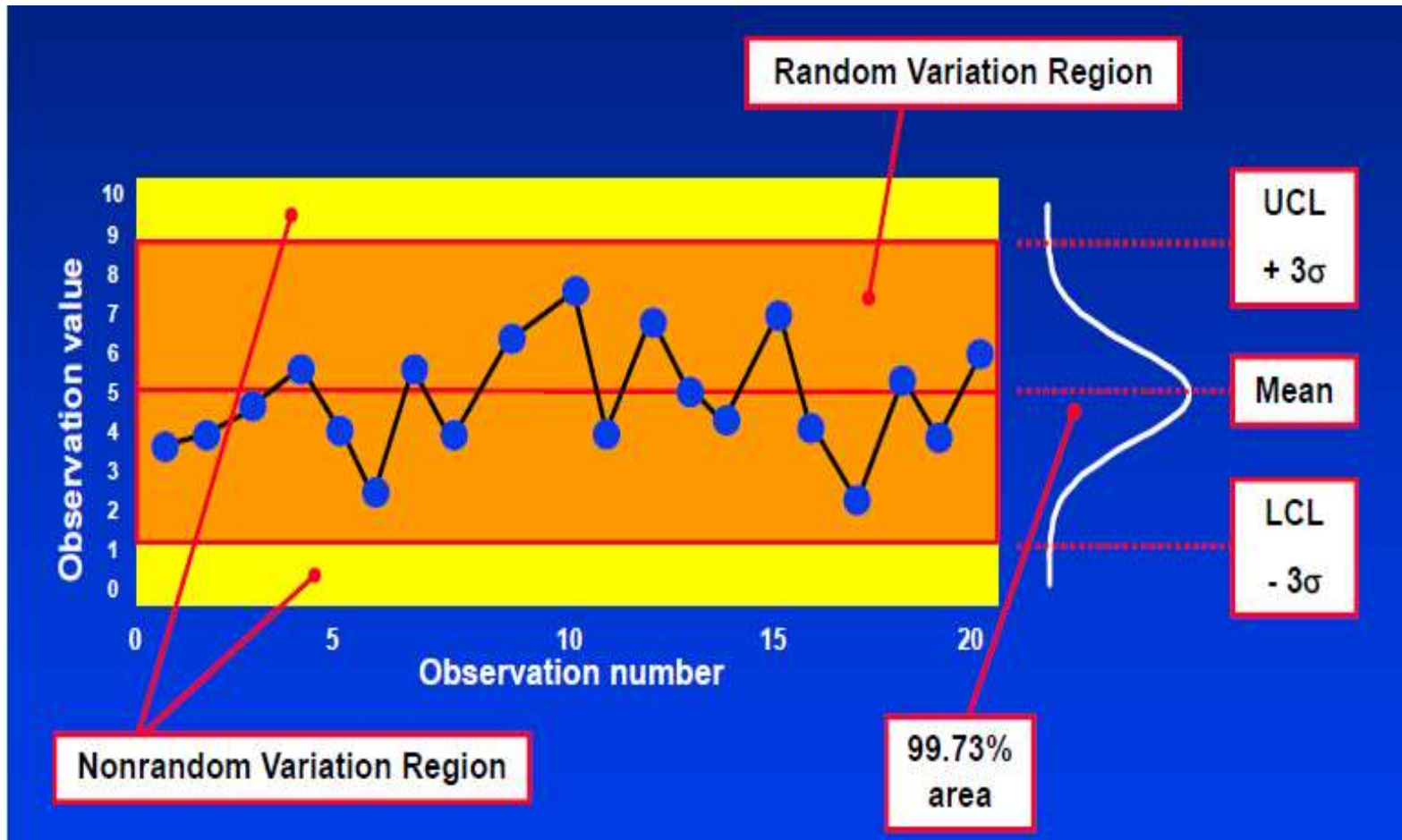
- Monitor your process through ongoing data collection
- Use control charts to plot the data and identify signals of special causes
- Use special cause and common cause thinking to reduce variation



# Anexas Consultancy Services

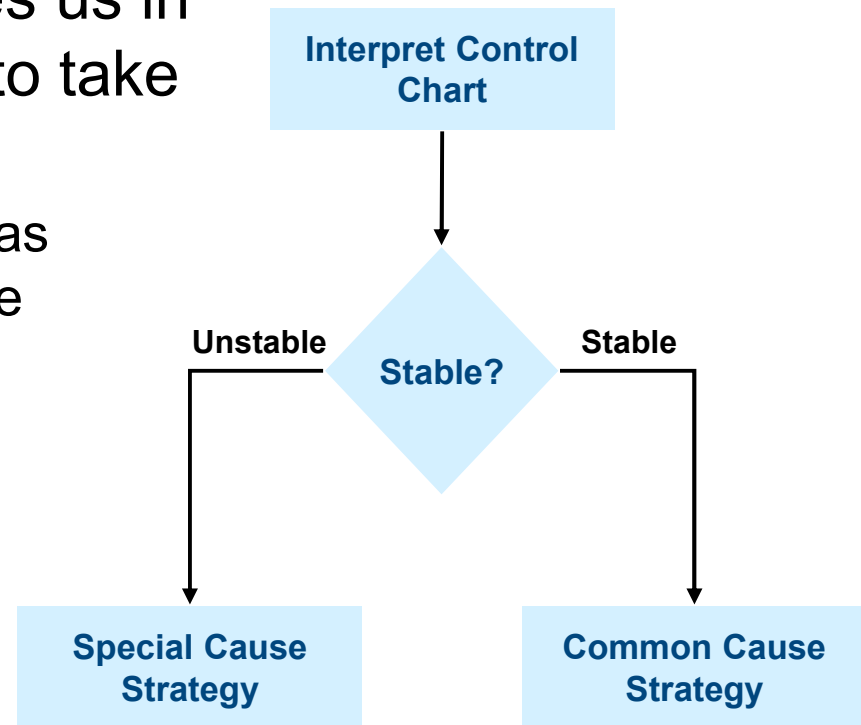
## Control Charts

# Statistics of a Control Chart











# Responding to Control Charts

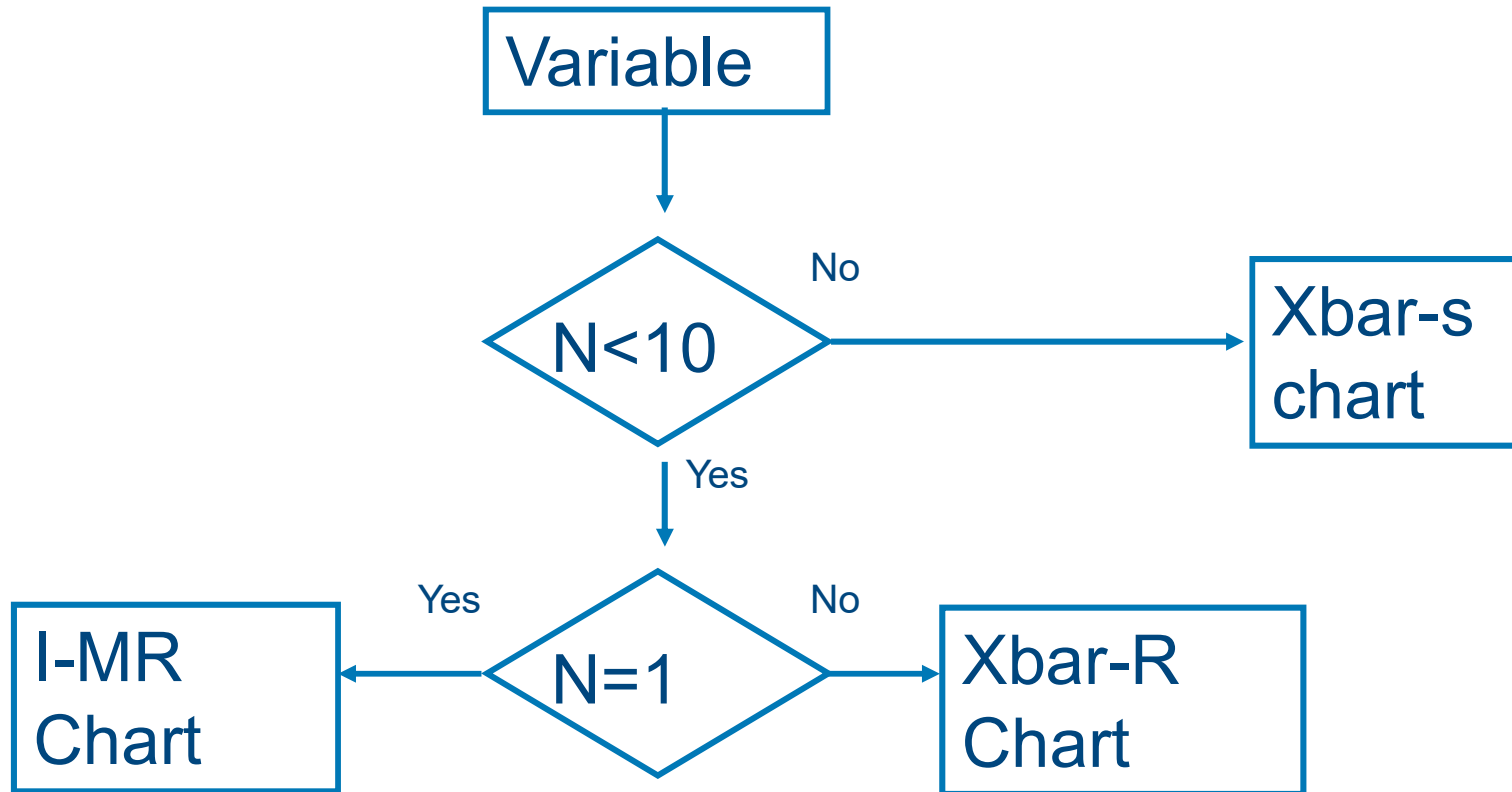
- Appropriate managerial actions are quite different for common causes than for special causes
- Control charts help us determine if we have special or common causes, which guides us in selecting what type of action to take and when:
  - The choice is an important one as inappropriate actions often make things worse; at the very least, they waste time, resources, and money



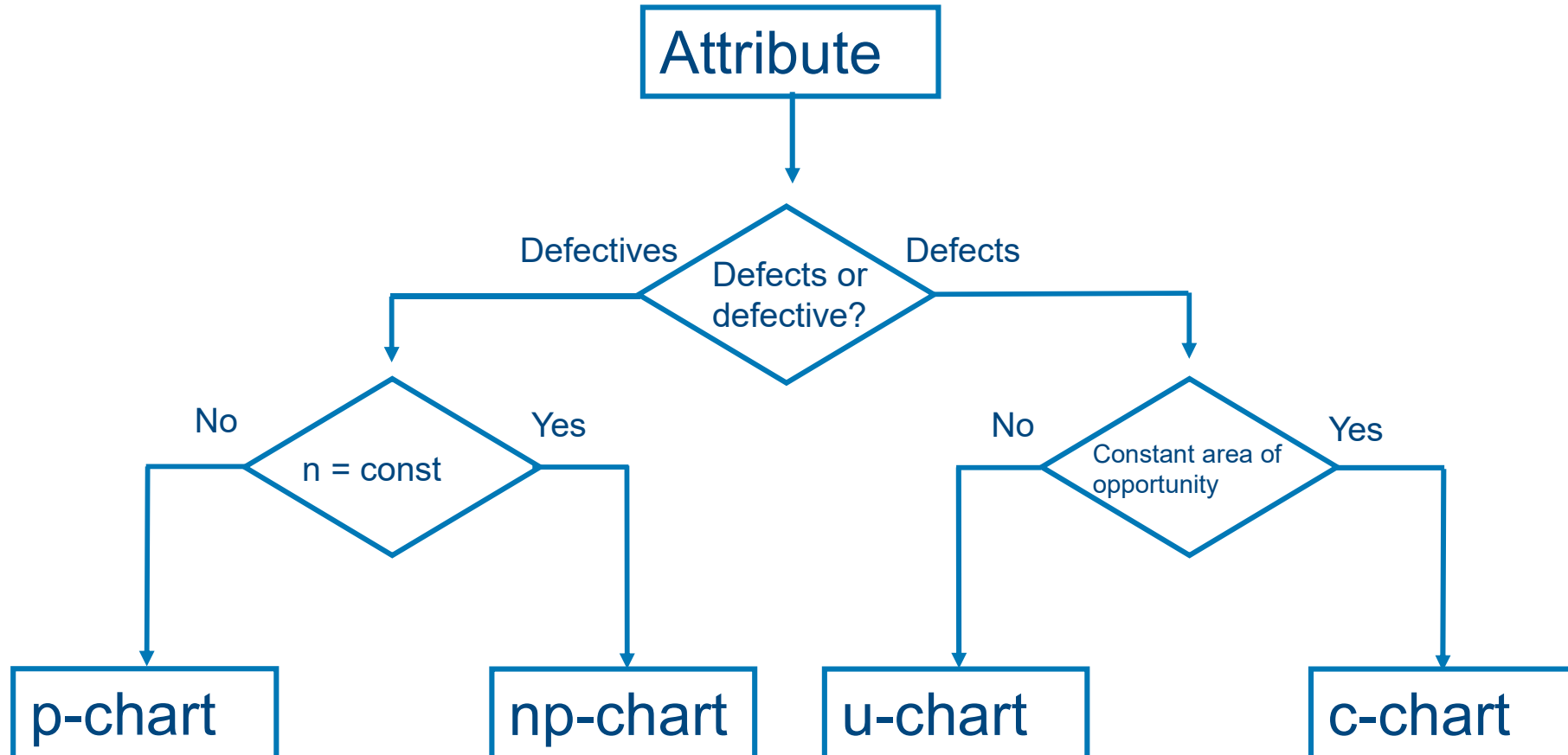
# Consequences of Actions Taken in Response to Control Charts

		Type of Action			
		Special Cause		Common Cause	
		Look for what was different between individual points	Take action based on the reported difference	Study all the data	Make basic changes to the process
Type of Variation	Common Cause	Waste time 	Increase variation 	Gain a better understanding of the system 	Reduce variation 
	Special Cause	Gain useful information 	Reduce variation 	Lose time in responding to the problem 	Loss of productivity, may increase variation 

# Control Chart Roadmap



# Control Chart Roadmap



# Constructing Control Charts for Discrete Data

Chart	Control Limit Calculations
p-chart	$\bar{p} \pm 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$
np-chart	$n\bar{p} \pm 3\sqrt{n\bar{p}(1-\bar{p})}$
c-chart	$\bar{c} \pm 3\sqrt{\bar{c}}$
u-chart	$\bar{u} \pm 3\sqrt{\frac{\bar{u}}{a}}$

- More information on how to calculate the limits by hand is beyond the scope of Green Belt Training
- We will use Minitab to construct the charts and obtain the control limits

# Summary of Assumptions for Control Charts, cont.

- **Binomial distribution:**

- The term “binomial” derives from the fact that any given item either will or will not have the attribute being studied—so there are only two possible outcomes for each item (yes, it has the attribute, or no it does not): Yes/No an item is defective or Yes/No a sale was lost:
  - The binomial distribution describes the probability that a given number of items from the population will possess a given attribute (such as a defect) when the overall proportion of the population with the attribute is described by  $p$ :
    - A binomial distribution may or may not be symmetrical
    - To use a binomial distribution, you need to be able to count both occurrences (Yes) and non occurrences (No)
  - When the binomial assumption holds, the  $p$  chart is preferred
  - If the binomial assumption does not hold, use an individuals chart on percentages



# Summary of Assumptions for Control Charts, cont.

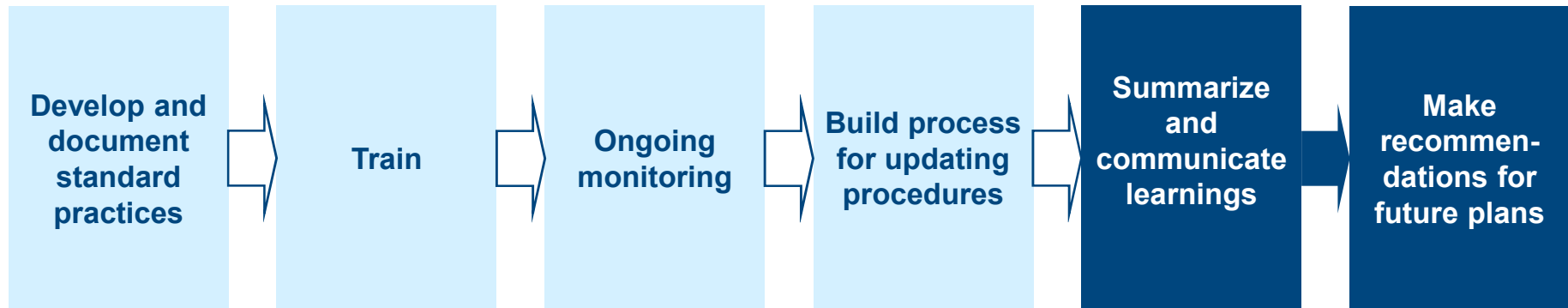
- **Poisson distribution:**

- When you can count occurrences but not non occurrences (such as counting the number of computer outages) and the probability of occurrence is relatively rare, your data are more likely to follow a Poisson distribution
- A good rule of thumb is that an occurrence is rare if it is likely to occur less than 10% of the time
- If the probability of occurrence is high, treat the data like individual data points and create an individuals chart based on the Normal distribution assumptions:
  - When the Poisson assumption holds, the c chart is preferred
  - If the Poisson assumption does not hold, use an individuals chart of counts

# Anexas Consultancy Services

## Communication and Future Plans

# You Are Here



# Goal

- Identify ways to bring a project to closure in your own organization

# Importance of Closure

- Recognize the considerable time and effort that went into the initiative
- Capture the learnings from the initiative:
  - About the problem or process being studied
  - About the improvement process itself
- Hand over responsibilities for standardization and monitoring to the appropriate people



# Project Closure

- Improvement must be continuous, but individual initiatives and project teams come to an end
- Learn when it is time to say good-bye
- Effective project closure weaves together the themes of:
  - Project purpose
  - Improvement methods
  - Team skills and structures
- Develop managerial systems to capture learnings and enable the organization to address system issues
- Documentation and recognition are two critical aspects of project team closure

# Exercise: Closure

**Objective:** Build awareness of how to successfully bring a project to a close

**Instructions:**

- Work in pairs or small groups to answer the following questions:
  - What would indicate that a team had ended well? What would indicate that a team had not formally closed or had not done it well? Explain
  - Think of teams that have completed their work—what methods of closure were used by these teams? How effective were they? How do you know?
- Be prepared to share your answers with the whole group

**Time:** 10 minutes

# Closure Checklist

1. Avoid needless continuation
2. Summarize learnings:
  - About the work process
  - About the team's process
  - About your results
3. Finalize documentation on improvements
4. Summarize future plans and recommendations
5. Communicate the ending
6. Celebrate



**6 steps to close a Project: Project Closure Process**



# 1. Avoiding Needless Continuation



## 2. Summarizing Learnings

- About your results
- About the work process
- About the team's process



## 3. Finalize Documentation on Improvements

- Finish your storyboard:
  - It should contain your final results and conclusions
- Present the completed document to:
  - The sponsors
  - The people whose jobs are changing as a result of the work
  - Customers of the change
  - Other interested people
  - Collect, catalog, and make the documentation available to others

## 4. Summarize Future Plans and Recommendations

- Have your team discuss the following issues and compile recommendations that you submit to your sponsor or guidance team:
  - What are your recommendations for maintaining the gains already put in place? What role would you like your team to play?
  - How much improvement is still needed to achieve the business goals?
  - What portions of the problem are still left to address? Which of these are the most urgent to address?
  - What would you and your team like to work on next, if approved by management? Where do you think management should devote resources next?

## 5. Communicate the Ending

- Communicating the team's results is a joint task for the project team and its sponsor or guidance team:
  - If not done already, identify the people who will be involved in implementing the improved methods
  - Which employees could benefit from your lessons learned?
  - How can you communicate to management? To the rest of the organization?
  - How can the end of this project sow the seeds for future projects?



## 6. Celebration and Recognition

- What is the appropriate way to celebrate this closure (lunch/dinner/dessert)?
- How will you say good-bye?





## 6. Celebration and Recognition, cont.

- Recognition is an important aspect of celebration and should reinforce intrinsic sources of satisfaction and motivation:
  - Examples:
    - A small souvenir related to the project work
    - A pizza lunch for everyone who was involved in the initiative, including those whose jobs have changed as a result of the initiative



# Exercise: Closure in Your Organization

**Objective:** Understand what parts of closure may be most challenging in your organization

**Instructions:**

- Work alone or in small groups
- Review at least one of the closure checklist items and discuss how it would work in your organization
- Be prepared to discuss your results with the whole group

**Time:** 10 minutes



# Anexas Consultancy Services

**CONTROL: Standardize  
and Make Future Plans**

Review

# You Were Here—CONTROL



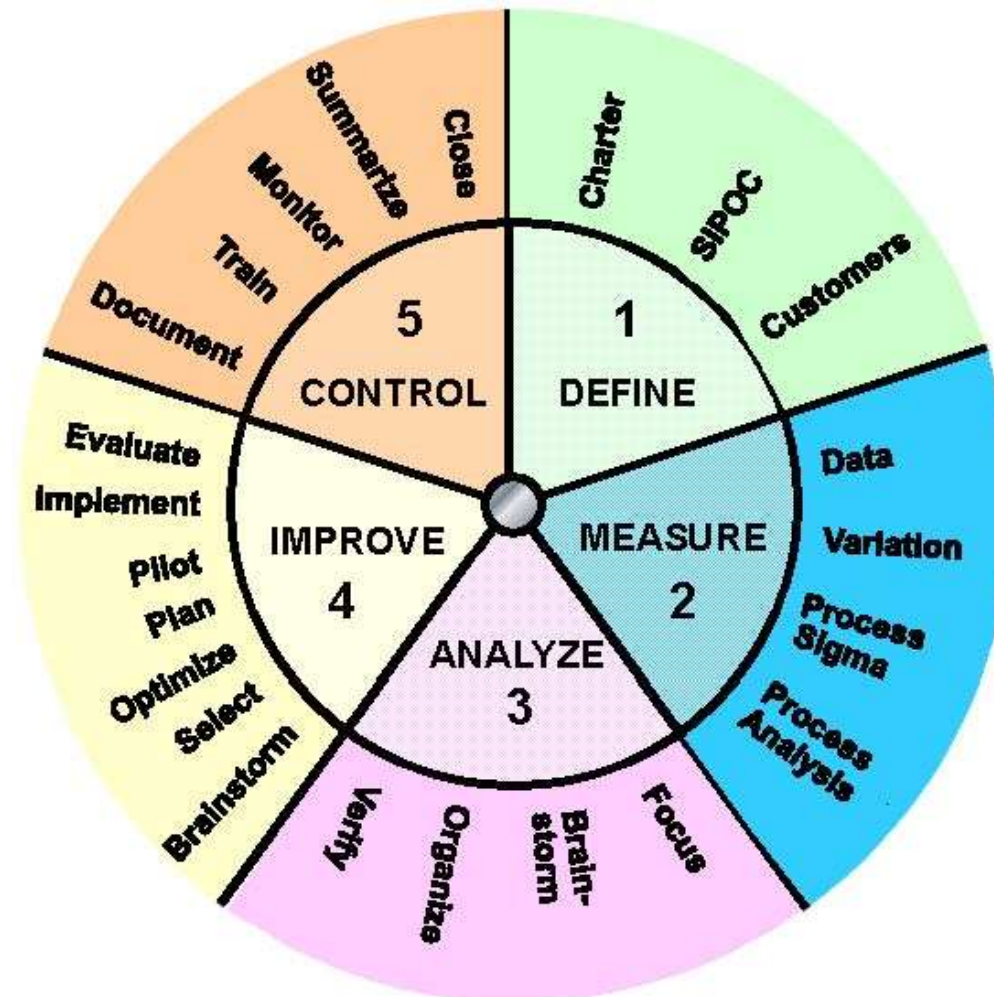
# CONTROL Phase Deliverables

- All **standard documentation**, policies and procedures, role profiles, performance management, training, etc., updated to standardize and integrate improved process
- **Monitoring and controls** identified to sustain improved performance
- Controls documented in control and reaction plan
- **Final project report** completed
- **Post closure audit plan** (if required)
- Storyboard completed
- Project discussed and sign off agreed with sponsor, coach, and process owner
- CONTROL deliverables completed, phase and project closure approved
- Project handed over to process owner

# CONTROL Checklist

- By the end of CONTROL, you should be able to demonstrate to your sponsor that everyone involved in the process is consistently using new methods designed to eliminate/minimize problems identified back in MEASURE:
  - Your sponsor may be interested in seeing:
    - Documentation of the new methods, and examples of how they are used (Standard Practice Worksheet, One-Point Lesson)
    - Examples of self-audits or other checks done to monitor implementation
    - Control charts regularly updated with new data and showing that process performance is within control limits (only common cause variation is present)
    - Method of ongoing control and acceptance of ownership by process owner
    - Recommendations (supported by data, if possible) for next actions involving this process or ideas for a spin-off from this project
    - Completed project documentation (include a final storyboard that shows the results of your data analysis)
    - Plans for (or results from) communicating achievements to the organization
    - Potential replication of solutions found to other similar lines/processes
    - Plans for celebrating your success

# DMAIC REVIEW



## Stay Connected!

- Please subscribe Anexas Europe YouTube channel to get access to all the training material on video.  
<https://m.youtube.com/channel/UCAk6lJsnPCsz7-rZTVx90uw>
- Please join Anexas Alumni Facebook Group for updates about latest developments and career opportunities for Green Belts and Black belts  
<https://www.facebook.com/groups/160848720633713>
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- Please join Anexas Europe Linked In Page for updates about latest developments and career opportunities for Green Belts and Black belts  
<https://www.linkedin.com/school/9379319/>
- Connect with the founder of Anexas Europe – Amitabh Saxena on his LinkedIn account to see his latest videos on quality topics  
<https://www.linkedin.com/in/amitabhsaxena/>

# Thank You!

- Please write to Anexas Team at [enquiry@anexas.net](mailto:enquiry@anexas.net) for any queries
- Anexas website: [www.anexas.net](http://www.anexas.net)



# Thank You!

- Congratulations on completing a milestone in your life!
- Best wishes for your Lean and Six Sigma journey!

- Amitabh Saxena  
CEO  
Anexas Europe